A colleague once complained to me that after his
daughter had successfully completed her last required
mathematics course from a highly regarded university, she reported to him that as a result of that course
she hated mathematics. My colleague was particularly
upset because his daughter had enjoyed mathematics
up to that point, and he was worried that the negative
effects of her last mathematics class might linger long
into her future. For many students studying math-
ematics in school, the beliefs or feelings that they carry
away about the subject are at least as important as the
knowledge they learn of the subject. A 2005 Associ-
ated Press poll (AP–AOL News, 2005) showed that
nearly 40% of the adults surveyed said that they had
hated mathematics in school, and although those
polled also acknowledged hating other subjects, twice
as many people said that they hated mathematics as
said that about any other subject. While students are
learning mathematics, they are also learning lessons
about what mathematics is, what value it has, how it is
learned, who should learn it, and what engagement in
mathematical reasoning entails.

To understand students’ experiences with school
mathematics, one must understand a central factor
in their experience: mathematics teachers. Two de-
cades have passed since Lee Shulman (1986) rejected
George Bernard Shaw’s infamous statement “He who
can, does. He who cannot, teaches” as overly simplis-
tic because it was not reflective of the highly special-
ized knowledge required of teachers. He introduced
the now famous term pedagogical content knowledge to
refer to the complex knowledge that lies at the inter-
section of content and pedagogy and that teachers
must possess to make the curriculum accessible to
their students. Shulman ended his landmark article
by suggesting that a more apropos saying would be
“Those who can, do. Those who understand, teach”
(p. 14). Researchers studying teachers’ knowledge,
beliefs, and affect related to mathematics teaching
and learning are still trying to tease out the relations-
ships among these constructs and to determine how
teachers’ knowledge, beliefs, and affect relate to their
instruction.

The focus of this chapter is a consideration of
what researchers have to say about teachers’ beliefs
and affect. A saying by Heller, “Be careful how you
interpret the world: It is like that,” indicates that the
way one makes sense of his or her world not only de-
defines that person for the world but also defines the
world for that person. Beliefs might be thought of
as lenses through which one looks when interpreting the world, and affect might be thought of as a disposition or tendency one takes toward some aspect of his or her world; as such, the beliefs and affect one holds surely affect the way one interacts with his or her world. Although few researchers have examined the relationship between mathematics teachers’ affect and their instruction, the existing research shows that the feelings teachers experienced as learners carry forward to their adult lives, and these feelings are important factors in the ways teachers interpret their mathematical worlds.

ORGANIZATION OF CHAPTER

I begin this chapter with a brief review of two chapters from the first Handbook of Research on Mathematics Teaching and Learning (Grouws, 1992): one on teachers’ beliefs, written by Alba G. Thompson, and a general chapter on affect, written by Doug McLeod. After summarizing the state of research on mathematics teachers’ beliefs and affect at the time of publication of the first handbook chapter, I review the research conducted on teachers’ beliefs since that time in two sections. In the first section, I consider what beliefs are, how they are measured, what stances are taken on the role of inconsistent beliefs, and how they are changed. In the second section, I review emerging areas of research related to mathematics teachers’ beliefs by looking to research on teachers’ beliefs about students’ mathematical thinking, teachers’ beliefs about curriculum, teachers’ beliefs about technology, and teachers’ beliefs about gender. I then review the research on teachers’ affect. I begin with a brief review of the relationship between affect and achievement and then summarize studies that address teachers’ affect. Because little research specifically addresses the topic of mathematics teachers’ affect and several researchers have proposed frameworks on students’ affect that might be helpful for considering teachers’ affect, I follow the research on teachers’ affect with a review of some of the student-focused frameworks. After providing a review of the research on teachers’ beliefs and affect, I consider researchers who have taken a broader look at the construct of beliefs by considering such constructs as teachers’ orientations, teachers’ perceptions, and teacher identity, and I highlight how studying identity provides one promising path by which to integrate teachers’ beliefs and affect. I end with some final comments.

WORKING DEFINITIONS/DESCRIPTIONS OF TERMS

Many of the terms in this chapter are not used in the literature in a uniform way. However, I recognize that readers can become confused about the relationships among terms, and so I have attempted to distill meanings that capture distinctions that emerge in usage by researchers, and I list them in Figure 7.1. I call these definitions/descriptions because they are based upon a combination of the literature usage and the dictionary definitions. These definitions/descriptions are not intended to stand alone as definitions, but instead I provide them to support the reader in drawing distinctions among the commonly used meanings of terms in this chapter. Each of these terms is discussed in the chapter, but I provide them at the beginning (see Figure 7.1) so that the reader can refer to them as needed.

THE STATE OF RESEARCH ON MATHEMATICS TEACHERS’ BELIEFS AND AFFECT AT THE PUBLICATION OF THE FIRST HANDBOOK OF RESEARCH ON MATHEMATICS TEACHING AND LEARNING

Brief Summary of A. G. Thompson’s 1992 Handbook Chapter on Teachers’ Beliefs and Conceptions

Twice in her review of the literature on teachers’ beliefs and conceptions, A. G. Thompson (1992) noted the importance for researchers studying mathematics teachers’ beliefs to make explicit to themselves and to others the perspectives they hold about teaching, learning, and the nature of mathematics, because these perspectives greatly affect researchers’ approaches to and interpretations of their work. In keeping with her own advice, A. G. Thompson stated her stance that researchers must consider the discipline of mathematics and the relationship between what a teacher thinks about mathematics and how the teacher teaches. She embraced a view of mathematics “as a kind of mental activity, a social construction involving conjectures, proofs, and refutations, whose results are subject to revolutionary change and whose validity, therefore, must be judged in relation to a social and cultural setting” (p. 127). The conception of mathematics teaching associated with this view of mathematics and reflected in several documents A. G. Thompson cited from the 1980s was “one in which
**Affect**—a disposition or tendency or an emotion or feeling attached to an idea or object. Affect is comprised of emotions, attitudes, and beliefs.

**Emotions**—feelings or states of consciousness, distinguished from cognition. Emotions change more rapidly and are felt more intensely than attitudes and beliefs. Emotions may be positive (e.g., the feeling of “aha”) or negative (e.g., the feeling of panic). Emotions are less cognitive than attitudes.

**Attitudes**—manners of acting, feeling, or thinking that show one’s disposition or opinion. Attitudes change more slowly than emotions, but they change more quickly than beliefs. Attitudes, like emotions, may involve positive or negative feelings, and they are felt with less intensity than emotions. Attitudes are more cognitive than emotion but less cognitive than beliefs.

**Beliefs**—Psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one’s view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual. Beliefs are more cognitive than emotions and attitudes. (I do not indent this definition under affect because, although beliefs are considered a component of affect by those studying affect, they are not seen in this way by most who study teachers’ beliefs.)

**Beliefs System**—a metaphor for describing the manner in which one’s beliefs are organized in a cluster, generally around a particular idea or object. Beliefs systems are associated with three aspects: (a) Beliefs within a beliefs system may be primary or derivative; (b) beliefs within a beliefs system may be central or peripheral; (c) beliefs are never held in isolation and might be thought of as existing in clusters.

**Conception**—a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences.

**Identity**—the embodiment of an individual’s knowledge, beliefs, values, commitments, intentions, and affect as they relate to one’s participation within a particular community of practice; the ways one has learned to think, act, and interact.

**Knowledge**—beliefs held with certainty or justified true belief. What is knowledge for one person may be belief for another, depending upon whether one holds the conception as beyond question.

**Value**—the worth of something. A belief one holds deeply, even to the point of cherishing, and acts upon. Whereas beliefs are associated with a true/false dichotomy, values are associated with a desirable/undesirable dichotomy. Values are less context-specific than beliefs.

**Figure 7.1** Working definitions/descriptions of terms (listed alphabetically, except that emotion, attitude, and belief are listed from least to most cognitive component of affect. I accept sole responsibility for these definitions while crediting many).

students engage in purposeful activities that grow out of problem situations, requiring reasoning and creative thinking, gathering and applying information, discovering, inventing, and communicating ideas, and testing those ideas through critical reflection and argumentation” (p. 128). The role of teachers’ mathematical conceptions was a recurring theme underlying her 1992 review chapter, no surprise to readers familiar with her research (e.g., A. G. Thompson, 1984; A. G. Thompson, Philipp, Thompson, & Boyd, 1994; P. W. Thompson & Thompson, 1994).

The term belief is so popular in the education literature today that many who write about beliefs do so without defining the term. A. G. Thompson (1992) found, “For the most part, researchers have assumed that readers know what beliefs are” (p. 129). Further, many educators contend that distinguishing between knowledge and belief is unimportant for research, but investigating how, if at all, teachers’ beliefs and knowledge affected their experience is important. Although Thompson used both beliefs and conceptions in the title of her chapter, in most of her chapter, she seemed to think of beliefs as a subset of conceptions, and her definition of conceptions included beliefs. And yet, at times she seemed to use the terms interchangeably. She referred to teachers’ conceptions “as a more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like” (p. 130). When using the term conceptions, Thompson, recognizing the important relationship between knowledge and beliefs, seemed less interested in drawing distinctions between these terms, and she stated, “To look at research on mathematics teachers’ beliefs and conceptions in isolation from research on mathematics teachers’ knowledge will necessarily result in an incomplete picture” (p. 131). However, early in her chapter she recognized two distinctions between beliefs and knowledge, conviction and consensuality; these distinctions are generally referred to in the field.

First, beliefs can be held with varying degrees of conviction, whereas knowledge is generally not thought of in this way. For example, whereas one might say that he or she believed something strongly, one is
less likely to speak of knowing a fact strongly. Second, beliefs are not consensual, whereas knowledge is. One is generally aware that others may believe differently and that their stances cannot be disproved, whereas with respect to knowledge, one finds “general agreement about procedures for evaluating and judging its validity” (A. G. Thompson, 1992, p. 130).

A. G. Thompson defined a belief system as “a metaphor for examining and describing how an individual’s beliefs are organized” (1992, p. 130). Drawing upon the work of Green (1971) and Rokeach (1960, cited in A. G. Thompson, 1992), she highlighted three aspects of beliefs systems identified by Green, all related to the notion that because a belief is never held in total isolation from other beliefs, considering how beliefs are held in relation to one another is useful. First, some beliefs serve as the foundation for other beliefs in a quasi-logical structure, meaning that some beliefs might be thought of as primary beliefs whereas others serve as derivative beliefs. Thompson’s example of a primary belief was a teacher’s belief that clearly presenting mathematics to students is important, and an associated derivative belief that might follow is that teachers should be able to readily answer any questions asked by students. A second dimension of beliefs systems is that beliefs can be either central, which means strongly held, or peripheral, which means less strongly held and more susceptible to change. Green contended that primary beliefs might not necessarily be more central than the associated derivative beliefs. In the example above, the derivative belief that a teacher must be prepared to answer questions may be more central (strongly held) to the teacher than the primary belief that a lesson must be clear. A teacher might even, perhaps as a result of a professional development opportunity, change her primary belief about presenting clear, sequential lessons without affecting the derivative, more central, belief about being able to answer any questions posed. Green’s third dimension of beliefs systems is that beliefs are held in clusters that are more or less in isolation from other clusters. One outcome of holding beliefs in this manner is that people can avoid confrontations between belief structures. Another outcome is that these beliefs systems may appear contradictory or inconsistent from the point of view of an observer.

The major portion of her chapter was a review of the literature about topics of study that, at that time, A. G. Thompson (1992) considered relatively new: teachers’ conceptions of mathematics, their conceptions of mathematics teaching and learning, and the relationship between these conceptions and instructional practice. She ended the section by discussing changing teachers’ conceptions. I briefly summarize these topics below.

She first defined a teacher’s conception of the nature of mathematics as “that teacher’s conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics” (A. G. Thompson, 1992, p. 132). Her examples of ways that scholars have highlighted teachers’ conceptions included Ernest’s (1988, cited in A. G. Thompson, 1992) three conceptions of mathematics as (a) a dynamic, problem-driven discipline, (b) a static, unified body of knowledge, or (c) a bag of tools. She described Lerman’s (1983, cited in A. G. Thompson, 1992) absolutist view of mathematics as universal, absolute, certain, value-free mathematical knowledge and the fallibilist view of mathematics as developing through conjectures, proofs, and refutations. She described Perry’s (1970, cited in A. G. Thompson, 1992) scheme of intellectual and moral development and Copes’s (1979, cited in A. G. Thompson, 1992) attempt to adapt this general scheme to mathematical knowledge. She also described Skemp’s (1978) ideas of instrumental and relational understanding. Her look at teachers’ conceptions of mathematics teaching and learning focused upon models of mathematics teaching, and she drew upon the work of Kuhs and Ball (1986, cited in A. G. Thompson, 1992) to highlight four views: learner-focused, content-focused with an emphasis on conceptual understanding, content-focused with an emphasis on performance, and classroom-focused. In considering the relationship between teachers’ conceptions and their instructional practices, Thompson noted that although some researchers found a strong relationship whereas others found more variability and inconsistency, in general, a higher degree of consistency was reported by researchers studying the relationship between teachers’ conceptions of mathematics and their instructional practices than by researchers studying teachers’ conceptions about teaching and learning and their instructional practices. She cautioned that inconsistencies between professed beliefs and instructional practice raise a methodological concern related to how beliefs or conceptions are measured, and she suggested that researchers must go beyond teachers’ professed beliefs and at least examine teachers’ verbal data along with observational data of their instructional practice or mathematical behavior” (A. G. Thompson, 1992, p. 135).

In the last major section of her chapter, on changing teachers’ conceptions, A. G. Thompson (1992) reported studies of preservice teachers who showed little change, and she noted that teachers often assimilate new ideas to fit their existing schemata
instead of accommodating their existing schemata to internalize new ideas. She suggested that studies providing in-depth, detailed analyses would be required to better explicate teachers’ difficulties in accommodating new ideas into their existing schemata. She described a promising paradigm and indicated that teachers’ beliefs and practices underwent large changes when teachers learned about children’s mathematical thinking (Carpenter, Fennema, Peterson, Chiang, & Loej, 1989).

A. G. Thompson (1992) ended her chapter with a few suggestions. First, she was concerned that too many researchers thought of beliefs systems as static entities to be uncovered whereas she viewed beliefs systems as dynamic mental structures that were susceptible to change in light of experience. She challenged researchers to look less at how beliefs affected practice and more at the dialectic between beliefs and practice. She reiterated the position put forth by some researchers that distinguishing between teachers’ knowledge and teachers’ beliefs is not useful and that, instead, researchers should investigate teachers’ conceptions encompassing both beliefs and any relevant knowledge—including meanings, concepts, propositions, rules, or mental images—that bears on the experience. She also noted that virtually no researchers had investigated the extent to which teachers’ and students’ conceptions interact during instruction.

**Brief Summary of McLeod’s 1992 Handbook Chapter on Affect**

McLeod defined the *affective domain* as referring “to a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition” (McLeod, 1992, p. 576), and he identified several specific terms that make up the affective domain. *Emotions, attitudes, and beliefs* were the three terms to which he devoted most attention. Emotions change more rapidly than attitudes and beliefs; they are also less cognitive and are felt more intensely than attitudes and beliefs. An example of a negative emotion is the feeling of panic experienced by some students when doing, or even thinking about doing, mathematics, whereas an example of a positive emotion is the satisfactory feeling of “Aha!” experienced by students who have an insight during mathematical problem solving. McLeod noted that emotional reactions had not been major factors in research on affect in mathematics education, and he conjectured that the reason for this lack of attention was that emotions, being prone to changing quickly, lack the stability necessary for researchers to think that they can reliably gather questionnaire data about them.

*Attitudes* refer to “affective responses that involve positive or negative feelings of moderate intensity” (McLeod, 1992, p. 581). Attitudes are more cognitive in nature and more stable than emotions; they are felt less intensely. Examples of attitudes students might experience toward, for example, geometry are liking, disliking, being curious about, or being bored by the subject. One connection between emotions and attitudes is that repeated emotional reaction to an experience related to mathematics can result in automatizing that emotion into an attitude toward that experience. McLeod provided the example of a student who has repeated negative experiences with geometry proofs. The initial emotional response the student experiences will, with time, become more automatic and less physiologically arousing and will eventually lead to the student’s forming a more stable response, an attitude, that can be measured by use of a questionnaire. For example, a student who is struggling in a geometry class may feel discomfort, or even illness, when faced with devising geometry proofs, and with time these feelings may result in the more general attitude that the student dislikes geometry, and, eventually, mathematics.

Beliefs are more cognitive in nature than attitudes (and, hence, also than emotions), are generally stable, and are experienced with a lower level of intensity than emotions or attitudes. McLeod concluded that beliefs tend to develop gradually and that cultural factors play a key role in their development. Four categories of beliefs addressed in McLeod’s chapter are beliefs about mathematics, beliefs about self, beliefs about mathematics teaching, and beliefs about the social context. These discussions were almost exclusively about students’ beliefs, not teachers’ beliefs.

Although his major focus on affect was on emotions, attitudes, and beliefs, McLeod addressed several other affect-related concepts, such as confidence, self-concept, mathematics anxiety, and learned helplessness. He also addressed concepts more closely related to cognition, such as autonomy, intuition, metacognition, and social context.

A major theme identified by McLeod in his 1992 chapter was that “all research in mathematics education can be strengthened if researchers will integrate affective issues into studies of cognition and instruction” (p. 575), and he drew heavily from Mandler’s theory that affective factors arise out of emotional responses to interrupted plans. According to Mandler, one approaches a task with a schema for how the task will be completed, and if the anticipated sequence of actions cannot be completed, the result is a physiological response. For Mandler, these affective factors were connected to one’s knowledge and beliefs, because
interpretations of interruptions vary from individual to individual. McLeod offered the example of a sixth-grade student solving a story problem. If the student believed that all mathematics problems should be solvable in a couple minutes but the student was unable to solve the problem in that period of time, the student might experience an arousal that he or she would interpret as negative. If these experiences were repeated frequently, the student might develop a negative attitude toward story problems, and, in many cases, such negative attitudes toward one aspect of mathematics generalize to negative attitudes toward mathematics in general or, perhaps even worse, toward the student’s view of himself or herself as a mathematical learner. If, however, students believe that story problems can challenge even good problem solvers and require a longer period of time to solve, then arousal at an inability to quickly solve the problem might not be interpreted as negative. The student’s interpretation of the experience, not the experience itself, determines the outcome.

In the upcoming section on affect, I relate the research since the publication of McLeod’s chapter to two conclusions he drew. First, little research in mathematics education integrated cognitive and affective factors, and McLeod called for more integration, which, unfortunately, is not generally reflected in the recent research. At the end of the chapter, however, I describe some promising directions for such research.

Second, although McLeod addressed affect generally, he noted that teacher affect in mathematics education had been studied little, a trend that seems to have continued. However, the emotional responses felt by students learning mathematics and the associated attitudes that develop are believed to linger into adulthood and may have important implications for teachers. I look to some research on comparison of children’s and adults’ affective responses, for example, research showing that mathematics test anxiety can equally affect children and prospective teachers, and I consider implications.

Summary of the State of Research on Beliefs and Affect at the Publication of the First Handbook

A. G. Thompson (1992) found that most research on teachers’ beliefs and conceptions was interpretive in nature, employed qualitative methods of analysis, and was comprised of in-depth case studies with small numbers of subjects. She suggested that more such in-depth studies were needed to explain why teachers did not accommodate new conceptions into their existing schemata. Methodological approaches used at the time for these in-depth studies included “interviews, classroom observations, stimulated-recall interviews, linguistic analysis of teacher talk, paragraph-completion tests, responses to simulation materials such as vignettes describing hypothetical students in classroom situations, and concept generation and mapping exercises such as the Kelly Repertory Grid Technique” (p. 131). Those who conducted experimental or pseudoexperimental studies of beliefs used Likert-type instruments, and they tended to measure beliefs isolated from knowledge.

McLeod, (1992) in his chapter on affect, addressed beliefs as a cognitive component of affect and called for affect to be included in the study of cognition and, therefore, of beliefs but found that little research had been conducted on teachers’ affect. A. G. Thompson (1992), in her chapter on teacher beliefs, addressed the importance of considering beliefs together with knowledge and referred to this construct as teachers’ conceptions. Thompson did not address affect. This lack of attention to affect was common throughout the first handbook. With one exception, all index references to affect from the first National Council of Teachers of Mathematics (NCTM) Handbook of Research on Mathematics Teaching and Learning (Grouws, 1992) were to the McLeod chapter on affect in mathematics education. The one exception was Schoenfeld’s (1992) mention, in his chapter on learning to think mathematically, that space constraints precluded his addressing the topic of affect and that readers should refer to the work by McLeod for “authoritative starting points” (p. 358). Absent from both chapters was the topic of technology, either as a tool to support the study of teachers’ beliefs and affect or as an object of study about teacher’s beliefs or affect with respect to technology.

Setting the Context for Mathematics Teacher Education Research Since 1992

Much has changed since 1992, when the first handbook was published. I note five occurrences that, at least indirectly, have affected research in mathematics education in the United States and, to some extent, abroad. These five occurrences are the general acceptance and infusion of ideas from the NCTM Standards’ documents (NCTM, 1989, 1991, 1995) into the educational arena; the increased number of outlets for publishing research in mathematics education; the increased politicization of United States education in general and educational research in particular; technological advances affecting the manner in which we obtain and report research and capture,
edit, and post video for use when assessing teachers’ beliefs or affect; and the emergence of sociocultural and participatory theories of learning.

**Occurrence 1: The Acceptance and Infusion of Ideas From the NCTM Standards’ Documents Into the Educational Arena**

Between 1989 and 1995 the National Council of Teachers of Mathematics published three important documents: *Curriculum and Evaluation Standards for School Mathematics* (1989), *Professional Standards for Teaching Mathematics* (1991), and *Assessment Standards for School Mathematics* (1995). (Although NCTM subsequently published the *Principles and Standards for School Mathematics* [2000], this document appeared too late to influence much of the research reviewed for this chapter.) The *Curriculum and Evaluation Standards* document, which emphasized mathematics as problem solving, communication, reasoning, and making connections, particularly affected curriculum development in the United States and, as a result, also affected research that was conducted (Hiebert, 1999). Although this document had been published before the first *Handbook* (Grouws, 1992), it had had no substantial influence on either practice or research by that time. However, in the next decade, that situation was to change, with publication of hundreds of research studies about mathematics education reform (Ross, McDougall, & Hogaboam-Gray, 2002). One might conclude from the abundance of studies on reform that schools were engaged in important and fundamental change. However, a peek into randomly selected American classrooms has led to the conclusion that the reform movement in the United States has not led to widespread change in mathematics instruction (Hiebert et al., 2005; Stigler & Hiebert, 1999). Whereas reform documents have had an effect on mathematics education research, they have had a much smaller effect on what takes place in American schools. When Lerman (2002) wrote that “a reform along the lines of that in the USA could not take place in the UK because of the current dominance of the official pedagogical field” (p. 237), he could, to some extent, also have been writing about the reform’s effect in U.S. schools (Becker & Jacob, 2000; Gregg, 1995; Smith, 1996). One reason that schools change so little is that teaching is cultural (Stevenson & Stigler, 1992), and major commitments are required to enact meaningful school-based change. I am reminded of the response of a mathematics curriculum specialist in the mid 1990s to my question about whether the reform movement had affected the teaching of mathematics in his district: He had ensured that every school received a copy of the *Standards* (NCTM, 1989). But even if major reform initiatives designed to implement innovative curriculum were funded, meaningful and lasting changes in schools would not occur without sustained professional development designed to change teachers’ beliefs. In their review of research studies published about mathematics education reform, Ross, McDougall, and Hogaboam-Gray (2002) concluded that the main obstacle to implementation was teachers’ beliefs about mathematics teaching. During the past 10 years, many other researchers have studied teachers’ beliefs as they relate to reform (e.g., Lloyd & Wilson, 1998; Steele, 2001; Sztajn, 2003), and some have developed instruments to measure teachers’ beliefs as they relate to reform (e.g., Ross, McDougall, Hogaboam-Gray, & LeSage, 2003). Although others studying teachers’ beliefs may not explicitly mention reform, their focus reflects beliefs that are in the spirit of the reform.

**Occurrence 2: The Increased Number of Publishing Outlets**

A second notable occurrence to affect research in mathematics education over the past decade has been the increased number of outlets for publishing research in mathematics education in general and research on mathematics teachers in particular. Journals that have begun publishing since publication of the 1992 *Handbook* (Grouws) include *Hiroshima Journal of Mathematics Education*; *International Journal for Mathematics Teaching and Learning*; *The International Journal for Technology in Mathematics Education*; *Canadian Journal of Science, Mathematics, & Technology Education*; *International Journal of Science & Math Education*; *Journal of Mathematics Teacher Education*; *Mathematics Education Review*; *Nordic Studies in Mathematics Education*; and *Zentralblatt Für Didaktik Der Mathematik (International Reviews on Mathematical Education)*. To indicate the effect such journals have had on research on teachers’ beliefs and affect, I consider more closely the *Journal of Mathematics Teacher Education*, first published in 1998 by Kluwer Academic Publishers. This journal is devoted to publication of research, critical analyses, and critiques related to mathematics teachers’ learning and development at all stages of their professional development. The journal provided a new outlet for research on teachers’ affect and beliefs, and many studies on these topics have been published in this journal. In his final editorial as the first editor for this journal, Cooney (2001), after reviewing the first four volumes of the journal, wondered whether the articles reflected a field that is generating subsequent research questions but failing to provide the necessary foundation for substantial progress. In acknowledging that mathematics teacher education is
inherently a field of practice, Cooney was concerned that we teacher educators not allow the practical orientation of our work to preclude the development of powerful and useful new constructs:

The problems we study may be inherently practical, but surely their solutions (in the sense of greater wisdom) lie in our ability to see teaching and teacher education outside the confines of the acts themselves. This makes life difficult. It is relatively easy for us to see teaching from a more abstract level if we are not part of the act. In teacher education, however, teacher educators are indeed part of the act. So how is it that we can educate ourselves to engage in Dewey’s notion of reflective thinking or to follow von Glasersfeld’s notion of reflection which requires us to step out of ourselves and see our actions from a vantage point beyond our actions themselves? It is only from this vantage point that we can begin developing schemes and constructs that capture the webs in which we are all entangled. Simply put, we cannot engage in an analysis of our own activity using only the language of that activity itself. (pp. 256–257)

Cooney emphasized the need to engage in theoretical case-study research that goes beyond reporting anecdotes to develop constructs that might be applied to help make sense of teaching and learning environments.

Occurrence 3: The Increased Politicisation of United States Education and Educational Research

Questions about the quality of research in mathematics education have not been isolated to concerns about mathematics-teacher education. Even the broader field of educational research has come into question. Some contend that educational research tends to lack influence or usefulness, partially because it does not lead to practical advances that can be readily applied by practitioners (Burkhardt & Schoenfeld, 2003). By comparing education to other applied fields, Burkhardt and Schoenfeld suggested that the research-based development of tools and processes used by practitioners found in, say, the medical field are lacking in education. Mathematics educators who raise questions about research in our field often take as given that qualitative research is the appropriate means by which to conduct research, and their questions are about the methodological, analytical, or interpretive issues related to qualitative research (Simon, 2004). However, with the political agendas of granting agencies often driving the research that is conducted (Silver & Kilpatrick, 1994), a call for “scientifically based research” has affected funding agencies (Eisenhart & Towne, 2003). Sadly, during the 1990s, many states in the United States experienced a “Math War” (Becker & Jacob, 2000) that pitted mathematics educators calling for innovation and reform against those who believed that the best way to proceed was through a “back to basics” approach. Although published research has generally not yet reflected this recent politicization of educational research, it is a third notable occurrence since 1992 to influence subsequent research on teachers’ beliefs and affect.

Occurrence 4: Technological Advances That Support Obtaining and Reporting Research, and Capturing, Editing, and Posting Video for Use When Assessing Teachers’ Beliefs and Affect

Many researchers conducting literature reviews today begin the task with their computers. Many articles are available through online journals or online libraries. Furthermore, authors routinely post presentations and papers on their websites or send pdf versions of papers upon request. These changes speed the process of conducting research but do not fundamentally change it. However, other technological advances have changed the process of assessing teachers’ beliefs and affect. For example, one group of researchers developed a web-based survey with embedded video requiring open-ended responses to be used when assessing elementary school teachers’ beliefs (Integrating Mathematics and Pedagogy, 2003), together with a user’s manual also available on-line (Integrating Mathematics and Pedagogy, 2004). Those completing the survey respond online, and their data are downloaded to a spreadsheet. Use of this survey enabled the researchers to conduct a large-scale experimental study (Philipp et al., 2005) that would have been impossible 20 years ago.

Occurrence 5: The Emergence of Sociocultural and Participatory Theories of Learning

A fifth development to affect the research on teachers’ beliefs and affect has been the increasing adoption by researchers of emerging theories of learning. By 1992, when the first handbook was published, theoreticians (e.g., Brown, Collins, & Duguid, 1989; Lave, 1988) had already raised questions about psychological theories for explaining the complexities associated with learning, and during the 1990s, sociocultural (Cobb, 1995) and participatory (Wenger, 1998) frameworks were increasingly adopted by mathematics education researchers. The researchers investigating teachers’ beliefs and affect since 1992 have increasingly adopted these theoretical frameworks.
RESEARCH ON TEACHERS’ BELIEFS

CONDUCTED SINCE 1992

I address, in the first part of the section on mathematics teachers’ beliefs, four important areas: defining teachers’ beliefs, measuring them, considering inconsistencies in beliefs, and changing beliefs. In the second part of the beliefs section, I review emerging areas of research related to mathematics teachers’ beliefs and how their beliefs are changed by looking to beliefs about students’ mathematical thinking, beliefs about curriculum, beliefs about technology, and beliefs about gender. Although all these areas were studied before publication of the first handbook chapter, they have received substantially more attention from researchers since that time.

What Are Beliefs?

The construct belief is of great interest to those attempting to understand mathematics teaching and learning. An ERIC-database search for 1990s’ articles based on the terms teacher and belief located 3,105 documents, and although the numbers of documents for the 1980s (1,382) and the 1970s (745) were smaller, clearly the subject of teacher beliefs has been popular.1 When the term mathematics was added to the terms teacher and belief, the 23, 123, and 407 items located for the 1970s, 1980s, and 1990s, respectively, though far fewer than the number of generic teacher-beliefs documents, still reflect an area of growing importance in our field. In all these studies, the term belief was used. However, as popular as the term belief is in education research, it is often undefined by those who study it (Pajares, 1992; A. G. Thompson, 1992). Since publication of A. G. Thompson’s 1992 Handbook chapter, the amount of research on mathematics teachers’ beliefs has increased, and the research has included efforts to define belief. However, in general, no clear agreement about the definition has been reached. Two constructs often closely related to beliefs are values and knowledge.

Belief Versus Values

If you must tell me your opinions, tell me what you believe in. I have plenty of doubts of my own.

—Johann Wolfgang von Goethe

In the 1950s, Edward R. Murrow hosted a radio show titled “This I Believe,” for which Americans wrote about the “rules they live by, the things they have found to be the basic values in their lives” (Murrow, 1951/2005). National Public Radio recently revived this show with modern-day Americans from all walks of life reading short essays about the core beliefs guiding their daily lives. Topics included belief in the power of love to transform and to heal; belief in one’s personal responsibility to positively affect society; belief in empathy; and belief that the presence of a person who often feels perplexed, torn by issues, and unsure of his or her beliefs can be an asset, especially “in periods of crisis, when passions are high and certainty runs rabid” (Gup, 2005). These statements of belief are examples of creeds people believe in. Murrow (1951/2005) referred to these as “basic values,” and I take that stance: A belief that is about beliefs, but a belief in is about values.

In a review of the literature on values in mathematics teaching, Bishop, Seah, and Chin (2003) addressed the differences and the similarities found in the ways researchers think about beliefs and values. One difference they identified is that beliefs tend to be associated with a true/false dichotomy whereas values are often associated with a desirable/undesirable dichotomy. They contended that beliefs are, therefore, more context-dependent than values, because whereas a true/false judgment must be made in reference to some object, desirable/undesirable dichotomies are associated with more general, less context-dependent, attributes. They provided the example of the belief that mathematics is fun, a true/false judgment made about a particular subject. This judgment must be made in context, because holding this belief does not imply that one finds all subjects, or even all mathematics, fun. One who views fun as a personally desirable quality in a more universal way is likely to seek fun throughout his or her life. For a person who values fun, the belief that mathematics is fun whereas literature is not fun would be more pertinent than for a person who values something else, say usefulness, more than he values fun. To summarize this distinction between beliefs and values, beliefs are true/false statements about constructs whereas the choice of the particular constructs one finds desirable or undesirable represents one’s more context-independent values. These more context-independent values are often viewed as more internalized than beliefs and, hence, harder to change.

1 This search was conducted in 2001, before the United States Department of Education closed the ERIC database in an effort to restructure the search engine to include only documents that met a particular standard of “evidence-based” research.
In another approach to distinguishing between beliefs and values, some researchers view values as a subset of beliefs. Rokeach (1973, cited in Bishop et al., 2003) viewed values as enduring beliefs, and Clarkson and Bishop (1999, cited in Bishop et al., 2003) viewed values as beliefs in action. Raths, Harmin, and Simon (1987, cited in Clement, 1999) presented several attributes of beliefs that, if present, would constitute a value. They viewed values as beliefs that one chooses freely from among alternatives after reflection and that one cherishes, affirms, and acts upon. Because life situations may invoke incompatible values, many researchers relax the constraint that one must act upon one’s values and instead suggest that so long as one is committed to a particular belief, it might be said that that belief is a value for the person (Clement, 1999). As such, values influence, rather than determine, the choice of possible actions available (Bishop et al., 2003).

The similarities in researchers’ views on values and beliefs may be greater than the differences, and the two terms are often used interchangeably by mathematics educators (Bishop et al., 2003). Just as people hold incompatible values, so too do they hold beliefs that may conflict. Values exist within more complex values systems, and individual values alone seldom determine one’s decisions, actions, and responses (Bishop et al., 2003). Because beliefs also exist within systems (A. G. Thompson, 1992), in assessing a teacher’s values or beliefs, one must recognize the role played by a person’s incompatible or conflicting values and beliefs. Consider, for example, a teacher who values students’ development of mathematical proficiency. She will want to support their developing a combina-

tion of conceptual understanding, procedural fluency, and problem-representing and -solving approaches, all built upon the capacities to reason logically about relationships and justify their reasoning and upon the disposition to view mathematics as useful, worthwhile, and attainable (National Research Council, 2001). If this teacher also values raising her students’ test scores as measured by a standardized test focused primarily upon procedural fluency, then she is likely to find herself in a quandary. If this teacher also teaches in a low-performing school, subject to strict local or federal guidelines (like those currently applicable in U.S. schools), then the requirement to immediately improve students’ test scores is likely to cause the teacher to feel pressured into pursuing a course of action different from the one she believes will lead her students to develop mathematical proficiency. This teacher will have to prioritize among competing values. These priorities will vary across situations and contexts (Bishop et al., 2003); for example, for some time before the standardized test, the teacher may feel more con-

strained to alter her usual curricular and pedagogical approaches to focus upon developing procedural fluency. Simply observing this teacher’s performance is unlikely to provide one an accurate assessment of her values or beliefs. Furthermore, this teacher may be unaware of how these conflicting forces are affecting her curricular and pedagogical decisions; thus, teasing out the relationships among her values and beliefs and her thoughts and actions would be difficult, even in conversation.

**Belief Versus Knowledge**

*Not . . . what opinions are held, but . . . how they are held: instead of being held dogmatically, [liberal] opinions are held tentatively, and with a consciousness that new evidence may at any moment lead to their abandonment.*

—Bertrand Russell, Unpopular Essays, 1950

Across disciplines of anthropology, social psychology, and philosophy, one finds general agreement in viewing beliefs as “psychologically held understandings, premises, or propositions about the world that are felt to be true” (Richardson, 1996, p. 103). The notion that a belief is thought to be true raises one of the more common distinctions drawn between belief and knowledge (Pajares, 1992; A. G. Thompson, 1992), with researchers often viewing knowledge as “belief with certainty” (Clement, 1999). This definition is similar to one that is more than 2,300 years old, Plato’s definition of knowledge as “justified true belief” (McDowell, 1987, cited in Furinghetti & Pehkonen, 2002, p. 42), but with one important difference. By using the term true, Plato implied the existence of an external reality of which one could know and be certain, and von Glasersfeld’s observation that for many people today “there reigns the conviction that knowledge is knowledge only if it reflects the worlds as it is” (von Glasersfeld, 1984, p. 20) captures the sense that the view of knowledge put forth by Plato continues to prevail.

The notion that knowledge must be associated with truth has influenced many definitions of knowledge and, therefore, the distinction between knowledge and belief. For example, Scheffler (1965, cited in M. S. Wilson & Cooney, 2002) viewed both belief and knowledge as cognitive constructs but viewed knowledge as a stronger condition than belief. For Scheffler, one knows **Q** if three conditions hold: One believes **Q**; one has the right to believe **Q**; and **Q** is, in fact, true. Under this definition, a belief that is true is not knowledge for an individual unless one has a warrant for believing it, that is, unless one’s believing is based upon evidence. Fu-
hence, under this definition, a belief is not knowledge unless it is true. This truth requirement alarms many, because the means by which truth is determined is debatable, and the history of science indicates that what was taken as true at one point in time was generally modified or subsumed into more encompassing theories at other times. In von Glasersfeld’s (1993) view of constructivism, the notion that knowledge is a representation of reality is rejected. Instead, the most one can say about ability to predict physical phenomena is that knowledge has proved to be viable under particular circumstances. Cobb, Yackel, and Wood (1992) noted important implications for teachers’ holding the position that to know is to accurately represent what is outside the mind. Teachers who hold this view are likely to believe that they can use materials as a means for presenting readily apprehensible mathematical relationships as if there is a direct mapping between the materials and the mathematics, and all that is necessary for students to construct this mapping is for a teacher to clearly present the mathematics by using relevant materials (Cobb et al.). For example, a teacher might believe that the place-value relationships that students so often struggle to comprehend may be obvious if the students use base-ten blocks.

Some are troubled by the view of radical constructivists, who reject the notion that one has access to truth. For example, Goldin (2002b), who thought that the radical constructivist epistemology was deeply flawed and was “entering the realm of passé” (p. 205), feared that defining knowledge as justified true belief is a grave mistake made by “cultural relativists” (Goldin, 2002a, p. 64), because doing so “leaves no convenient word to distinguish beliefs that are in fact true, correct, good approximations, valid, insightful, rational, or veridical, from those that are in fact false, incorrect, poor approximations, invalid, mistaken, irrational, or illusionary” (Goldin, 2002a, p. 65).

This debate about the relationship between knowledge and belief is unlikely to cease, and I suspect that researchers will continue to disagree about the affordances and constraints of various relationships between the two. More important for researchers is to take clear stances on how they are viewing beliefs, stances that are reflected in their operationalization of the construct. For example, M. S. Wilson and Cooney (2002) dropped the truth condition and modified Scheffler’s second condition so that they suggested that X knows Q if X believes Q and X has reasonable evidence to support Q. M. S. Wilson and Cooney acknowledged that although in using this definition, they dodged the claim to know reality, the major question of knowing what constitutes evidence is unresolved. Using this definition, researchers can, however, acknowledge that what is evidence for one may not be evidence for another.

Educational researchers attempting to understand teachers’ knowledge and beliefs can benefit by recognizing how teachers look at these constructs and by drawing distinctions between knowledge and belief according to the world view of the subjects holding the knowledge or belief. The important question for researchers studying mathematics teachers’ beliefs and knowledge is not whether some conception is true in an ontological way but how a teacher views the conception. As a researcher, I have found the following stance useful when I attempt to understand how a person holds a particular conception: A conception is a belief for an individual if he or she could respect a position that is in disagreement with the conception as reasonable and intelligent, and it is knowledge for that individual if he or she could not respect a disagreeing position with the conception as reasonable or intelligent. By this definition, agreement upon what constitutes “a reasonable, intelligent position” is unnecessary; instead, the principles are left for the subject to apply and for the researcher to reveal. Under this definition, one person’s belief may be another person’s knowledge. For researchers to know how one holds a notion may be as important as knowing what one holds as the notion. Two people who hold contradictory beliefs about something may have more in common with each other than with a person who holds one of those conceptions as knowledge. For example, consider the following three people and the notions they hold about students’ learning of concepts and procedures: Person A holds as belief the notion that students are better served by being taught concepts before procedures; Person B holds as belief the notion that students are better served by being taught procedures before concepts; Person C holds as knowledge the notion that students are better served by being taught concepts before procedures. On the one hand, Persons A and B disagree about the beliefs they hold, but because they hold the notions as beliefs, they may find important points of agreement in a discussion, because they could imagine reasonable, disagreeing positions. On the other hand, Persons A and C, who hold the same notion but one questions it and the other does not, may experience unexpected difficulty when holding a meaningful discussion about their positions. The real roadblocks to meaningful dialogue are created when at least one of party of a disagreement holds a notion as knowledge and, hence, does not respect the position of those who disagree as reasonable or intelligent. Unfortunately, in mathematics education, this lack of ability to find common ground for discussion...
has fueled the fires in what have come to be known as The Math Wars (Becker & Jacob, 2000).

**Summary**

General distinctions have been drawn among values, beliefs, and knowledge, with values and knowledge often defined in terms of beliefs. Values are generally viewed as a type of belief to which one is deeply committed. Values are also viewed as preferences that are not associated with truth values, whereas beliefs are held by an individual as true/false dichotomies with the understanding that others may disagree. If one takes the ontological view that truth exists and people have access to it, then knowledge might be viewed as true belief. If one takes a view that truth, though it may exist, is not accessible to humans and instead the best one can hope for is viability, then knowledge is belief with certainty. Although for researchers to understand what beliefs teachers hold is important, perhaps equally important is understanding how teachers hold these beliefs.

**A Comment About Terminology**

A. G. Thompson, in her handbook chapter (1992), used the terms conception and belief, and although at times she seemed to use the terms interchangeably, more often she used conception to refer to a general construct that included beliefs as a component. Other researchers have used the term conception in their work, but with little agreement on its meaning. For example, Tirosh (2000) used conception as interchangeable with knowledge whereas Knuth (2002) used the term to represent knowledge and belief “in tandem” (p. 85). Andrews and Hatch (2000) noted that for some the term conception is essentially cognitive whereas for others it is affective. Although attempts to unify the definitions of constructs might seem critical for moving the field forward, I am not convinced of this need, because the constructs that researchers apply when studying the integration of teachers’ beliefs, knowledge, and practices are operationalized as much by the research methodologies used as by a particular definition. Throughout this chapter, I use the terms as the researchers used them in their studies and attempt to tease out similarities and differences in their approaches.

**Measuring Beliefs**

However one chooses to define (or not define) beliefs, when studying the beliefs of others, researchers must draw inferences from what people do or say (Pajares, 1992). Mathematics education researchers have typically approached the study of teachers’ beliefs in one of two ways: by using case-study methodology or by using beliefs-assessment instruments. Perhaps the more common approach has been to use case-study methodology to provide detailed descriptions of the beliefs of a small number of teachers by relying upon rich data sets that include some combination of classroom observations, interviews, surveys, stimulated-recall interviews, concept mapping, responses to vignettes or videotapes, and linguistic analyses. These data are often collected over a period of time and triangulated. These rich data sets are important for theory building, inasmuch as they enable researchers to consider interrelationships in the complex world of teachers. Such studies also enable researchers to meet the challenge put forth by A. G. Thompson (1992) to investigate the dialectic relationship between teachers’ conceptions, including their beliefs and knowledge, and teachers’ practices. Researchers in most studies I examined took this case-study approach to studying beliefs.

**Likert-Scale Surveys**

The case-study approach to studying teachers is powerful for building theory, but testing theory often requires tools for measuring the beliefs of larger groups of teachers. Typically, beliefs of large numbers of mathematics teachers are measured using Likert scales, and I present four examples of Likert scales used for measuring mathematics teachers’ beliefs.

As part of their long-term study of teachers’ use of knowledge about children’s mathematical thinking, referred to as Cognitively Guided Instruction, Carpenter, Fennema, and colleagues developed a beliefs survey, the Mathematics Belief Scale, comprised of 48 Likert-scale items with four subscales to assess Role of the Learner, Relationship Between Skills and Understanding, Sequencing of Topics, and Role of the Teacher (Fennema, Carpenter, & Loef, 1990; Peterson, Fennema, Carpenter, & Loef, 1989). The Role of the Learner subscale measured the belief that children are able to construct their own knowledge instead of being receivers of knowledge. The Relationship Between Skills and Understanding subscale measured the belief that skills should be taught in relation to understanding and problem solving rather than in isolation. The Sequencing of Topics subscale measured the belief that children’s natural development, rather than the logical structure of formal mathematics, should guide the sequencing of topics. The Role of Teacher subscale measured the belief that instruction should facilitate children’s construction of knowledge rather than consist of teachers’ presenting materials. An example of an item from The Role of the Learner subscale is “It is important for a child to be a good lis-
tender in order to learn how to do mathematics.” This beliefs survey has been used by researchers working with prospective teachers (Vacc & Bright, 1999) and with practicing teachers (Fennema et al, 1996).

Zollman and Mason (1992) developed a beliefs instrument based upon the National Council of Teachers of Mathematics (1989) Curriculum and Evaluation Standards for School Mathematics. Their instrument was designed to assess teachers’ beliefs about the Standards, and they selected 16 items to reflect 16 of the 54 standards included in the NCTM document. The items were stated as nearly direct quotes or the inverse of direct quotes from the Standards document, and each item met the criteria that the answer should not be “intuitively obvious” (Zollman & Mason, 1992, p. 359) and the item could be incorporated into a single sentence easily stated in a positive or negative manner. During their pilot work developing the survey, the authors found that respondents appeared to emphasize aspects of the items that were distracting, and so to ensure that respondents focused attention on the intent of the item, select words were capitalized. An example of a positively stated item is “Students should share their problem-solving thinking and approaches WITH OTHER STUDENTS”; a negatively stated item is “Children should be encouraged to justify their solutions, thinking, and conjectures in a SINGLE way” (Zollman & Mason, 1992).

Researchers seeking to use Likert-scale beliefs surveys to study mathematics teachers’ beliefs use existing surveys, as Vacc and Bright (1999) used the survey developed by Fennema et al. (1990), develop a new survey, as Zollman and Mason (1992) did, or create a hybrid, drawing upon an existing survey. Hart (2002) administered a three-part beliefs’ survey to prospective elementary school teachers before and after they had completed an integrated content/methods course; the first two parts were adapted from existing instruments, and the third part was Hart’s creation. The first part was a form of Zollman and Mason’s (1992) Standards’ Belief Instrument; the second was adapted from Schoenfeld’s (1989, cited in Hart, 2002) Problem-Solving Project. A positively stated item from this second part, which Hart used to assess change in teachers’ beliefs about teaching and learning mathematics within and outside the school setting is “Good mathematics teachers show students lots of different ways to look at the same question”; an item that was negatively stated is “Good math teachers show you the exact way to answer the math question you will be tested on.” Hart’s third part was comprised of two items on teacher efficacy, with the first stating, “I am very good at learning mathematics,” and the second stating, “I think I will be very good at teaching mathematics.”

Enochs, Smith, and Huinker (2000) developed a Mathematics Teaching Efficacy Belief Instrument designed to be used with prospective teachers. Their beliefs survey was comprised of 21 Likert-scale items, 13 on the Personal Mathematics Teaching Efficacy subscale and 8 on the Mathematics Teaching Outcome Expectancy subscale. An item on the former subscale was “Even if I try very hard, I will not teach mathematics as well as I will most subjects,” and one from the latter was “The mathematics achievement of some students cannot generally be blamed on their teachers.”

One concern about self-report surveys is whether teachers’ reports are accurate. Although Ross et al. (2003) provided evidence that the self-report data for elementary school teachers’ degrees of implementation of standards-based mathematics teaching in Ontario, Canada, were reliable and valid, whether teachers’ self-reports of their practices are different from their self-reports of their beliefs remains an open question. Is a Likert-scale survey a valid measure on which to base inferences about teachers’ beliefs? In the next section, I present an example from a group of researchers whose concern about the validity of Likert-scale beliefs surveys led them to develop an alternative survey that can be administered to large numbers of subjects.

An Alternative to Likert-Scale Beliefs Surveys for Large-Scale Data Collection

A group of researchers engaged in a large-scale research and development project referred to as IMAP (Integrating Mathematics and Pedagogy, 2003; see, also, Ambrose, Clement, Philipp, & Chauvat, 2004) developed a web-based survey requiring open-ended responses to overcome three problems they had identified with use of Likert-scale surveys. They noted, first, that inferring how a respondent interprets the words in Likert-scale items is difficult. For example, in the item “It is important for a child to be a good listener in order to learn how to do mathematics” (Fennema et al., 1990), how do researchers know how the respondent interprets the idea of good listener? Might a respondent think differently about this statement depending upon whether she was thinking of a child listening to a teacher demonstrating a procedure, a teacher presenting a problem situation, or a child sharing a solution strategy? Individual respondents are not able to explain their responses for Likert items, so researchers can only infer what considerations may have guided the respondents. A second difficulty the IMAP researchers identified with Likert items is that
responses provide no information for determining the importance of the issue to respondents (Ambrose et al., 2004). McGuire (1969) stated, “When asked, people are usually willing to give an opinion even on matters about which they have never previously thought” (p. 151). For example, although teachers who build their instruction around having children solve problems and share solutions with other students are likely to agree strongly with the statement “Students should share their problem-solving thinking and approaches WITH OTHER STUDENTS,” teachers who have not thought much about basing instruction upon their students’ sharing their thinking with other students may, when reading this statement, think that it makes sense and may agree or even strongly agree. The third concern the IMAP researchers raised is that Likert scales tend to provide little, and often no, context. For example, Collier (1972) developed a Likert-scale survey containing the item “In mathematics, perhaps more than in other fields, one can find set routines and procedures.” Respondents might interpret such a comment in multiple ways, and their responses give no indication of their interpretations. Some respondents may differentiate between middle-school algebra and primary-grade arithmetic and believe that one has set routines and procedures and the other does not. Because no context is associated with the item, respondents may feel compelled to consider mathematics in general when, in fact, they view different levels of mathematics in different ways.

The IMAP team set out to create a beliefs survey by identifying four characteristics of beliefs identified in the literature as accounting for the critical role that beliefs play in teaching and learning and, thus, were important for the way the researchers attempted to measure the beliefs. First, beliefs influence perception (Pajares, 1992). That is, beliefs serve to filter some complexity of a situation to make it comprehensible, shaping individuals’ interpretations of events (Grant, Hiebert, & Wearne, 1998). Beliefs might be thought of as serving as a type of constructed model (von Glasersfeld, 1993) or theory (Nisbett & Ross, 1980) through which one looks, so that beliefs affect what one sees or notices (Mason, 2002). The IMAP team addressed this issue in their web-based survey by providing respondents with complex situations to interpret (Ambrose et al., 2004). Second, beliefs are viewed as drawing one toward a position or, at least, predisposing one in a particular direction. Allport (cited in McGuire, 1969) considered a belief as a readiness to respond, and Rokeach (1968) thought of a belief as a disposition to action. The notion that beliefs are dispositions toward action has played a key role in some important research on beliefs (e.g., Cooney, Shealy, & Arvold, 1998), because one’s beliefs can be inferred by attending to the manner in which one is disposed to act in a particular situation. Because the IMAP team intended their beliefs survey to be used by practicing and prospective teachers, they took into account the fact that many prospective teachers are not yet positioned to act and thus many means available to researchers for measuring practicing teachers’ dispositions toward action are unavailable for use with prospective teachers. They addressed this issue in their beliefs survey by providing respondents with scenarios in which they were asked to make teaching decisions. The prospective and practicing teachers’ dispositions to act in these situations provided evidence from which to infer their beliefs. A third characteristic the IMAP team identified as pertinent for their work was that beliefs are not all-or-nothing entities; they are, instead, held with differing intensities (Pajares, 1992, citing Rokeach, 1968). To address this characteristic in their survey, they provided tasks with multiple interpretation points; the order in which questions were posed was carefully constructed to elicit views that were important to the respondents before “giving away” the “preferred” answers. For example, in one task, after being shown a video clip, respondents were asked, “What stands out to you in this video?” before they were asked, “What are weaknesses of the teaching in the video?” Those who identified weaknesses before being prompted for weaknesses were scored as holding the designated belief more strongly than those who did not mention weaknesses until asked to do so. Furthermore, because of the electronic administration of the survey, respondents could not return to change their responses to previous items. To allow for the differing intensities with which individuals hold beliefs, the team devised the scoring rubrics to differentiate among strong evidence, evidence, weak evidence, and no evidence for a respondent’s holding a belief (Integrating Mathematics and Pedagogy, 2004). Furthermore, they were careful not to claim that an individual lacked a particular belief but instead stated that they found no evidence for the belief in the responses the individual provided. Fourth, beliefs tend to be context specific, arising in situations with specific features (Cooney et al., 1998). The IMAP team addressed this issue by situating survey segments in contexts, and they inferred a respondent’s belief on the basis of his or her interpretation of the context (Ambrose et al., 2004).

This beliefs survey was used in an experimental study of prospective elementary school teachers enrolled in a mathematics course and randomly assigned to (a) concurrently learn about children’s mathematical thinking by watching children on video or working directly with children, (b) concurrently visit elemen-
tary school classrooms of conveniently chosen or specially selected teachers, or (c) a control group that had no relevant additional instructional activities outside the mathematical course (Philipp et al., 2005). Those who studied children’s mathematical thinking while learning mathematics developed more sophisticated beliefs about mathematics, teaching, and learning and improved their mathematical content knowledge more than those who did not study children’s thinking. Furthermore, beliefs of those who observed in conveniently chosen classrooms underwent less change than the beliefs of those in the other groups, including those in the control group. The researchers concluded that by developing a context-based open-ended beliefs survey, they were able to measure prospective elementary school teachers’ beliefs related to mathematics and to mathematics learning and teaching years before these prospective teachers were in the position to act as teachers. Beliefs of PSTs could be changed over the course of a semester, although the authors did not speculate whether these changes would remain with time (Philipp et al., 2005).

Summary

The predominant approach used by researchers to measure beliefs is to employ qualitative measures, including teacher interviews, classroom observations, responses to simulated materials, concept generation and mapping, paragraph completion, or Kelly Repertory Grid techniques. Although these qualitative approaches provide rich data sets, they are expensive to employ across large numbers of participants; researchers who need to collect large data sets often rely upon less expensive approaches, such as Likert-scale surveys. Some have identified problems with Likert-scales surveys, and one example was provided of an alternative beliefs survey that was developed and successfully used to survey large numbers of prospective elementary school teachers without relying upon Likert-scale responses.

Inconsistent Beliefs and the Role of Context

Because of the complexity of teachers’ beliefs systems, researchers may find that teachers hold beliefs that appear to be inconsistent with their teaching practices. One approach taken by researchers to explain inconsistencies is to examine whether particular beliefs within a beliefs system are more central or primary, and hence play a greater role in influencing practice, than other beliefs. Another approach is to study whether a teacher’s perspective on his or her practice might help explain the apparent contradiction. In this section I share examples of both approaches and conclude by considering the stances researchers take regarding inconsistent beliefs.

In a study of six novice elementary school teachers, Raymond (1997) investigated the inconsistency between one teacher’s mathematics beliefs and teaching practice. For each teacher, data, collected during the 10-month period between March of their first year of full-time teaching and December of their second year, included an introductory phone interview, six 1-hour audiotaped interviews, five monthly classroom observations, an analysis of several samples of lesson planning, a concept-mapping activity in which teachers presented their views of the relationships between mathematics beliefs and practice, and a take-home questionnaire on mathematics beliefs and factors that influence teaching practice. Raymond analyzed these data within four categories—teachers’ beliefs about the nature of mathematics, teachers’ beliefs about learning mathematics, teachers’ beliefs about teaching mathematics, and teachers’ mathematics-teaching practices—on a 5-level scale ranging from traditional to nontraditional (traditional, primarily traditional, even mix of traditional and nontraditional, primarily nontraditional, and nontraditional). The focus of her case study was a fourth-grade teacher, Joanna, selected, first, because her expressed beliefs were similar to those of the other five teachers and, second, because her case was dramatic in that although her beliefs about the nature of mathematics were fairly traditional, her beliefs about learning and teaching mathematics were placed at the nontraditional end of the scale.

Joanna expressed the view that students should discover mathematics without being shown and that students learn mathematics better when solving problems. Joanna also stated the beliefs that teachers could be effective without following a textbook and that they should use various activities from various sources; she was a strong advocate of using manipulatives to achieve hands-on learning. Raymond noted a major inconsistency between Joanna’s nontraditional beliefs about mathematics teaching and learning and her traditional practices. Joanna’s class was run in an orderly way with her students’ working exclusively at their desks in a quiet atmosphere, reflecting strict discipline. Joanna established herself as the authority, presenting information as teacher-directed instruction; although some teacher-student dialogue took place, on no occasion was student-to-student dialogue observed. Joanna rigidly followed her mathematics textbook, and the students worked quietly on problems from the textbook without access to manipulatives.

Joanna’s beliefs about mathematics were categorized as traditional—at the opposite end of the continuum from her beliefs about mathematics teaching
and learning but on the same end as her practice. Joanna identified her own negative experiences as a mathematical learner as the primary influence on her beliefs about the nature of mathematics but her own teaching practice as the main influence on her beliefs about teaching and learning. Despite her negative view of mathematics as a student, and perhaps because of her negative experiences, Joanna wanted her own students to experience mathematics in a more positive light than she had experienced, a desire that may account for her nontraditional beliefs about learning and teaching mathematics. But if Joanna held these nontraditional beliefs she espoused, why was her practice traditional?

Raymond (1997) concluded that this inconsistency arose from the effects on Joanna’s teaching of time constraints, scarcity of resources, concerns over standardized tests, and students’ behavior. As a novice teacher, Joanna was particularly concerned with classroom management and discipline. Raymond reported also that, in at least one case, Joanna seemed to view her mathematics-teaching practice in terms of what she wanted to do, or thought she should do, rather than what she accomplished. She claimed that her belief in teaching mathematics through manipulatives led to her using them, but the only time Joanna actually used manipulatives was during a class she described as chaotic.

Raymond (1997) concluded, further, that her study provided evidence that the teacher’s beliefs and practice were not wholly consistent and that her practice was more closely related to her beliefs about mathematics content than to her beliefs about mathematics learning and teaching. She also suggested that during teacher education, beliefs of prospective elementary school teachers might be more explicitly addressed so that teachers become aware of the beliefs with which they enter and attend to how these beliefs begin to change during this important period of growth.

Hoyles (1992) reflected upon how her changing view of beliefs affected her thinking about inconsistent beliefs. During her work in the early 1980s on a research project designed to identify characteristics of good secondary-school teaching practice, she and her colleagues approached teachers’ beliefs as a construct that researchers might access from outside the classroom. Assuming that beliefs could be assessed in a de-contextualized manner and that a linear relationship existed between teachers’ beliefs and their classroom practices, the researchers first assessed teachers’ beliefs and then observed teachers in the classroom. By the early 1990s, after observing the changes teachers experienced while interacting with a classroom innovation involving computers, Hoyles drew upon work in situated cognition (Brown et al., 1989) and subscribed to the notion that situations coproduce beliefs through activities. She came to view beliefs as situated and dialectically constructed from the relationships among activity, context, and culture and, thus, teachers’ beliefs about mathematics or mathematics teaching and learning as being affected by factors such as grade level, students’ level, textbook, or computer use.

For Hoyles (1992), viewing all beliefs as situated reconciled the apparent inconsistencies between teachers’ beliefs and actions reported by researchers. Once the embodied nature of beliefs is recognized, the notion of inconsistent beliefs is an irrelevant consideration that can be replaced by considering the circumstances and constraints within settings. When Hoyles reconsidered data from the 1980s study with this new lens, she noted “the remarkable harmony between what the teacher wished to ‘deliver’ and ‘what the students wanted’—an example of beliefs constructed in practice” (p. 40). Hoyles continued, “Such harmony exemplifies a good practice—a different blend of variables in the setting might produce alternative pictures of success. The notion of situated beliefs allows us to cope with this diversity” (p. 40).

Skott (2001) also considered the influence of context on teachers’ beliefs, and although his conclusion about context differed from Hoyles’s, he shared her conclusion about inconsistencies. He investigated the relationship between teachers’ beliefs about mathematics and mathematics teaching and learning and the teachers’ classroom practices. Skott introduced the term school-mathematics images to “describe teachers’ idiosyncratic priorities in relation to mathematics, mathematics as a school subject and the teaching and learning of mathematics in schools” (p. 6). On the basis of responses to a questionnaire comprised of open and closed questions, administered to Danish student teachers two months before their graduation, Skott selected 11 student teachers, representing a variety of school-mathematics images, to interview after graduation. Four teachers who presented school-mathematics images inspired by current reform efforts in school mathematics were videotaped teaching their mathematics classes for 2 to 3 weeks during their first 18 months of teaching; they participated in informal discussions after each lesson and in a comprehensive interview after the videotaped lessons were completed. Skott selected one of the four teachers, Christopher, as the subject of a research study.

Christopher’s responses to the questionnaire and the first interview indicated that his school mathematics images were in contrast to those he remembered experiencing as a high school student in the Danish
gymnasium, in which teachers lectured at the board. He favored process aspects of mathematics; he viewed his role as initiating and supporting investigative activities so that his students could assume responsibility for their own learning. Christopher was committed to school-mathematics reform but also to his students’ developing broad educational, nonmathematical skills: learning to plan, reflecting on their own learning, working cooperatively, relating critically to information, and independently finding solution strategies and models.

After graduation, as a music and mathematics teacher in a typical Danish folkeskole (a municipal school for Grades 1–10), Christopher enjoyed freedom in his choice of teaching methods and curricula. He was observed teaching his Grade 6 mathematics class, in which he had introduced a textbook different from the traditional book used by the previous teacher. Because as a novice teacher Christopher felt overwhelmed with lesson preparation and grading, he relied upon the textbook for the aims, the content, and the tasks for instruction, but he strongly influenced the flow of the classroom by the methodology he chose. He lectured little and addressed the whole class primarily to initiate small-group or individual work. He sought student explanations and elaborations, and he encouraged students to explore in the noisy, but productive, classroom.

Skott (2001) described two visits to Christopher’s classroom. The first visit took place during the second day of a unit on area; the students were cutting congruent triangles to make parallelograms, cutting parallelograms to make rectangles, and measuring areas of desks, blackboards, and other items. When two students called for help with a problem, Christopher listened to the weaker student’s approach and asked the student questions, although he recognized that the approach would be nonproductive, before he asked whether they could approach the problem another way. He then pursued the stronger student’s approach with the two students. Skott viewed this episode as being consistent with Christopher’s school-mathematics image, because he attempted to initiate and support the students’ learning by letting them devise suggestions and by refraining from deflecting suggestions that might be wrong or insufficient.

The second visit took place during the next lesson. When two other boys asked for help to find the areas of rectangular lawns drawn with different scales, Christopher intervened and led the two students through a series of computationally oriented questions in such a way as to deplete the task of any mathematical challenge and replace it with carrying out simple, prescribed computations. Skott (2001) viewed this episode as being inconsistent with Christopher’s school-mathematics image and Christopher’s actions as replacing his general reformist school-mathematics priorities with traditional values in mathematics education.

When confronted with the roles he had played in the two episodes, Christopher, who was often critical of his own teaching, was satisfied with both. Skott (2001) suggested that although Christopher’s second interaction appeared to reflect inconsistencies both with his action during the first class and with his stated beliefs, the relationship between Christopher’s school-mathematics images and his actions could explain his seemingly inconsistent behavior. Christopher’s evaluation of each episode encompassed aspects of his approach to teaching beyond students’ mathematical learning, and Skott identified three such aspects that became apparent in the two episodes and in the interview. First, Christopher consistently mentioned the need to think of the children as learners generally, not solely as mathematical learners, for example, as needing to develop self-confidence in mathematics and in general. Second, Christopher encouraged and expected his students to interpret and solve tasks on their own and to discuss strategies and results among themselves. In keeping with this goal, he provided minimal support before they started working, and as a result, the students were in constant need of assistance from the teacher during group work. Third, Christopher’s constructivist view of learning did not lead to a particular method for teaching, and he spoke about the need to play multiple roles when teaching. Understanding these three aspects helped Skott understand Christopher’s actions. In the first episode, he delved into the first student’s suggestion because he believed that the student’s self-confidence was an important consideration. In the second episode, he ensured that the students arrived at the correct answer because of his concern for the general management of the class, realizing that one of the two boys was extremely demanding and would require excessive attention unless the two boys believed that they had solved the problem. Skott concluded that teachers have “multiple and sometimes conflicting educational priorities, and that in this case the priorities related to Christopher’s SMIs [school-mathematics images] were dominated by others concerned with managing the classroom with broader educational issues” (p. 18).

Skott (2001) concluded that Christopher’s school-mathematics image shaped and filtered the objects of his reflection and framed his interpretation of what he saw but was insufficient for explaining all his actions, sometimes being overshadowed by the more general educational priorities of building students’
confident and managing the class. Skott also noted how the teacher’s classroom actions tended to replicate themselves. Christopher used the same calculational approach (A. G. Thompson et al., 1994), seemingly incompatible with his school mathematics image, that he had used to move the class along by appeasing these two students with a number of other groups of students, even when the motives for which it was originally used, building student confidence and managing the class, no longer were issues. The way the teacher coped with the original problem became a “prototypical action type” (Skott, 2001, p. 22) that was applied even when the original reasons for applying the action type were no longer present.

Skott’s (2001) conclusion that the relationship between the teacher’s school-mathematics images and his classroom practices varied according to the classroom situation seems to support Hoyles’s (1992) notion that beliefs are situated. Although both Skott and Hoyles considered context critical to assessment of teachers’ beliefs, they differed in their views on the situativity of beliefs. Skott contended that Christopher’s beliefs did not change with the situation but that Christopher continued to believe in the importance of calculational processes and the teacher’s role in initiating and monitoring student activity to facilitate both the processes and the products. The change was in his specific goals during the two episodes—from mathematical learning of the two students in the first to class management in the second. Skott (2001) elaborated upon how this view differed from that of Hoyles (1992):

> It is not the situation that produces a new set of school mathematical priorities. Situations that are outwardly different—for example, a research interview as opposed to a classroom setting—do not necessarily create or radically change a teacher’s beliefs. Rather, the situations, that is, very specific situations each concerned with a specific interaction between student(s) and teacher, create or co-produce competing objects and motives of the teacher’s activity. It is these competing objects and motives that form the basis of what apparently, but only apparently, is a new set of priorities produced under new and slightly different circumstances. In other words, the contextual embeddedness of Christopher’s activity did not necessarily lead to a similar contextualisation of his school mathematical priorities. (Skott, 2001, pp. 24–25)

Skott acknowledged one general conclusion about beliefs research: No general conclusions should be expected in beliefs research. Furthermore, because teachers and the contexts in which they work differ, the relationship between classroom practices and teachers’ beliefs may require more detailed, long-term investigations of teachers’ approaches in specific classrooms over several studies to begin to generate preliminary understandings of theoretical constructs.

Sztajn (2003) also considered the relationship between teachers’ beliefs and practices as mediated by the contexts in which teachers were working, but instead of analyzing one teacher in two contexts as Skott had done, she analyzed two teachers who held similar beliefs but taught in different contexts. She observed each of two elementary school teachers a week at a time for four weeks spaced throughout the semester and conducted five semistructured interviews with each teacher. When visiting the teachers, she observed throughout the day, including during lunch, breaks, and staff meetings. Other classroom data included the teachers’ lesson plans, grading books, students’ report cards, mathematics worksheets and completed assignments, and materials from activities in other subjects. To triangulate her data related to the class context, Sztajn interviewed the principal, other teachers, and parents from each school.

Because she set out to study beliefs that go beyond mathematics and mathematics teaching and learning experiences, Sztajn (2003) initially coded the data using seven descriptive categories: the teacher, the students, the classroom, the school, the parents, general educational goals, and mathematics. The two public-school teachers she investigated taught classes in small, midwestern towns, one a third-grade and the other a fourth-grade class; each class was comprised only of Caucasian children. The teachers held similar beliefs about mathematics, believing that problem solving and basic skills were important but that basic skills had to be mastered before children could engage in problem solving. They believed that some children bring basic-skills knowledge from home but that the others lack both basic and social skills. In spite of the similarity of their beliefs about mathematics, their instruction differed; to explain the differences, Sztajn turned to their beliefs that go beyond mathematics.

Sztajn (2003) found that the concept of students’ needs, which included teachers’ beliefs about children, society, and education, emerged as an important factor in the teaching of both teachers and accounted for the differences in their instruction, because the two schools differed with respect to the manner in which the children were viewed. The students of the third-grade teacher, Theresa, were at a lower socioeconomic level than the students of the fourth-grade teacher, Julie. At Theresa’s school, 40% of the students received free or reduced-cost lunches, compared to only 10% of the students at Julie’s school. The educational levels of the parents, as estimated by the principals and teachers, differed; most...
parents of the students at Theresa’s school held low-income, manual-labor jobs whereas in Julie’s school, many middle-income parents were doctors, lawyers, and university professors. Although Theresa believed in the importance of students’ engaging in problem solving and developing higher order thinking skills, because she saw her students as coming from unstable, chaotic homes—poor environments for children—basic facts, drill, and practice were at the core of what she thought her students needed to prepare for their roles in the workplace. Most important for Theresa, whose teaching goal was to form responsible citizens, was that “learning and following rules in a responsible and organized way is what her students need in order to find their place in society” (p. 64). The fourth-grade teacher, Julie, held beliefs about mathematics and about students’ needs that were similar to Theresa’s, but because she thought that she had to deal with few problems and worried little about teaching basics, she structured her classroom around problem solving and projects. In Julie’s class, even for routine mental-computation tasks, students shared various solution strategies. Although at the time of the study Julie and Theresa approached teaching differently, the approach Julie described having used previously, at a school in which children were more like Theresa’s students, was very similar to Theresa’s approach. Both teachers believed that teaching social skills was a main role of school. For example, Julie said that responsibility was important to teach, but because she thought that her students learned responsibility at home, they needed less practice on social skills and could explore more sophisticated mathematical content.

Sztajn (2003) contended that both teachers are the heroes of the stories presented—never the villains. They are not, in a Machiavellian way, trying to hold students back, lessen their chances of going to college, or ensure that they will remain where they are in the socioeconomic spectrum. Quite the contrary, both of them use their best judgments when defining what their students need, and once they form this concept, they direct their work and all their efforts toward meeting these needs. (p. 71)

Sztajn concluded that because these students came from different socioeconomic backgrounds, they were learning different mathematics. It was the teachers’ beliefs about children, society, and education, not their beliefs about mathematics, that accounted for the differences in instruction.

Summary
Researchers who have investigated apparent inconsistencies between teachers’ beliefs and practices have attempted to explain them. Raymond found two factors that accounted for such inconsistencies. First, a teacher’s practices were more in line with her beliefs about mathematics than her beliefs about mathematics teaching and learning. Second, as important as beliefs about mathematics were, observed inconsistencies between a teacher’s espoused beliefs and her practice were explained by general educational issues, such as time constraints, resources, standardized tests, and students’ behavior.

Hoyles (1992) found that when she viewed beliefs as decontextualized, she observed inconsistencies, but when her view of beliefs became more contextualized and situated, she could explain apparent inconsistencies by considering the circumstances and constraints within settings.

Skott (2001) found that a teacher’s beliefs about mathematics and about mathematics teaching and learning were important for understanding the teacher’s practice but that these beliefs were often overshadowed by the more general educational priorities of building students’ confidence and managing the class. But unlike Hoyles, who concluded that beliefs were situated, Skott concluded that not the teacher’s beliefs but the teacher’s goal for the particular activity changed, and after understanding the goal, the researcher could explain apparent contradictions between the teacher’s beliefs and actions.

Sztajn (2003) studied two teachers who held similar beliefs about mathematics but taught in very different contexts and differed in their teaching. She found that attending to the teachers’ beliefs about mathematics was insufficient for explaining the teachers’ practices; only after considering the teachers’ beliefs about children, society, and education was she able to account for the differences in the teachers’ practices.

A Stance on Inconsistent Beliefs
What are we mathematics education researchers to make of inconsistencies? Is resolving inconsistencies important for the work we do? Within mathematics, consistency is so central that, historically, when mathematicians have faced a new way to consider some aspect of mathematics that seemed to them unimaginable or even untrue, they would, often after much resistance, accept this new way so long as no inconsistencies arose. As a classic example, 19th-century mathematicians realized that rejecting Euclid’s fifth postulate led to consistent, albeit previously unimaginable, possibilities, resulting in mathematicians’ accepting the findings and inventing new non-Euclidean geometries that led to productive ways for considering the physical world. Other examples are the
expansion of the number systems to include irrational numbers, negative numbers, and complex numbers and Cantor’s theory of transfinites, which engendered new ways of thinking about infinity. Although inconsistencies are unacceptable in logical domains, people, in general, have come to accept that inconsistencies abound in human affairs. So what are the implications for how we choose to consider inconsistencies when studying teachers’ beliefs?

Most researchers cited in this section found that the inconsistencies that arose when documenting teachers’ beliefs and actions ceased to exist after they better understood the teachers’ thinking about some aspects of their contexts. I propose that as a research stance in studying teachers and their beliefs, we researchers assume that contradictions do not exist. Taking this stance when we observe apparent contradictions, we would assume that the inconsistencies exist only in our minds, not within the teachers, and would strive to understand the teachers’ perspectives to resolve the inconsistencies. Inconsistencies should still present problems, but for researchers instead of teachers. I do not propose that this position is ontological; that is, I do not suggest that inconsistencies are nonexistent or can all be explained away. I suggest, instead, that researchers who assume that inconsistencies do not exist will attempt to better understand teachers’ beliefs systems and that the circumstances surrounding the teachers’ practices will often, as for the researchers cited in this section, lead to resolution of the inconsistencies.

Changing Beliefs

Two general approaches are considered for the relationship between the change in beliefs and the change in behavior. One notion, supported by research, is that to change behavior, one must first change one’s beliefs, because beliefs act as filters that affect what one sees (Pajares, 1992) and people have difficulty seeing what they do not already believe. However, if beliefs must change before behavior, then how do beliefs change? Can one change his or her beliefs merely by reflecting upon them? Or might reflection and action form a dialectic that can contribute to teachers’ learning and growth? If Socrates was right that the unexamined life is not worth living, perhaps it is equally true that the unlived life is not worth examining.

Guskey (1986) presented an alternative model predicated on the notion that “significant change in teachers’ beliefs and attitudes is likely to take place only after changes in student learning outcomes are evidenced” (p. 7). He offered a linear model, proposing that teachers who implement an instructional change that leads to students’ success will then alter their related beliefs. In 1993, von Glasersfeld, in stating, “If one succeeds in getting teachers to make a serious effort to apply some of the constructivist methodology, even if they don’t believe in it, they become enthralled after five or six weeks” (p. 29), also observed that changes in teachers’ beliefs follow changes in their instruction.

In his research on teacher change, Guskey (1986) examined classroom teachers’ success in implementing what he described as a relatively minor alteration in their practices. Grant, Hiebert, and Wearne (1998) wondered how Guskey’s model might hold for substantive changes to teachers’ practices, such as those called for in the effective-schools literature. They noted that implementers of past programs had often incorrectly assumed that simply telling teachers how they should teach would lead to the desired changes in practice and wondered whether, instead of telling, showing teachers models of teaching would be effective? What would teachers notice when observing reform-oriented instruction applied with their own students, and would what the teachers noticed be related to the beliefs they held? To investigate these questions, nine teachers observed reform-oriented lessons for 6–12 weeks. The teachers’ beliefs about what mathematics should be learned and how to teach mathematics were placed on a continuum (from skills/teacher-responsibility end to the process/student-responsibility end). The researchers then compared the positions of the nine observing teachers’ beliefs along the continuum with their evaluations of the alternative instruction, and they discovered a clear relationship between the teachers’ beliefs and what they observed during the instruction. Beliefs of four of the nine teachers were on the skills/teacher-responsibility end of the beliefs continuum, and these four teachers tended to focus on one particular aspect of the reform-oriented lesson or to notice features that the researchers believed were tangential, not the intended point of the instruction. For example, whereas the intended purpose for using manipulatives was to provide opportunities for teachers to promote multiple strategies that might link representations for the students, the teachers on the skills/teacher-responsibility end of the continuum thought that their purpose was to present the only way to solve problems.

Three of the nine observing teachers, whose beliefs were placed in the middle of the beliefs continuum, recognized some features of the instruction but did not connect these features with the larger intended goals. For example, one teacher noted the importance of explaining strategies, doing story prob-
lems, and engaging in critical thinking, but she juxtaposed these against her view of the teacher’s role as the provider and demonstrator of strategies. This teacher seemed to be driven by an image of a teacher as one who removes obstacles from her students’ paths. When working with story problems, she tried to support her students’ success by helping the students focus on key words. One teacher who was classified in the middle-ground category later taught the place-value unit she had observed. She explained that except for making a little change, she “pretty well stuck to their lesson plans.” The change she made was that instead of encouraging students to solve the problems as they chose, she explained the rule of solving by adding from the one’s column.

Beliefs of two of the nine observing teachers were placed at the process/student-responsibility end of the beliefs continuum; these teachers recognized the features of instruction important to the researchers and understood the underlying goals of the instruction. For example, even after hearing other teachers say that the approach was based on use of manipulatives, one of these two teachers noted that the approach was less about manipulatives and more about having children describe their thinking processes and solve problems in different ways.

The authors concluded that their study provided evidence for the position that the beliefs teachers hold filter what they see and, consequently, what they internalize. Teachers whose beliefs are at odds with a particular instructional approach may not change their thinking or their practices solely by observing the approach. The researchers suggested that observations combined with other activities designed to support the teachers’ reflecting upon the experience, such as collegial discussions of the instruction, are more likely to lead to change.

In a study of 14 third-grade teachers undergoing professional development, Borko, Mayfield, Marion, Flexer, and Cumbo (1997) found a similar result. When teachers’ beliefs were incompatible with the goals of the staff developers, the teachers generally either ignored the new ideas or inappropriately assimilated the ideas into their existing practices. Borko et al. concluded that beliefs served as filters through which ideas were perceived, and teachers needed to be challenged to reflect upon their beliefs.

Although Benbow (1995) acknowledged that beliefs act as filters through which teachers interpret their school experiences and influence their classroom practices, he rejected the linear model—that beliefs change must precede change in practice—in investigating the relationship between prospective elementary school teachers’ mathematical beliefs and their classroom teaching in the context of an early field experience. On the basis of data from three measures of beliefs, two Likert-type surveys and one open-ended questionnaire, he concluded that the preexisting beliefs held by the preservice teachers played key roles in their planning and implementation decisions for their mathematics lessons. At the same time, the preservice teachers’ interpretations of the instructional outcomes affected their beliefs and, therefore, their subsequent practices. The early field experience had little effect on the preservice teachers’ beliefs about mathematics, but it did affect their beliefs about pedagogical issues, such as their becoming more open to alternative points of view. The most dramatic belief change measured by the researcher was in the preservice teachers’ personal mathematics-teaching efficacy, that is, their confidence in their own abilities to help students learn mathematics. Benbow concluded that the preservice teachers’ core beliefs underwent little change and that, overall, the early field experience was primarily a confirming experience for the participants.

Reflection and Changing Beliefs

Cooney et al. (1998) also found that reflection played an important role in the growth of prospective secondary-school teachers over their last year in an undergraduate teacher preparation program. They applied a rich theoretical perspective, drawing heavily from the work of Green (1971, cited in Cooney et al., 1998) to attend not only to what beliefs the prospective teachers held but also to the ways in which the beliefs were held. They considered the three aspects of Green’s belief structures: Beliefs can be held as primary or derivative beliefs; they then can be held with differing degrees of strength; and they can be held more centrally or more peripherally in relation to other beliefs. Another important factor for Green was the role that evidence plays in the ways one holds beliefs, because nonevidentially held beliefs are impervious to change through reason or evidence whereas beliefs held evidentially can be changed through teacher reflection. A related construct is teacher authority, because for one who holds beliefs nonevidentially, the role of an outside authority figure is important for the construction of beliefs. Cooney et al. applied two frameworks for considering authority, one based upon Perry’s (1970, cited in Cooney et al., 1998) scheme, which was developed by investigating how college males think and come to know, and the other based upon women’s ways of knowing (Belenky, Clinchy, Goldberger, & Tarule, 1986, cited in Cooney et al., 1998). Cooney et al. “were interested in capturing the meanings each of the teachers ascribed to his or her
Greg was open to others’ perspectives and enjoyed the exchange of diverse opinions, whereas Henry tended to either accept or reject an opinion, holding tightly to his own, and felt frustrated when his beliefs were challenged. He held a more authoritarian view in which assimilation, but not accommodation, may be possible. Greg held a reflective stance toward the world whereby he was able to hold onto his core ideas while reformulating his core beliefs when faced with perturbations. The two students who changed the most were the two who were most reflective and who were open to teaching as a problematic activity.

The authors presented four characterizations for how the preservice teachers held their beliefs, characterizations that were further elaborated by Cooney (1999): isolationist, naive idealist, naive connectionist, and reflective connectionist. Henry was classified as an isolationist, one who tends to have beliefs structured in such a way that beliefs remain separated or clustered away from others. Accommodation is not a theme that characterizes an isolationist. For whatever reason, the isolationist tends to reject the beliefs of others at least as they pertain to his/her own situation. (Cooney, 1999, p. 172)

The naive idealist “tends to be a received knower in that, unlike the isolationist, he/she absorbs what others believe to be the case but often without analysis of what he/she believes” (Cooney, 1999, p. 172). The last two characterizations are considered to be connectionist positions that emphasize reflection and attention to the beliefs of others as compared to one’s own. “The naive connectionist fails to resolve conflict or differences in beliefs whereas the reflective connectionist resolves conflict through reflective thinking” (Cooney, 1999, p. 172). Cooney et al. (1998) considered Greg a reflective connectionist, and they suggested that this position set the stage for the preservice teachers to emerge into reflective practitioners (Schön, 1983). The researchers concluded that a goal of teacher education was to produce reflective connectionists, but they could not conceive of some of the students in their study moving in that direction. They suggested that understanding the belief structures of students might help teacher educators make decisions about effective activities.

We submit that understanding the belief structures as evidenced by these four young teachers can provide a basis for conceptualizing the nature and activities of teacher education programs. The inculcation of doubt and the posing of perplexing situations would seem to be central to the promotion of movement from being a naive idealist or even isolationist to being a connectionist. Inciting doubt and making the previously unproblematic problematic can have significant impact on a person’s world and lead to varied and perhaps unsettling responses. It is not enough to make mathematics and teaching problematic for teachers. We need to understand the effect of this inculcation of doubt and also understand the kind of
support teachers need to make sense of it. (Cooney et al., 1998, pp. 330–331)

The authors ended by suggesting that the research community needs to develop a better understanding of the linkages between what teacher educators provide to preservice teachers and the effect these activities have on the preservice teachers’ belief systems.

Mewborn (1999) investigated reflection and its effect on four preservice elementary school teachers in a field-based mathematics methodology course. She noted that reflection and action must go together, and she used the terms *verbalism* and *activism* to refer to reflection without action and action without reflection, respectively. Mewborn created a community of learners among four preservice elementary school teachers, herself, and a fourth-grade teacher in whose class they observed and worked. Over the course of the 11-week experience, the preservice teachers observed and discussed lessons in the fourth-grade class, observed and discussed videotaped examples of children’s thinking, interviewed children and discussed the experience, and taught small groups of children and discussed the experience. Mewborn studied what the four prospective teachers found problematic, and she classified the concerns into four somewhat-ordered categories of issues related to classroom management and the classroom context; issues related to teaching mathematics; issues related to children’s mathematical thinking; and, to a lesser extent, mathematics content. She found that the preservice teachers were able to engage reflectively, but they needed support in learning to observe mathematics teaching and learning environments and, in particular, in developing an internal locus of authority for pedagogical ideas. Initially, the students held an external locus of authority whereby they would turn to the classroom teacher or to Mewborn for answers. Later, they moved to a second stage in which the locus of authority was both internal and external, and during this stage, they moved beyond asking questions and began to state problems and generate hypotheses. During the third stage, which was reached while the students immersed themselves with children’s mathematical thinking, the locus of authority was internal, and the preservice teachers became fully reflective when they not only presented hypotheses but also searched for evidence by which to test their hypotheses. Mewborn identified five elements of the design of the field experience that she considered critical to successfully helping the students become reflective about teaching and learning mathematics: (a) The field experience was approached from an inquiry perspective; (b) the students, teacher, and teacher educator participated as a community of learners; (c) the community was nonevaluative; (d) the preservice teachers were given time to reflect; and (e) the field experience was subject specific.

Many have noted the need to support teachers in their becoming reflective practitioners, but we educators must be careful lest we think that this support, alone, will be sufficient to create major change in our schools. The organization and structure of schools present a formidable challenge to the kind of change called for in the reform, even if we create reflective preservice teachers. Gregg (1995), in a case study of a beginning high school teacher’s acculturation into the school mathematics tradition, concluded that encouraging teachers to view their current practices as problematic may not be sufficient to promote reform, because the school culture has provided explanations designed to institutionalize the problems as matters to be coped with, not resolved. Gregg noted, for example, that the school mathematics tradition has separated the acts of teaching and learning so that a teacher can be considered to have met the obligation to teach even though the students may not have met their obligation to learn. This view is perpetuated by the commonly accepted conception of ability as capacity, because the teacher cannot be accountable for failing to successfully teach students who are unable to learn. Gregg provided the example of designating a test as “too hard” or certain questions as “unfair” because many students scored poorly on them, so that teachers can blame the test, even if they constructed it, instead of blaming their instruction; blaming their instruction might lead to questioning other of their fundamental beliefs and practices related to the teaching and learning of mathematics. Gregg doubted that “a teacher in the school mathematics tradition would question or reflect on the taken-as-shared beliefs and practices of this tradition as a result of students’ poor test performance” (p. 463). He concluded that to be successful, a reform movement “must challenge the classroom, school, and societal obligations that characterize teachers’ roles in the school mathematics tradition” (p. 463). In other words, Gregg raised the prospect that looking to change teachers’ beliefs and knowledge within the current school mathematics tradition will, for the vast majority of teachers, fail.

*Two Obstacles to Changing Beliefs: Teachers’ Caring and Teachers’ Belief That Teaching is Telling*

Cooney (1999) suggested that to help teachers change their beliefs, teacher educators must find ways to support teachers to become more reflective while they unravel their notions about teaching and rebuild them in a rational way. But he identified two
major roadblocks to attaining this effect: teachers’ overwhelming propensities to be caring teachers, and teachers’ orientations toward teaching-as-telling. Cooney recommended supporting preservice teachers while they transform their notions of caring and telling into notions that encourage attention to context and reflection by helping them move beyond their concerns for children’s personal comfort levels to consider students’ intellectual needs and by integrating content and pedagogy in mathematics teacher education to support the transition of teachers toward reflection. These two obstacles are related, because the challenges teachers face when they change their role in the class affect their perceived sense of their effectiveness in meeting the needs of their students.

Nearly all mathematics educators seem to agree that effective teaching is more than telling, a view reflected in the U.S. national reform documents (NCTM, 1989, 2000). However, even expert instructors grapple with the difficulties of developing and maintaining a view of teaching as more than telling. In a study during which a researcher and a teacher collaborated on the curriculum that they each taught to separate ninth-grade mathematics classes, Romagnano (1994), the researcher, described difficulties he experienced when he encouraged his students to solve problems independently without first being told what to do, whereas the teacher with whom he was working avoided such difficulties by being more directive with the class. Neither Romagnano’s nor the other teacher’s approach was without challenges, because whereas the teacher’s approach removed much of the conceptual mathematics for the students, Romagnano’s approach often led his students to frustration so that they avoided participating in classroom activities. He concluded that the differences between his and the teacher’s views of mathematics, of how it is learned, and of the role of the teacher in the process led to his facing the Ask Them or Tell Them dilemma, whereas the teacher faced no such dilemma.

Chazan and Ball (1999) described challenges they experienced when trying not to “tell” while teaching a high school algebra lesson and a third-grade class, respectively.

Chazan described how in the algebra lesson he had taught to a lower track high school class the students disagreed on whether, when determining the average raise given to employees of a company, they should include a person who received no raise. When the discussion became heated and several students became excited, Chazan found himself wondering how to strike a balance between his desire for his students to develop greater confidence in their abilities to discuss their way through a mathematical problem and his desire for the students to reach some common ground from which they could stand back and reflect upon the mathematics. Chazan wondered whether, or how, he should intervene. During a lesson Ball taught to third graders, the students seemed to agree on an incorrect way of labeling the number line, and when her attempts to induce disequilibrium were unsuccessful, she was left wondering how best to support the students’ learning. Whereas Chazan was concerned that his students might have engaged in an unproductive disagreement, Ball was concerned that her students were engaged in an unproductive agreement. Chazan and Ball were aware that telling can take different forms and that the type of telling must be considered with the context. For example, a teacher’s telling through demonstration of a mathematical procedure for the students to practice is different from a teacher’s telling by attaching conventional mathematical terminology to a distinction the students are already making. In spite of this knowledge and their vast experience teaching without simply telling, Chazan and Ball, two expert teachers, faced difficult challenges in their lessons. They warned that the commonly accepted reform exhortation not to tell is a statement about what not to do, but it contributes nothing toward supporting teachers while they consider what they might do. They described three factors they balanced in considering how to proceed during the class discussion: the mathematics under consideration, the nature and direction of the class discussion, and the social and emotional climate of the class.

If expert teachers face difficulties with teaching without telling, one could expect that many teachers struggle even more. Smith (1996) suggested that a teacher’s efficacy, originally defined as “the extent to which the teacher believes he or she has the capacity to affect student performance” (Tschanen-Moran, Hoy, & Hoy, 1998, p. 202), is being challenged by a reform movement that calls for teachers to approach teaching from an entirely different perspective. Smith pointed to the existence of a core set of beliefs that characterize how most mathematics teachers in the United States view mathematics, mathematics learning, and mathematics teaching. For most teachers, school mathematics is a fixed set of facts and procedures for determining answers, and the authority for school mathematics resides in the textbooks, with the teacher serving as the intermediary authority between the textbooks and the students. For them, teaching mathematics requires telling, or providing clear, step-by-step demonstrations of these procedures, and students learn by listening to the teachers’ demonstrations and practicing these procedures. These beliefs support teachers’ senses of efficacy, because the con-
exception of mathematics as a fixed set of facts and procedures restricts the content teachers must know; thus they can think that they have mastered the necessary content. The notion of teaching-as-telling provides a detailed but attainable model that teachers can hope to master. Telling students how to perform procedures also supports teachers’ senses of efficacy, because the conventional nature of procedures is such that students cannot be expected to know them until the teacher shows them, and so the students’ successes in mastering the procedures can be attributed to the teacher. Furthermore, the belief that students learn by listening, watching, and practicing clearly specifies to the students and the teacher what the students should be doing in class.

Teachers of mathematics, like all teachers, need to believe that their teaching actions have significant causal impact on their students’ learning. Telling, irrespective of its pedagogical strengths and weaknesses, provides a clear model for teachers of mathematics to develop a sense of efficacy. (Smith, 1996, p. 393)

Smith went on to suggest that the reform movement has substantially changed what teachers must know and do to succeed in helping their students learn. How can teachers who have learned in a telling environment achieve a sense of efficacy consistent with the reform? Two things must happen. First, teachers must learn about the reforms and attempt to change their practices accordingly; a growing body of research on teachers’ knowledge, beliefs, and practices in the context of reform has addressed this need. Second, according to Smith, teachers’ success in making the changes to their practices must bring about a re-conceptualization of their senses of efficacy, and this transformation has received no explicit focus. Now, nearly a decade since Smith’s appeal, I think that the movement to consider teachers’ identity, to be discussed later in the chapter, might address this call for research into teacher efficacy.

Summary
Researchers, teacher educators, and professional developers are generally not interested in just measuring teachers’ beliefs; they want to change beliefs. The research indicates that the beliefs teachers hold affect their views of the instruction they observe, complicating attempts to answer the question of whether beliefs’ change precedes or follows change in instruction. Determining which changes first is less important than supporting teachers to change their beliefs and practices in tandem, and reflection is the critical factor for supporting teachers’ changing beliefs and practices. Through reflection, teachers learn new ways to make sense of what they observe, enabling them to see differently those things that they had been seeing while developing the ability to see things previously unnoticed. While teachers are learning to see differently, they challenge their existing beliefs, leading to associated beliefs change. Change in teachers’ beliefs may not lead to change in their practices, or vice versa, but I conjecture that the most lasting change will result from professional development experiences that provide teachers with opportunities to coordinate incremental change in beliefs with corresponding change in practice. Teacher educators and professional developers must better understand not only what beliefs teachers hold but also how they hold them, because the ways that teachers hold their beliefs affect the extent to which existing beliefs can be challenged. Two impediments to changing teachers’ beliefs are concern for the well-being of children that often inhibits teachers’ willingness to challenge students and difficulty in overcoming the classroom challenges that derive from moving beyond their role as the teacher as one whose job it is to tell students how to be successful.

TEACHERS’ BELIEFS, PART II: FOUR AREAS OF RESEARCH

Three major areas of research on teachers’ beliefs are their beliefs about students’ mathematical thinking, about curriculum, and about technology. These three areas are important because of their potential role in changing teachers’ beliefs. One other area I briefly review in this section is teachers’ beliefs about gender.

Teachers’ Beliefs Related to Students’ Mathematical Thinking

For more than 25 years, mathematics educators have gained substantial knowledge about children’s mathematical thinking within specific content domains, and this research has served as an important base for supporting professional development of prospective and practicing teachers (Grouws, 1992; National Research Council, 2001). Prospective elementary school teachers care fundamentally about children, but not necessarily about mathematics (Darling-Hammond & Sclan, 1996), so helping prospective teachers learn about children’s mathematical thinking has been an important aspect of teacher development. Likewise, professional development based on children’s thinking can help teachers create rich instructional environments that promote mathematical inquiry and understanding, leading to docu-
mented improvement in student performance (S. M. Wilson & Berne, 1999). I now turn to recent research on teachers’ beliefs about children and the relationship of teachers’ beliefs about mathematics, teaching, and learning to teachers’ understanding of children’s mathematical thinking.

**Elementary School Teachers’ Beliefs Related to Students’ Mathematical Thinking**

Fennema et al. (1996) conducted a longitudinal study of 21 primary-grade teachers over a 4-year period during which the teachers participated in a Cognitively Guided Instruction (CGI) (Carpenter, Fennema, Franke, Levi, & Empson, 1999) professional development program focused on helping teachers understand the development of children’s mathematical thinking by interacting with a specific research-based model. They studied changes in the beliefs and in the instruction of the teachers and growth in the children’s learning. Data sources for measuring teachers’ beliefs were comprised of audiotape transcriptions of classroom observations, interviews, Mathematics Belief Scale (Fennema et al., 1990) scores, and field notes from many informal interactions with the teachers. As noted previously, the beliefs survey was comprised of 48 Likert-scale items assessing four subscales: Role of the Learner, Relationship Between Skills and Understanding, Sequencing of Topics, and Role of the Teacher. The Role of the Learner subscale measures the belief that children are able to construct their own knowledge instead of being merely receivers of knowledge. The Relationship Between Skills and Understanding subscale measures the belief that skills should be taught in relation to understanding and problem solving rather than in isolation. The Sequencing of Topics subscale measures the belief that children’s natural development, rather than the logical structure of formal mathematics, should guide the sequencing of topics. The Role of the Teacher subscale measures the belief that instruction should facilitate children’s construction of knowledge rather than consist of teachers’ presenting materials.

The researchers engaged in an extensive 2-year process to define levels of teachers’ instruction and levels of teachers’ beliefs. These two multilevel scales (that later were collapsed into one) were scored on a scale of 1, 2, 3, 4A, and 4B. The belief levels were based on the extent to which teachers held the belief that children can solve problems without instruction and the belief that what the teachers know about their students’ thinking should inform the teacher’s curricular decisions. Level 1, the lowest level, reflected the belief that children cannot solve problems without instruction. Level 2 teachers were struggling with the belief that children can solve problems without instruction. Level 3 reflected the belief that children can solve problems without instruction, but the teacher believed only in a limited way that his or her students’ thinking should be used to make instructional decisions. The highest level was divided into Levels 4A and 4B; Level 4A reflected the belief that children can solve problems without instruction in specific domains and that teachers should use knowledge of their students to guide interactions with them; Level 4-B reflected the belief that children can solve problems without instruction across mathematics content domains and that teachers should use knowledge of their individual children’s thinking to inform their decision making, regarding both interactions with the students and curriculum design.

The researchers used the same Levels (1, 2, 3, 4A, and 4B) to characterize levels of cognitively guided instruction. Level 1, the lowest level of cognitively guided instruction, was characterized by the teacher’s providing few or no opportunities for students to engage in problem solving and share their thinking. The highest level, Level 4-B, was characterized by the teacher’s providing opportunities for children to engage in various problem-solving activities and eliciting children’s thinking, attending to the thinking children shared, and adapting instruction according to what was shared. That is, at Level 4B, instruction was driven by the teacher’s knowledge of individual children.

Results (Fennema et al., 1996) indicated that over the course of the longitudinal study, 18 of the 21 teachers increased their levels in terms of sophistication of their beliefs, and 18 of the 21 teachers increased their levels in terms of cognitively guided instruction. Overall, 17 teachers increased on both levels. At the beginning of the study, 2 teachers were categorized as holding beliefs at Level 4 and 2 teachers were categorized for instruction at Level 4 (I do not know whether these were the same teachers), and by the end of the study, 11 of the 21 teachers were categorized as holding beliefs at Level 4, and 7 of the 21 teachers were characterized for instruction at Level 4. The authors noted that in spite of the obvious relation between levels of instruction and levels of belief, they were unable to compare relationships, because a teacher’s beliefs and instruction were not always categorized at the same level, and they found no overall pattern as to whether a teacher’s level was higher for beliefs or for instruction. In short, the researchers found no consistent relationship between change in beliefs and change in instruction. Regarding the 17 teachers whose final ratings were higher than their initial ratings on both beliefs and instruction, 6 teachers’ beliefs changed first, 5 teachers’ instruction changed first, and the change
occurred simultaneously for 6 teachers. The authors stated that the exploration of the relationships between change in beliefs and change in instruction was complex and could be understood only in terms of specific teachers. In stating their conclusions, the authors noted that although many factors contributed to the teachers’ change, the two that seemed most critical were that the teachers learned the specific research-based model about children’s thinking and used that model in their classrooms.

One other important finding in this study (Fennema et al., 1996) was that “gains in students’ concepts and problem-solving performance appeared to be directly related to changes in teachers’ instruction” (p. 430). The authors attributed the changes in the students’ achievement to the facts that teachers provided the students more opportunities to engage in problem solving, encouraged their students to share their thinking, and adapted instruction to the problem-solving abilities of the students. Although the authors expected that more problem-solving emphases by the teachers would lead to an increase in problem-solving performance by their students, they found striking that the shift in emphasis from skills to concepts and problem solving did not result in a decline in students’ computational-skill performance.

In a follow-up to the original CGI study (Peterson, Fennema, Carpenter, & Lof, 1989), Knapp and Peterson (1995) conducted phone interviews with 20 of the original 40 CGI teachers 3–4 years after their initial introduction to CGI; they found that all but one of the teachers reported using CGI in some way in their current classroom practices. They identified three patterns of CGI use: (a) 10 teachers reported “steadily, if often gradually, developing their use of CGI” (p. 47) and currently using it as the main basis for their instruction; (b) 6 teachers reported having used CGI more extensively when they first learned about it but currently using it only supplementally or occasionally; and (c) 4 teachers reported having never used CGI more than supplementally or occasionally. Knapp and Peterson found that “the relationship between changes in teachers’ beliefs and their classroom practices was often interactive, and in many cases, beliefs seem to have followed practice” (p. 61). They reported that the teachers’ beliefs in CGI principles began to develop during the workshop but deepened over time when their students generated solutions to complex mathematical problems.

In a longitudinal study of 496 German elementary school students in 27 classrooms, Staub and Stern (2002) also found a relationship between teachers’ beliefs as measured by the CGI Belief Scale (Fennema et al., 1990) and students’ achievement. Staub and Stern found that students of third-grade teachers who scored higher on the CGI Belief Scale displayed higher achievement in solving mathematical word problems than did students of third-grade teachers who scored lower on the CGI Belief Scale. Furthermore, as in the study by Fennema et al. (1996), they found that the students of teachers who scored higher on the CGI Belief Scale did no worse on mathematics-facts achievement than the students of teachers who scored lower on the survey. In fact, the authors concluded, “Teachers with a cognitive constructivist view of learning tend to be more successful in fostering math fact achievement” (Staub & Stern, 2002, p. 354).

**Preservice Elementary School Teachers’ Beliefs Related to Students’ Mathematical Thinking**

Vacc and Bright (1999) investigated the effects that introducing Cognitively Guided Instruction had on preservice elementary school teachers’ changing beliefs about teaching and learning and on their instructional use. Thirty-four preservice teachers enrolled in a 2-year undergraduate cohort for preservice teachers participated in the 2-year study starting with the beginning of the students’ junior year when they began the professional development program and continuing through the end of student teaching at the end of their senior year. On four occasions all students completed the CGI Belief Scale (Fennema et al., 1990); Vacc and Bright (1999) also conducted eight on-site, formal observations of each prospective teacher. In-depth case studies of two students were conducted, and data for these case studies comprised reflective journal entries written during the mathematics methods course and student teaching, videotapes of four mathematics lessons conducted during the student teaching, and three open-ended interviews. Vacc and Bright applied the Levels of Instruction framework and Levels of Belief framework used by Fennema et al. (1996) for determining the levels of the two student teachers.

Results of the study (Vacc & Bright, 1999) indicated that although the preservice teachers’ beliefs-scale scores changed little during the first year of the 2-year program of professional course work, which included a 10-hour/week internship in a professional development school, the scores increased significantly during the third semester during their enrollment in the mathematics methodology course, about one third of which was devoted to instruction related to CGI. The students’ belief-scale scores continued to increase significantly during the fourth semester during their student-teaching experiences. On the basis of their analysis of the case-study data, Vacc and Bright categorized one of the two preservice teachers at the end of
student teaching at Level 3 for belief and Level 3 for instruction; that is, they found that she believed that children can solve problems without instruction and, in a limited way, that children’s thinking should be used to make instructional decisions; also, she was putting these beliefs into practice. The other teacher was categorized at between Levels 2 and 3 for belief and at Level 2 for instruction; therefore, she was providing limited opportunities for her children to engage in problem solving or share their thinking, and she was, in a limited way, eliciting and attending to children’s thinking. The authors concluded that counter to previous research indicating that preservice teachers’ beliefs are resistant to change, this study provided evidence that preservice teachers’ beliefs did change. The researchers were cautious about drawing conclusions regarding either the depth of the changes in beliefs or the contributions of various factors to the change. They concluded, “The data indicate the possibility that intensity of experience and a focus on children’s thinking in the mathematics methods course may be keys for helping preservice teachers change their views” (p. 108).

Ambrose (2004) studied the beliefs of prospective elementary school teachers engaged, in Southern California, in an early field experience linked to a mathematics course. The purpose of the early field experience, described by Philipp, Thanheiser, and Clement (2002), was to motivate students to recognize the importance of learning mathematics by exploring the complexities involved when interviewing and tutoring elementary school children. In her theoretical framework for beliefs, Ambrose (2004) considered two primary sources for beliefs: emotion-packed experiences and cultural transmission. An example of an emotion-packed experience, described later in this chapter in the section on affect, is the prospective elementary school teacher Jo’s negative childhood experiences with timed multiplication tests (Walen & Williams, 2002). The second source of beliefs mentioned by Ambrose (2004), cultural transmissions, might be thought of as the hidden curriculum in which people are engaged every day; beliefs created as a result of these experiences are held at a subconscious level. Examples of hidden curricula at the secondary school level were provided by Schoenfeld (1988): He found that a “well-taught” high school geometry course perpetuated the views that (a) all problems can be solved in just a few minutes; (b) the answer is what counts; (c) students are passive consumers of other people’s mathematics; and (d) the world of deductive geometry is separate from the world of constructive geometry. Ambrose (2004) noted that beliefs have a filtering effect on people’s thinking, so that, through time, beliefs become more resilient. Ambrose provided the example shared by a colleague, who, in an effort to help her prospective elementary school teachers understand that children come to school with a great deal of knowledge on which teachers can build, had them interview kindergarten children during the second week of school. These preservice teachers were amazed at some of the problem-solving skills the students demonstrated, but instead of taking these skills as evidence that students bring much informal knowledge to school, some mentioned how impressed they were with how much the teacher had taught the children during the first week of school! The students’ belief that one’s mathematical knowledge is a result of school learning colored their view of the experience in such a way that they misinterpreted it.

Ambrose (2004) presented four mechanisms by which students’ beliefs systems might change; the first two of these were mentioned as mechanisms for creating new beliefs:

(a) they can have emotion-packed, vivid experiences that leave an impression; (b) they can become immered in a community such that they become enculturated into new beliefs through cultural transmissions; (c) they can reflect on their beliefs so that hidden beliefs become overt; (d) they can have experiences or reflections that help them to connect beliefs to one another and, thus, to develop more elaborated attitudes. (p. 95)

Ambrose (like Cooney, 1999, mentioned earlier) noted that the two beliefs central to prospective teachers related to teaching as explaining and teaching as caring about students; the study was designed to provide opportunities for the prospective teachers to challenge their belief about teaching-as-telling by building upon their caring of children.

The 15 prospective elementary school teachers who participated in the study were simultaneously enrolled in a mathematics course and in a Children’s Mathematical Thinking Experience. Data sources were surveys, interviews, prospective teachers’ written work, field notes, and a computerized beliefs survey completed at the beginning and end of the semester. Ambrose (2004) conducted an intense analysis of one student, Donna, and compared the emergent themes from Donna’s case with experiences of the other 14 students. Ambrose learned that for Donna the most powerful experiences were her three sessions working with a fifth-grade student on fractions, assessing the child during the first session and trying to expand the child’s fraction understanding during the second and third sessions. The second session was a high point for most of the prospective teachers, because they felt
confident that they had successfully taught the children something about fractions, but during the third session, most were surprised to discover that the children with whom they had previously worked experienced difficulty with the same fractions topic the prospective teachers thought they had taught during the previous lesson. The language the preservice teachers used captured the intensity of the experience; they described feelings of being stoked or excited by the events of the second session, whereas words they used to describe the third session included shocked, disappointed, and aggravating. Ambrose concluded that for many of the preservice teachers, their initial beliefs were undifferentiated and were thus held in an unreflective manner. The intense experiences in working with the students resulted in their beliefs’ becoming more salient to them. Whereas many teacher educators hold belief reversal as the goal for work with prospective teachers, Ambrose found that instead of letting go of their old beliefs, the preservice teachers in her study held onto their old beliefs while forming new ones, and hence the preservice teachers engaged in actions with their children that reflected a combination of beliefs about mathematics teaching and learning. Ambrose suggested that the students’ experiencing a “failed teaching experiment” may have been an important component in their learning experience, and she concluded by stating, “Providing preservice teachers with intense experiences that involve them intimately with children poses a promising avenue for belief change” (p. 117).

D’Ambrosio and Campos (1992) studied five preservice teachers enrolled in their senior year of the mathematics program at the Catholic University of São Paulo, Brazil, and, like Ambrose (2004), they found that providing preservice teachers with opportunities to learn about children’s mathematical thinking led to their reflecting upon conflicting situations that arose. These situations created a state of disequilibrium for the preservice teachers, leading them to question normally accepted instructional practices and generally to develop characteristics of reflective practitioners (Schön, 1983).

**Secondary School Teachers’ Beliefs**

**Related to Students’ Mathematical Thinking**

Most researchers investigating the relationship between teachers’ beliefs and students’ thinking have focused at the elementary school level. Nathan and Koedinger (2000b), however, studied 67 secondary school mathematics teachers from the United States and 35 mathematics education researchers to determine their predictions of problem-solving difficulty for a set of arithmetic and algebra problems. Nathan and Koedinger constructed six types of problems using three presentation formats (story problems, word-equation problems, and symbolic-equation problems), and for each format, they wrote two problems, one with the start unknown and the other with the result unknown. Their previous research had indicated that students found result-unknown story problems and result-unknown word equations to be the easiest; result-unknown equations, start-unknown story problems, and start-unknown word equations of easy-to-medium difficulty; and start-unknown equations to be the most difficult of the problems. On the one hand, Nathan and Koedinger suggested that this research supported the view that the development of students’ verbal reasoning precedes the development of their symbolic reasoning, and they developed a model they named a **Verbal-Precedence Model** to reflect the student data. On the other hand, the teachers incorrectly predicted that the students’ symbolic-reasoning skills would develop first, with word-problem-solving ability developing later, and the researchers referred to the model they developed to reflect the teachers’ view as the **Symbolic-Precedence Model**. On the basis of their data, they concluded, “High school mathematics teachers hold beliefs that cause them to systematically misjudge students’ symbolic- and verbal-reasoning abilities” (Nathan & Koedinger, 2000a, p. 212). The researchers found that in two commonly used mathematics-textbook series the development seemed to match the teachers’ **Symbolic-Precedence Model**, and they suggested that through repeated exposure to textbooks, “teachers internalize the symbolic precedence view as a basis for their predictions of problem difficulty for students” (Nathan & Koedinger, 2000b, p. 181).

In a follow-up study, Nathan and Koedinger (2000a) asked 107 Grade K–12 teachers to rank order the six mathematics problems used in the previous study (Nathan & Koedinger, 2000b); each teacher also completed a 47-item Likert-scale assessment designed to assess each of six constructs related to teachers’ views of mathematics, mathematics instruction, and student learning:

1. Algebraic procedures are “best” for effectively solving mathematical problems.
2. Invented solution methods are effective for solving problems.
3. Arithmetic problems are easier than algebra problems and should be presented first, and mathematics problems presented in words are most difficult and need to appear later in the curriculum (symbolic-precedence view).
4. Students may enter the classroom with valid ways of reasoning, and teachers should encourage invented strategies.
5. Correct answers are more important than reasoning processes.
6. Alternative solution strategies (such as arithmetic, guess-and-check, and other nonsymbolic methods) demonstrate gaps in students’ knowledge.

Results indicated that across grade levels, the teachers reflected recent mathematical reform views, and they tended to disagree with views that challenged reform-based views. The elementary school teachers were most likely and high school teachers were least likely to agree with reform-oriented views. Teachers across all grade levels correctly predicted that result-unknown problems would be easier than start-unknown problems. High school teachers incorrectly ranked symbolic equations as easier than verbal problems, and the rank orders of the elementary school teachers were similar to those of the high school teachers, although elementary school teachers were more likely to rank problems on the basis of algebraic and arithmetic structure than on presentation format. Middle school teachers stood out as a group because their rank orderings were most consistent with the research on students’ performance. That is, “middle school teachers were most accurate in predicting students’ problem-solving performance in contrast to the view held by many high school teachers that symbolically presented problems were easier to solve than verbally presented problems (Nathan & Koedinger, 2000a, p. 226). In their discussion, the authors suggested that middle school teachers, who work with students who have not typically received formal algebra training, may have occasions to observe students’ use of more informal and invented methods, and thereby the teachers may have more opportunities to observe the transition from arithmetic to algebraic reasoning. They suggested, further, that elementary school teachers, who held the strongest reform-based views of learning and teaching, are relatively unfamiliar with students’ algebraic reasoning and may have therefore expected that students would operate with a traditional vision of algebra curriculum and instruction. High school teachers were least aware of the efficacy of students’ invented algebra solution strategies, and the authors suggested that because high school teachers have greater expertise in their content areas, they are personally more distant from their novice students’ difficulties. The authors introduced the term expert blind spot, a feature they attributed to teachers with high levels of content knowledge but low levels of awareness of alternative interpretations for symbolic equations.

Nathan and Petrosino (2003) studied 48 prospective secondary school teachers enrolled in a “nation­ally acclaimed teacher education program” (p. 911) in the United States to test their expert-blind-spot hypothesis, stipulated as follows:

Educators with advanced subject-matter knowledge of a scholarly discipline tend to use the powerful organizing principles, formalisms, and methods of analysis that serve as the foundation of that discipline as guiding principles for their students’ conceptual development and instruction, rather than being guided by knowledge of the learning needs and developmental profiles of novices. (p. 906)

Participants were categorized in one of three ways, on the basis of their subject-matter knowledge: (a) 13 were placed in a “basic math” category because they had not completed a precalculus course; (b) 16 from the general population of teacher education students who were pursuing licensure to teach elementary school who had completed at least one calculus course were placed into the HiMath knowledge group; (c) 19 who were enrolled in a specialized program for mathematics and science majors and had completed many higher level mathematics courses were placed in the MathSci category. Using problems similar to those used in previous studies (Nathan & Koedinger, 2000a, 2000b), Nathan and Petrosino (2003) asked all participants to rank order six problems as they would predict beginning-level algebra students would experience ease or difficulty in solving them. The participants also responded to the 47-item beliefs survey used in the previous study (Nathan & Koedinger, 2000a). Results of the predictions of problem difficulty (Nathan & Petrosino, 2003) indicated that the preservice teachers with more advanced mathematics knowledge (the MathSci and HiMath groups) were far more likely to follow the symbolic-precedence view of algebra development than the preservice teachers with less mathematics knowledge (the Basic Math group). Results of the beliefs survey supported this finding, with a greater percentage of those preservice teachers in the two higher mathematics knowledge groups agreeing that using algebraic formalisms is best for solving complex problems, whereas students in the Basic Math group were more likely to believe that instruction should build on students’ intuitions and invented methods. Nathan and Petrosino (2003) conjectured,

Educators with greater subject-matter knowledge tend to view student development through a domain-centric lens and consequently tend to make judgments
about student problem-solving performance and mathematical development that differ from actual performance patterns in predictable ways. (p. 918)

Summary

Researchers measuring teachers’ beliefs about children’s mathematical thinking have correlated these beliefs to teachers’ instruction and to students’ learning. The Cognitively Guided Instruction group (Carpenter et al., 1999) developed an instrument to measure teachers’ beliefs about children’s abilities to solve problems without instruction and about teachers’ beliefs about the role that knowledge of children’s thinking should play when one makes instructional decisions, and they used this instrument in a study in which they found that teachers’ beliefs were related to instruction reflecting a focus on children’s mathematical thinking, and, furthermore, teachers’ instructional changes were related to gains in students’ understanding and problem-solving performance (Fennema et al., 1996). Knapp and Peterson (1995) found that most of the original CGI teachers who had made instructional changes to focus upon children’s mathematical thinking continued to apply this focus several years later. Staub and Stern (2002) used the same beliefs survey and found a direct link between teachers’ beliefs about children’s mathematical thinking and students’ achievement. Preservice elementary school teachers provided with opportunities to learn about children’s mathematical thinking can change their beliefs, and the intensity of the experience associated with focusing upon children’s thinking was identified as a cause for these changes (Ambrose, 2004; Vacc & Bright, 1999). Although teachers’ beliefs about children’s mathematical thinking have been studied more extensively at the elementary school level than at the secondary school level, Nathan and his colleagues have investigated secondary school mathematics teachers’ beliefs about students’ mathematical thinking. Nathan and Koedinger (2000b) found that the development of students’ verbal reasoning skills preceded the development of their symbolic reasoning but that high school mathematics teachers believed just the opposite, and they coined the term expert blind spot (Nathan & Koedinger, 2000a) to refer to the relationship between teachers’ higher levels of content knowledge and lower levels of awareness of students’ understanding. Nathan and Petrosino (2003) found that the expert-blind-spot characterization also applied to prospective secondary school teachers. Although researchers studying teachers’ beliefs about students’ mathematical reasoning have noted the complex relationship between beliefs and practices, several studies provide evidence that teachers’ beliefs often change as a result of observing children’s mathematical reasoning (Ambrose, 2004; D’Ambrosio & Campos, 1992; Knapp & Peterson, 1995; Vacc & Bright, 1999).

Teachers’ Beliefs Related to or Changed by Use of Mathematics Curricula

Any mathematics class has two curricula: the intended curriculum and the enacted curriculum. The intended curriculum, that which is stated or explicit, is comprised of the materials of a class, including textbooks, curriculum guides, and course descriptions. Although many think of these materials as defining the curriculum, one cannot understand the experience students have in a class without attending to how materials are enacted by a teacher, with students, in a particular context (Ball & Cohen, 1996). Just as the knowledge and beliefs teachers hold about mathematics, teaching, and learning affect the ways in which they enact curricula, so too should we expect that teachers might learn from the curricula they use. That is, teachers not only adapt and change curricula but also are changed by the curricula they use. The relationships between teachers’ beliefs and the innovative curricula they use are addressed in this section.

Lloyd (2002) highlighted two aspects of reform-oriented curricula designed to support teacher learning. First, by focusing upon the exploration of solutions to real-world problems, reform-curricula designers emphasize a more inquiry-oriented approach to mathematics than is found in traditional curricula, and the materials are formatted to support these mathematical and pedagogical differences. Second, reform-oriented curricula include more extensive information for teachers than traditional curricula. For example, they provide historical information, details about different representations, and examples of students’ reasoning. Lloyd (1999, 2002) found that using the reform-oriented Core-Plus curriculum over several years supported a high-school teacher’s learning or relearning of mathematical subject matter. Furthermore, as a result of becoming more familiar with the curriculum and interacting with students, this teacher’s beliefs about the graphical representations of functions became richer and more nuanced.

Collopy (2003) reported on a study designed to investigate two elementary school teachers’ changes in beliefs related to teaching and learning mathematics and their changes in instructional practices resulting from their first year’s use of Investigations in Number, Data, and Space, a reform-oriented curriculum. The two teachers were experienced elementary school teachers working in similar contexts, schools
comprised of a high percentage of at-risk students described as “below average in mathematics.” Both teachers sat on their district mathematics committees, both had recently used a district-mandated traditional Addison-Wesley mathematics textbook, and both had volunteered to pilot the Investigations curriculum.

Collopy (2003) collected data about the teachers’ beliefs and practices in three stages. During the first stage, she collected data on the teachers’ backgrounds and baseline data on their beliefs and knowledge in an extended formal interview and on their initial use of the curriculum through 7 or 8 classroom observations. During the second stage, she focused on investigating the teachers’ decisions about curriculum use and changes in beliefs and practices through six days of observations, in two-day sets, a month apart. During the third stage of data collection, Collopy collected data on teachers’ reflections; use of the curriculum; and beliefs, knowledge, and instructional practices by observing each class for two consecutive weeks during the final weeks of the school year and conducting a final interview of each teacher. Collopy conducted two data analyses: a thematic analysis and a segment analysis. In the thematic analysis of the interviews and field notes, she developed categories of codes to explore the stability, changes, and relationships among the teachers’ beliefs about mathematics, students, pedagogy, curriculum, and themselves as learners and teachers. She presented her interpretations during each final interview; both teachers concurred with her interpretations. In the segment analysis, Collopy tracked the format and focus of each teacher’s practice across the year by coding each observed lesson into segments and coding each segment for length in minutes, instructional format, the teacher’s role and focus, student behavior, and teacher’s expectations for students’ thinking.

From her detailed case studies, Collopy (2003) concluded that the two teachers differed dramatically in what they had learned from the materials and ways they engaged with the materials as a support for learning. Ms. Clark held a tightly integrated set of beliefs about mathematics, student learning, the teacher’s role, the purposes of mathematics instruction, and her own mathematical efficacy, and her instructional practice was consistent with these beliefs. She believed that computational speed and accuracy distinguished successful students from unsuccessful students, that her students needed to learn the rules of mathematics, and that mathematics topics should be presented systematically from easier to harder. Ms. Clark agreed with the emphasis in the Investigations materials on students’ understanding mathematical concepts but not with the definition of understanding as “familiarity with the magnitude of numbers, mathematical relations, and the meaning of mathematical operations and situations” (p. 296); Ms. Clark defined understanding as the memorization and correct execution of standard algorithms. Eventually she became frustrated with the Investigations materials, in which speed and rote memorization were downplayed, and she adapted lessons to compensate, changing their essence by omitting alternative problem-solving strategies and mathematics discussions she considered distractions; her frustration level rose until January, when she shelved the Investigations curriculum and returned to her more traditional curriculum. Collopy’s segment analysis indicated that in Ms. Clark’s use of the Investigations curriculum, 61.8% of the observed time was devoted to procedures and correctness, 35.4% to organizational and management issues, and 2.8% (consisting of 24 minutes during one lesson) to conceptual understanding.

The other teacher, Ms. Ross, approached the Investigations curriculum differently than Ms. Clark, significantly changing her approach to instruction in shifting from her early focus on procedures and correctness (94% of the early observations to 21% for November through May) to conceptual understanding and mathematical reasoning (from no early focus to more than 75% in later observations). Unlike Ms. Clark, whose actions were driven by beliefs about mathematics as a set of facts and procedures that one learns through exposure to direct and clear instruction, Ms. Ross held beliefs about the importance of developing students’ confidence for learning mathematics by building upon their prior knowledge. Ms. Ross saw no clear structure for mathematics, so she relied upon her curriculum materials and objectives for the state’s standardized tests to direct the content of and emphasis in her instruction. Ms. Ross followed the curriculum carefully, reading and using the student notes, and she learned mathematics while challenging her beliefs. For example, in the process of asking students to demonstrate, write, and discuss their ideas, she came, for the first time, to see that problems can be solved in many ways.

Although the two teachers in this study had similar backgrounds, taught in similar situations, and willingly piloted the same reform-oriented curriculum, their enactment of the curriculum provided different opportunities for them to learn, differences Collopy (2003) explained in terms of the beliefs most integral to the teachers’ identities. Ms. Clark’s identity as a teacher and learner of mathematics was centered in her belief that mathematics is a set of rules and procedures that students learn by being shown, incompatible with the beliefs underlying the curriculum, so in
her curriculum adaptation, she lost most opportunities to learn from the materials. The beliefs most integral to Ms. Ross’s identity as a teacher were not about mathematics but, instead, were about developing students’ confidence for learning mathematics and were compatible with those espoused in the curriculum. Thus, she was able to use the materials as they were intended and was, thereby, poised to learn much from their use. Collopy’s findings show that while considering how to build opportunities for teachers to learn from their curriculum, writers must also consider the beliefs held by teachers using the materials.

Remillard and Bryans (2004) extended the work of Collopy (2003) by studying the way the reform-oriented Investigations curriculum supported the learning of urban elementary school teachers in one school over several years. In this study, seven of the eight teachers who volunteered to participate and attend monthly study-group meetings and related research activities attended for two years. During the first year, each teacher was observed 2–4 times and interviewed at least twice; during the second year, six of the seven teachers were observed 7–8 times and interviewed three times. Two of the teachers’ classrooms were designated as focus classrooms, which were observed daily over 2-week periods four times during the second year; each focus teacher was interviewed approximately five times. Data were comprised of audiotapes of the lessons and written field notes, which were later used to complete a predesigned observation instrument. The researchers conducted segment analyses of these data, documenting the time devoted to each task, the teacher’s aim and focus of the task, and ways the students engaged with the task.

Three broad categories of teachers’ use of the Investigations curriculum were identified by Remillard and Bryans (2004): intermittent and narrow, adopting and adapting, and thorough piloting. Teachers whose use was intermittent and narrow used the materials minimally and relied primarily on their own teaching routines and other resources to guide their curricula over the year. Adopting and adapting described use of the materials as a guide for determining what topics to teach and how to sequence them, but with teachers’ adapting the materials to fit their familiar strategies and approaches to teaching. Thorough piloting referred to use of all parts of the curriculum guides. Although teachers whose use was in this category continued to draw upon activities and strategies from other sources, when they used the Investigations curriculum, these teachers followed the lessons as suggested in the guide and studied and grappled with most of the information provided for the teacher.

To address each teacher’s relationship with the Investigations curriculum during the first 2 years of use, Remillard and Bryans (2004) introduced the construct orientation toward curriculum,

a set of perspectives and dispositions about mathematics, teaching, learning, and curriculum that together influence how a teacher engages and interacts with a particular set of curriculum materials and consequently the curriculum enacted in the classroom and the subsequent opportunities for student and teacher learning. (p. 364)

They presented a dynamic model designed to capture the interactive relationships among teachers’ perspectives and beliefs, the use of curriculum materials, the enacted curriculum, and consequent opportunities for teachers to learn.

To illustrate orientation toward curriculum, Remillard and Bryans (2004) described three veteran teachers, Jackson, Reston, and Kitcher, who first used the Investigations curriculum during the study. Jackson held beliefs about mathematics teaching and learning incompatible with those underlying the curriculum, and his curriculum use was categorized as intermittent and narrow. Jackson not only seemed intent upon using his own materials to emphasize procedural skills but also, in his use of the curriculum, failed to use the suggestions to facilitate students’ work and to have students explain their reasoning or discuss their strategies. The other two veteran teachers, Reston and Kitcher, shared views about mathematics teaching and learning that seemed compatible with the curriculum, and so one might have expected their use of the materials to differ from Jackson’s. Kitcher’s use was as intended (thorough piloting), but Reston’s use, in spite of her views of mathematics, teaching, and learning, was similar to Jackson’s (intermittent and narrow). Remillard and Bryans found that Reston’s “general mistrust of published materials trumped any potential compatibility between her beliefs about mathematics and those represented in the book” (p. 366). That is, her orientation toward curriculum, not her beliefs about mathematics, teaching, and learning, explained her skeptical approach to the curriculum and led her to use the curriculum only as a collection of useful activities.

Use by four of the eight teachers in the study was categorized as thorough piloting of the curriculum, and although these four teachers shared beliefs about mathematics, teaching, and learning that were consistent with those reflected in the Investigations curriculum, others in the study did as well, leading the authors to conclude that their shared ideas about the role of curriculum materials, not their beliefs, were
Standards-based curricula are not a panacea, but they can play an important role in fostering reform-based practices. However, without additional support for teachers, the impact of these curriculum materials is likely to be unpredictable and varied. Teachers using curricula like Investigations would benefit from opportunities to explore the content of the materials and have conversations with others about how they use them. (p. 386)

Clearly, curricula may be an important site of learning for practicing teachers. What of prospective teachers? Spielman and Lloyd (2004) attempted to use reform-oriented curricula to change prospective elementary school teachers’ beliefs. They arranged for two classes of mathematics for elementary school to be taught by the same instructor using different curriculum models. The textbook section was taught using a popular mathematics textbook and pedagogical strategies the researchers believed reflected the philosophy of the textbook authors. The class met weekly for 3 hours, and a typical lesson began with a homework review during which groups of students (who were prospective teachers) were assigned to discuss problems then present solutions to the class, followed by the instructor’s comments on the homework review and lecture on the new material. Following the lecture, the instructor showed a short video or students worked, individually or in groups, on a problem or activity at their tables, often with manipulatives available. Student volunteers shared outcomes of their work with the class until a consensus was formed. During the final hour of class, after the instructor made final comments or summary statements, students worked alone or in groups on homework, with the teacher available to answer questions.

The curriculum-materials section used units from two reform-oriented middle school curricula, Mathematics in Context and Connected Mathematics Project, selected to correspond to the mathematical topics in the textbook section (Spielman & Lloyd, 2004). Whenever possible, instructional decisions were made to support the intent of the authors of the curricula. For example, the curricula materials rarely offered rules or solutions to sample problems, and the instructor refrained from providing rules or solutions. A typical lesson began, as in the textbook section, with the students’ reviewing homework by working in groups and sharing their solutions. However, instead of providing solutions or explanations when students had questions, the instructor referred the students to other students for support, often redirecting or rephrasing questions. During the lively homework discussions, students responded to one another’s questions without being called upon, and although the students expressed frustration with the instructor’s refusal to offer rules, procedures, or solutions, “the students appeared to become accustomed to this type of classroom” (p. 35). After the homework review, the students worked together at their tables on problems or activities, and they would occasionally watch video. Although the videos were the same for both classes, the problems and activities differed both in type and presentation. In the curriculum materials section, no instructor presentation or lecture preceded work on problems, and the instructor’s role was to clarify and pose questions. During the last hour of the class students worked on their homework, but instead of answering students’ questions, the instructor posed other questions designed to facilitate students’ finding their own solutions.

Students completed a Likert-scale teaching-beliefs survey and a mathematics-content instrument as pretests and posttests. No between-group differences were found on the content test, but differences were found on the beliefs survey. The students in the textbook section saw the instructor as more of a classroom authority than did the students in the curriculum-materials section. When asked whether the textbook, other students, or the instructor had been most beneficial to learning, only about 10% of the students in either group selected the textbook, but more than 68% of the students in the textbook section selected the instructor whereas more than 82% of the students in the curriculum-materials section selected other students. A paired-sample t-test comparing students’
pretest and posttest responses on the percentage of time students thought should be devoted to teacher lecture and explanation or to group work and discussions showed significant differences for the students in the curriculum-materials group, but no significant differences were found for students in the textbook section. Over the course of the semester, the amount of time students in the curriculum-materials group thought should be devoted to teacher lecture and explanation decreased (from 30.6% to 22.2%) whereas the amount of time they thought should be devoted to group work and discussion increased (from 54.1% to 63.7%). Spielman and Lloyd (2004) concluded that prospective elementary school teachers’ beliefs about textbooks, teaching, and learning changed as a result of using reform-oriented middle school textbooks to learn mathematics.

Summary

Several mathematics curricula have been designed to promote a reform-oriented approach (NCTM, 2000; NRC, 2001) to mathematics teaching and learning. The curriculum designers of these materials recognized the role that curriculum might play in supporting teachers’ learning, and they included instructor-support materials designed to provide guidance for classroom instruction. These curricula are said to speak to teachers, rather than through them (Remillard, 2000). As important as curriculum materials are for supporting teachers in the classroom, assuming that teacher-proof curriculum can, or should, be used would oversimplify the matter; the research indicates that if teachers’ beliefs about mathematics, teaching, and learning are not consistent with the beliefs that serve as the foundation of the reform-oriented curriculum, the teachers do not use the materials as they were intended (Collopy, 2003; Remillard & Bryans, 2004). Holding particular reform-oriented beliefs is a necessary, but not sufficient, condition for teachers to use a curriculum as intended, because Remillard and Bryans (2004) found that another factor, teachers’ orientation toward curriculum, played a significant role in curriculum use even for teachers who held reform-oriented beliefs about mathematics, teaching, and learning. According to Collopy (2003), a teacher who viewed mathematics as a set of procedures and rules was less open to implementing the curriculum as intended than was the teacher who lacked such a clear structure for mathematics. The less experienced teachers who had yet to develop classroom routines for teaching mathematics were more likely than the experienced teachers to read and learn from the supportive materials in a study by Remillard and Bryans, indicating that providing these curricular materials to teachers early in their careers may be a promising means of supporting their growth as teachers. Spielman and Lloyd (2004), studying prospective teachers enrolled in a course of mathematics for elementary school, found that prospective teachers need not yet be in their own classrooms to accrue the educational benefits of reform-oriented curricula; beliefs of students in a course based on a reform-oriented curricula changed more than the beliefs of students who used a conceptually oriented college textbook.

Teachers’ Beliefs About Technology

Prospective teachers believe that computers are important in education, and they want to use them in their preservice programs, but except at a few institutions, students do not learn to use technology effectively (Willis & Mehlinger, 1996). This failure to prepare teachers is reflected in the fact that elementary school teachers generally do not like to teach with computers, and when they do use them, they most commonly do so for drill and practice activities (Cummings, 1998). Using a Likert-scale instrument to survey 33 elementary school teachers about their uses of technology, Cummings found that although most teachers thought that they possessed expertise about using computers, they cited time spent preparing for computer instruction as the greatest barrier to computer use.

Schmidt (1998) investigated beliefs of teachers in Grades 4–6 about the use of calculators and concluded that they reflected their more traditional views about mathematics. In a follow-up study, Schmidt (1999) provided the teachers with an in-service during which they spent one week using calculators to explore fifth- and sixth-grade concepts and applications. After the in-service, during which teachers kept reflective journals; evaluated teaching materials; read and discussed journal articles about a variety of topics, including research about calculators; and participated in and led discussions, the teachers continued their professional development by engaging in action-research projects and sharing the results with colleagues, attending the state mathematics conference, and cohosting a conference for teachers and principals about using calculators. Before and after the in-service, 32 teachers completed a two-part questionnaire: In the first part, they gave personal and professional characteristics and answered three open-ended questions related to what would facilitate the integration of calculators into mathematics, what most hindered the use of calculators in mathematics, and how students in their classes currently used calculators. In Part 2, comprised of 29 Likert-items, respondents provided one of six re-
sponses (strongly agree, agree, slightly agree, slightly disagree, disagree, and strongly disagree) to items such as “If calculators are used in school mathematics programs, students no longer need to know paper-pencil computing techniques” (Schmidt, 1999, p. 24), a statement with which the teachers strongly disagreed on both administrations of the questionnaire. On another item, “Calculators are tools that allow students to focus more attention on mathematics concept development and understanding” (p. 25), the teachers strongly agreed on both the pretest and posttest. After finding little change in the teachers’ beliefs about calculator use as a result of the in-service, Schmidt suggested that the “teachers’ perspective and philosophy of teaching and learning mathematics constrained their beliefs about calculators” (1999, p. 32).

Wiegel and Bell (1996) investigated the integration of computers into six mathematics content classes for prospective elementary school teachers—three sections each of the first and second courses. Data for the first course consisted of precourse surveys and a topic-specific survey, part of a required but ungraded assignment, for which students wrote about their attitudes and beliefs about mathematics and the integration of computer activities into the mathematics content course. Data for the second course consisted of one essay, weekly reflections from all students, and anecdotal comments collected during the computer labs. Used in the first mathematics course were two interactive microworlds, one for geometry and the other for fractions, created as part of a research grant directed by Les Steffe and John Olive. The students in the second mathematics course used a spreadsheet, a probability microworld, LOGO, and Geometer’s Sketchpad. The authors concluded that the students and instructors all rated the overall experience positively, and that the classroom atmosphere, which was tense at the beginning of the semester, seemed to relax when students used computers, working in small groups and communicating with one another. The students reported appreciating the opportunity to use the computer, an activity they came to see as a welcome change in the class routine. Although the authors of this mid-1990s’ study found that many of the students needed to learn to operate computers and manipulate programs and that often the students’ attention to the basic mechanical operation of the computer detracted from their learning the mathematics, college students today would have more expertise with computers. Also, because their computer work was to be ungraded, the students wanted to stop using the computers toward the end of the semester to prepare for the paper-and-pencil items they expected on the final examination.

Tharp, Fitsimmons, and Ayers (1997) conducted a study designed to determine whether providing teachers with instruction on the use of graphing calculators would have an effect on the teachers’ beliefs about computers or on the teachers’ instruction. Five 3-hour, monthly in-services designed to help teachers integrate graphing calculators into their instruction were provided to 261 Grades 6–12 teachers (168 mathematics teachers, 72 science teachers, and 21 teachers of other subjects). Topics covered during these in-services included solving linear equations; studying mathematical modeling, exponential growth, projectile motion, and parametric equations; and connecting multiple (graphic, tabular, and numeric) representations. Teachers watched videotaped lessons offered by experienced teachers using calculators in their classes. All the teachers were required to provide instruction using graphing calculators in their own classrooms. Data included pre and post questionnaires to assess how teachers’ attitudes toward graphing calculators and mathematics shifted because of the instruction and the use of technology. In journals, teachers described their initial use of the graphing calculators during instruction. Likert-scale items, such as “Calculators should ‘only’ be used to check work,” and “When doing mathematics, it is only important to know how to do a process and not why it works,” were used to measure the teachers’ views of calculator use and mathematics learning, teachers’ access to calculators, teachers’ efficacy, student efficacy, and administrative support. Results indicated that teachers’ beliefs about the use of calculators changed significantly to favor using calculators as an integral part of instruction. A significant positive correlation was found between teachers’ views of mathematics and their views on the use of calculators in the classroom, with those teachers holding more rule-based views of mathematics tending to hold the view that calculators do not enhance instruction and those with less rule-based views of mathematics more likely to view calculators as integral for mathematics and science instruction. After coding and analyzing all the teachers’ journal entries, the authors found that although all the teachers attempted to integrate the use of the graphing calculator into their instruction, the rule-based teachers tended to quickly return to limiting the amount and type of calculator use in their classrooms. Whereas the rule-based teachers tended to focus upon the students’ emotional reactions, the non-rule-based teachers were more likely to concern themselves with the students’ conceptual understanding and thinking. The authors concluded
that technology can be used to support changes in teachers' instruction, followed by changes in teachers' beliefs.

Fleener (1995) used a Likert scale to measure the attitudes toward the use of technology held by 233 elementary, intermediate, and high school classroom teachers and by 78 preservice teachers enrolled in a mathematics methodology course for K–3 teachers. Items on the 29-item scale included “Calculators make mathematics fun” and “Students should not be allowed to use calculators until they have mastered the concept.” Fleener’s previous research had shown that a critical issue (which she labeled the mastery issue) that divided teachers related to whether they believed that students should be allowed to use calculators prior to having mastered the concept. In the 1995 study, Fleener investigated whether teachers with different views on the mastery issue also differed with respect to their responses to other items on the survey. Results showed that 55% of the preservice teachers were in the mastery group (i.e., believed that students should attain conceptual mastery before being allowed to use calculators), and Fleener found no differences between the mastery views of preservice and practicing teachers. She did, however, find differences in the ways both preservice and in-service teachers in the mastery group tended to respond to other items compared with teachers not in the mastery group. Finally, she concluded that the mastery issue may be key for affecting teachers’ beliefs about using calculators.

Walen, Williams, and Garner (2003) investigated the relationship between students’ use of calculators in college mathematics courses for prospective elementary school teachers and their views of appropriate use of calculators in elementary school classrooms. Although technology has been identified as an essential component of teaching and learning mathematics (NCTM, 2000), Walen et al. (2003) reported that many college mathematics faculty limit the use of calculators for fear that their students will grow to depend upon them or because calculators enable students to “get answers that they don’t ‘know’” (p. 447). The researchers contended that this position, held by many faculty, reflects a view of mathematics as primarily comprised of a set of rule-governed calculations and procedures. They studied four college mathematics classes in which calculators were incorporated into a collaborative setting dedicated to students’ learning with understanding (NCTM, 2000). They hoped that because calculators were readily available for students, they would use them “naturally and unreflectively to accomplish mathematical tasks” (Walen et al., p. 449).

To determine whether college students who learned mathematics in such an environment developed beliefs about how calculators should be used in elementary school classrooms different from the beliefs of others, Walen et al. (2003) collected informal participant-observation notes, classroom artifacts, and questionnaire responses from students in four sections of a mathematics content course designed for prospective elementary school teachers. They also conducted four case studies of students enrolled in the course. Their results indicated that when asked to solve computational tasks, the students used more mental approaches or paper-and-pencil approaches than calculator solutions. In other words, their students did not overuse the calculator by using it on simple problems. When analyzing the students’ responses to three open-ended questions, the researchers found that the three most common responses to the question “When do you use a calculator?” were when accuracy or confidence in the answer was needed, when the problem was large or complex, or when trying to save time, with each answer given by about one third of the respondents (although a person may have given more than one answer). Overall, the researchers concluded that their students reported positive experiences using the calculators in their mathematics classes.

The researchers (Walen et al., 2003) had hoped that the classroom experiences provided to their students would positively influence their attitudes toward calculator use in elementary classrooms and their decisions to model the experiences by using calculators with their subsequent students. However, they were disappointed to see that their “success towards this goal was limited” (p. 457). The second open-ended question was “When do you think students should use a calculator?” The most common answer, given by about half the students (32 of 66), was that students should use the calculator only after they knew the operations; 8 students responded that children should never use calculators because they will handicap students’ learning. The most common response (by 42 of 66) to the third open-ended question—“Is there a time when a calculator shouldn’t be used?”—was that calculators should not be used while students are learning the basics. The researchers concluded that although some students’ responses regarding their subsequent teaching with calculators paralleled their descriptions of their positive experiences using calculators, many students’ responses did not. Instead, the prospective teachers in this study held “seemingly contrasting views that it is acceptable for them to use a calculator to do an arithmetic problem, but it is not acceptable for their students” (p. 459). The researchers suggested that their students found themselves in two different worlds, one in which, as doers of mathematics, they should use calculators whenever the
need arises and the other in which, as teachers helping children learn mathematics, they juxtapose the use of calculators against a set of beliefs about mathematics teaching: “that basic arithmetic skills must be learned; that the task of their future students was not to do mathematics, but to learn mathematics”; and the calculator was neither an integral nor welcome part of this context. The students’ view of mathematics as something that must first be learned and then done caused them to view calculators as inappropriate for children before they learn the mathematics.

Summary of Teachers’ Beliefs About Technology

Teachers need support to effectively use computers or calculators in their classes, but even when teachers are comfortable using computers or calculators for their own learning, they may not believe that using technology with their students is appropriate. Teachers’ beliefs about appropriate use of technology for children are constrained by their beliefs about mathematics (Tharp et al., 1997; Walen et al., 2003) and by their beliefs about teaching and learning mathematics (Fleener, 1995; Schmidt, 1999). One issue that appeared in several studies was the belief that calculators should be used only after students have learned the mathematics (Walen et al., 2003) or have mastered the concepts (Fleener, 1995).

Mathematics Teachers’ Beliefs Related to Gender

Results of the Third International Mathematics and Science Study (TIMSS) indicated that, although performance of boys and girls worldwide is comparable at the 4th- and 8th-grade levels, by the last year of senior high school, in 18 of 21 countries participating in the testing, boys perform significantly better on mathematics assessments than girls, and even in the remaining 3 countries (including the United States), the differences favored males (Mullis, Martin, Fierros, Goldberg, & Stenler, 2000). In the United States, males and females take similar mathematics classes at the K–12 level, but males participate in mathematics more than females after high school (Levi, 2000). Although boys and girls at the early elementary school level have differed little in achievement-test scores, when researchers extended the focus of their investigations to the types of strategies students used, gender differences between boys and girls appeared as early as first grade, with boys consistently using more abstract strategies than girls used to solve problems (Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). Such early differences may explain why major differences appear at the senior high school level, because students who learn to make sense of mathematics early on by virtue of applying mental computation and estimation strategies are demonstrating the kind of understanding of numbers and operations that will help them be more successful when studying more advanced mathematics (Sowder, 1998).

Some proposed reasons boys consistently outperform girls in mathematics at the senior high school level and some differences favoring boys that appear even much earlier relate to the views that students have about mathematics. For example, both female and male students in the United States, Australia, and Japan have stereotyped mathematics as a male domain, and the males held this stereotype more than the females (Keller, 2001). Might these students’ beliefs be affected by teachers’ beliefs? Although much remains to be learned about how teachers’ verbal and nonverbal behaviors affect students, the finding that teacher expectancy affects pupils’ performance, sometimes known as the educational self-fulfilling prophecy, has been clearly established (Rosenthal, 2002). Less clear, however, is what the research on teachers’ beliefs about boys, girls, and mathematics has shown.

Li (1999), in a review of literature on teachers’ beliefs related to gender, found that although results were inconclusive, the literature indicated that teachers had different beliefs about male and female students and that they tended to stereotype mathematics as a male domain. Teachers tended to overrate male students’ mathematics capabilities and had more positive attitudes about male students. Li found no differences between male and female teachers’ beliefs.

Helwig, Anderson, and Tindal (2001) found little evidence of a relationship between third- and fifth-grade teachers’ ratings of their students’ mathematics skills and gender. They also agreed with an earlier conclusion by Brophy (1983, cited in Helwig et al., 2001) that teachers’ perceptions of their students were accurate.

In urban and rural Kansas, Leedy, LaLonde, and Runk (2003) used a survey to assess mathematics attitudes of 4th-, 6th-, and 8th-grade girls and boys participating in a regional mathematics contest; their parents; and their teachers. The students were selected on the basis of their interest in the mathematics competition and their mathematical abilities. Using 12 Likert-type items drawn from the Fennema-Sherman Scales (Fennema & Sherman, 1976), the authors assessed the students on six factors; they modified the items to assess parents’ and teachers’ views of mathematics as a male domain and usefulness of mathematics. The teachers were comprised of two groups, those mathematics teachers who served as coaches for the mathematics teams and those who did not. Results indicated
that the boys, fathers, parents of sons, and noncoaching mathematics teachers more strongly supported the belief that mathematics is a male domain than the girls, mothers, parents of daughters, and mathematics coaches. The most strongly endorsed individual statement related to gender bias was “Men are naturally better at mathematics than women,” whereas the most strongly endorsed nonbiased statement was “It is just as appropriate for women to study mathematics as for men” (p. 289). The authors questioned whether agreement with the first statement is an indication of gender bias or of “real, inherent differences between girls and boys, men and women, in how they learn and think about mathematics” (p. 289).

The belief that mathematics is a male domain was also found among teachers in Switzerland. Keller (2001) drew from TIMSS data and used a Likert-type scale she designed to assess teachers’ and 6th-, 7th-, and 8th-grade students’ aptitude concepts, such as mathematics as a male domain. Results indicated that students and teachers stereotyped mathematics as a male domain, and evidence showed that teachers’ views affected the students’ stereotyping. Keller concluded that the assumption, put forth by Fennema, “that teachers transmit their views of mathematics during instruction and that students adopt such views is more plausible than the assumption that teachers adopt the views of students” (p. 171).

Tiedemann (2000) studied the beliefs of Grades 3 and 4 German elementary school students and their teachers. The teachers were each asked to name three girls and three boys from their classes, one of each gender performing in mathematics at the low, medium, and high levels, and then the teachers completed a questionnaire for each identified child. The 21-item questionnaire addressed six categories, including estimation of the child’s competence, attribution of improvement, and presumed self-concept of the child. The results indicated that even for equally achieving, average boys and girls, teachers rated mathematics as more difficult for the girls than the boys. Teachers attributed unexpected failures more to low ability and less to lack of effort for girls than for boys. Tiedemann also reported similar results for third- and fourth-grade Russian teachers, who thought of mathematics as more a male domain than a female domain. Tiedemann concluded, “If teachers’ beliefs are important influences on how they interact with and teach students, these teachers’ beliefs could be seen as an influence on the development of gender differences in mathematics” (p. 205).

Summary of Teachers’ Beliefs Related to Gender

Researchers raise more questions than they answer related to teachers’ beliefs about boys, girls, and mathematics and the effects of these beliefs on children’s performance. Apparently, teachers continue to hold the stereotype that mathematics is a male domain, and evidence indicates that this teacher belief affects students’ beliefs. Most research in this area has been conducted using Likert scales, although case-study research has also proven to be useful.

RESEARCH ON TEACHERS’ AFFECT SINCE 1992

One can better understand the state of the research on teacher affect in 1992 by considering the intersection of McLeod’s handbook chapter on affect and A. G. Thompson’s handbook chapter on teachers’ beliefs and conceptions. Neither author wrote about teachers’ affect. In writing about affect, McLeod focused on students’ affect when solving mathematics problems, and in her chapter, Thompson focused on teachers’ beliefs and conceptions but not on their affect. McLeod called for researchers to begin to incorporate affect into research on cognition, but most of those responding have not focused specifically on teachers.

I begin by sharing some of the little research that has addressed teachers’ affect as it relates to mathematics teaching and learning and then turn to work that addresses either affect in general or affect-related frameworks developed to describe children’s experiences, because I contend that the frameworks may be applicable for work with prospective and practicing teachers learning or engaged in mathematics. But first, I address a more general question: Does affect matter in the learning of mathematics? I report two reviews of research that clearly indicate that, at least for students, affect is related to achievement.

Relationship Between Affect and Achievement

The relationship between affective factors and achievement appears to be important for researchers in mathematics education. Ma and Kishor (1997) conducted a meta-analysis of 113 studies to investigate this relationship between students’ attitudes toward mathematics and their achievement in mathematics. Their results indicated an overall mean effect size of 0.12, which they concluded was “statistically significant but not strong for educational practice” (p. 39). They found that gender differences were too weak to
have practical implications and that whereas the relationship between attitude toward mathematics and achievement in mathematics was not strong at the elementary school level, it “may be strong enough for practical considerations at the secondary school level” (p. 40). They concluded that the junior high school level may be the most important period during which students shape their attitudes toward mathematics as these attitudes relate to students’ mathematics achievement and that by high school, many students’ attitudes are fixed or stable, which may result in their having less effect on or being less affected by achievement in mathematics. They also found a relationship between attitudes toward mathematics and the means by which researchers measured achievement in mathematics. The researchers found that attitudes toward mathematics could predict achievement measured using complex conceptual and procedural mathematics tasks, such as applications and problem solving, better than achievement measured using less mathematically complex tasks. They concluded that researchers should work toward substantially refining their measures for affect. They also called for studies in which researchers differentiate among students’ ability levels, take into account students’ grade levels and ethnicities, and examine the relationships between varying school characteristics (e.g., school size, school mean socioeconomic status, school policies) and mathematics achievement.

Ma (1999) conducted a meta-analysis of 26 studies narrowly focused on the relationship between anxiety toward mathematics and achievement in mathematics and reported a significant correlation of –.27. Although several definitions for mathematics anxiety were used by the authors of the studies analyzed, Ma drew upon McLeod’s (1992) definitions to conclude that affect was a general construct and mathematics anxiety was a distinct construct generally associated with the more intense feelings students exhibit in mathematics classrooms, including tension, helplessness, dislike, worry, and fear. Ma also summarized theoretical models put forth for the negative relationship between mathematics anxiety and mathematics achievement. According to one model, when arousal states are either below or above an optimal state, performance decreases. Under this inverted-U-curve model depicting a curvilinear relationship between anxiety and performance, some arousal is beneficial to performance, but when the arousal level increases too much, performance drops. Although this model is popular with arousal theorists, Ma found that most researchers tended to consider the relationship between arousal and achievement to be linear, with increased arousal associated with decreased achievement. Ma cited researchers who have identified this linear relationship among children at the elementary and secondary level, among adults in general, and particularly among college students.

A primary reason posited for the negative relationship between mathematics anxiety and mathematics achievement stems from the theory of test anxiety. Researchers who adhere to this notion regard mathematics anxiety “as a kind of subject-specific test anxiety” (Ma, 1999, p. 522). In his meta-analysis, Ma quantified the potential improvement in mathematics achievement when anxiety toward mathematics is reduced, reporting that when an average student who is highly anxious about mathematics is able to decrease that level of anxiety, improvement from the 50th to the 71st percentile in mathematics achievement may occur. Ma thought that this meta-analysis resulted in more definitive results than the study of the relationship between attitude and achievement (Ma & Kishor, 1997) because the instruments used to measure mathematics anxiety are more effective than those used to measure the more general construct of attitude, perhaps because “anxiety toward mathematics is easier to measure than attitude toward mathematics in that it is more operationally definable for researchers and more verbally expressible for students” (Ma, 1999, pp. 533–534). Although Ma identified several measures of mathematics anxiety, almost half the studies (12 of 26) used the same one—the Mathematics Anxiety Rating Scale (MARS) (Richardson & Suinn, 1972); he found that the research results were similar regardless of the anxiety-measurement instrument. As in the 1997 study, Ma found that the type of instrument the researchers used to measure mathematics achievement made a difference. Researchers who used standardized achievement tests tended to report weaker relationships than those who used researcher-made achievement tests or mathematics teachers’ grades. No gender differences were found in this study.

Mathematics Teachers’ Affect

The meta-analyses described previously indicated that affect in general and mathematics anxiety in particular impede students’ mathematical learning. Clearly, this is an important finding about students, but if the anxiety dissipates by the time these students enroll in college, we need not concern ourselves with mathematics anxiety of prospective or practicing teachers. Unfortunately, this dissipation does not occur, as the following studies indicated.

Harper and Daane (1998) investigated the causes of mathematics anxiety among prospective elementary school teachers enrolled in a U.S. midsized southeastern university before and after they completed
their mathematics methodology courses. Fifty-three prospective teachers enrolled in three methodology courses completed three assessments. Mathematics anxiety was assessed using the 98-item Mathematics Anxiety Rating Scale (MARS) (Richardson & Suinn, 1972) test as a pretest and a posttest. The MARS test is a self-rating Likert-scale comprised of items the respondent rates in terms of the degree of anxiety aroused. Items include “Thinking about beginning a math assignment,” “Figuring out your monthly budget,” and “Doing a word problem in algebra” (Bessant, 1995, pp. 334–335). Factors influencing mathematics anxiety (FIMA) were assessed with a 26-item checklist designed by the authors and used only as a pretest. Respondents completing the FIMA assessment were asked to either agree or disagree with each item on the assessment, and if they agreed, they then indicated whether they believed that the factor had caused them to experience any mathematics anxiety. Examples of items on this survey are “There was an emphasis on drill and practice,” “I lacked an interest in math,” and “I had embarrassing or negative experiences in past math classes.” To further assess the influence of the methods course, the authors developed a 7-item methods-course reflection prospective teachers completed at the end of the course; it was designed to assess effects on students of (a) working with a partner, (b) working in cooperative groups, (c) working with small groups or in centers, (d) using manipulatives, (e) working on problem-solving activities, (f) writing about mathematics in journals, and (g) doing fieldwork in local schools. Finally, 11 of the 53 students were interviewed. Results indicated that mathematics anxiety persists in prospective elementary school teachers and that, often, the anxiety originated in elementary school. Causes for these students’ mathematics anxiety included an emphasis on right answers and the right method, fear of making mistakes, insufficient time, and word problems and problem solving. The mathematics methods course led to decreased levels of mathematics anxiety for 44 of the 53 prospective teachers, but 9 of them experienced increased levels of mathematics anxiety during the course.

Mapolelo (1998) conducted a case study of one Botswanian preservice teacher’s beliefs and attitudes toward mathematics. In Botswana, mathematics teaching was, at that time, procedurally oriented, and many students had difficulty with it in elementary school. These difficulties continued into teacher training to the extent that more than one third of the 490 prospective teachers at the teacher-training colleges failed the mathematics final examination. Mapolelo selected the case-study student because she expressed encountering difficulty in mathematics and had previous teaching experience. Dudu was 22 years old and an average student with “less than impressive overall class performance, particularly in mathematics” (p. 339) and two years of primary-level teaching experience in a school in a remote area. Dudu, like most of the preservice teachers, reported that she had had unpleasant experiences with mathematics at the junior and senior secondary levels. Results indicated that Dudu had been frustrated by an educational system in which memorizing facts and procedures without understanding the memorized procedures was emphasized. She came to view mathematics as something that was right or wrong, and she saw little place for reasoning when engaged in mathematics. Dudu was obsessed with failing examinations and attributed passing any examination to luck. Mapolelo concluded that to develop the better mathematics teachers needed in Botswana, mathematics educators needed to attend to the beliefs and affect of preservice teachers.

Philippou and Christou (1998) reported that Greek prospective teachers “brought very negative attitudes to Teacher Education” (p. 196). For example, 24% of the prospective teachers agreed with the statement “I detest mathematics and avoid using it at all times,” and 62% agreed with the statement “I do not think mathematics is fun, but I have no real dislike of it” (p. 197). Efforts to change these attitudes were successful over the course of the program, during which prospective teachers completed two mathematics content courses based on the history of mathematics (with particular attention paid to the contributions of the Greek culture to mathematics) and a methods course. Attitudes that changed were the prospective teachers’ satisfaction from mathematics and their views of its usefulness. The prospective teachers’ deeply rooted anxieties about mathematics did not seem to change, and the researchers concluded that students’ emotions are resistant to change.

Bessant (1995) studied attitudes of 173 Canadian university students enrolled in the introductory statistics courses in the mathematics, psychology, or sociology department. The author administered a 35-item mathematics-attitude scale the first week of class, followed a week later by a slightly reduced 80-item version of the Mathematics Anxiety Rating Scale (MARS) (Richardson & Suinn, 1972). He also administered a questionnaire designed to identify students’ learning approaches. Using factor analysis, the author identified six factors that accounted for 54.9% of the total variance: general evaluation anxiety, everyday numerical anxiety, passive-observation anxiety, performance anxiety, mathematics-test anxiety, and problem-solving anxiety. The most im-
In their study designed to investigate students’ emotional responses to timed tests, Walen and Williams (2002) provided a deeper look into issues that might explain Bessant’s findings related to the role of general test anxiety. In the only study that I found to establish a relationship between affect experienced by children and adults, Walen and Williams conducted two case studies of college students and one of a third-grade child. They built their work on Mandler’s (1989) framework of emotion as resulting from cognitive analysis and physiological response. In particular, visceral arousals can be produced by some discrepancy or the interruption of some ongoing action, and these emotional responses can substantially affect cognitive functioning. One experiencing such stress tends to focus on aspects of a situation deemed important by that individual, but Walen and Williams (2002) pointed out that although this focus may appear to be helpful, often “what an individual considers important in a given context may not lead to a resolution of the perceived stress” (p. 364). Timed mathematics tests are, for some people, such a context.

Walen and Williams juxtaposed this notion of stress induced by interruptions or blockage of activity, an occurrence that varies from individual to individual, with the more culturally based view that people have of time and temporality. Whereas in the Western culture time is seen as “objective, continuous, universal, linear, and infinitely reducible” (2002, p. 364), many in polychronic cultures view time as more subjective and variable in speed. For such people, “time is not something life is measured against, but rather something that conforms to life” (p. 364). Walen and Williams cited the American Indian culture as including a view of time that often clashes with the Western view. For example, whereas in the American Indian culture, careful, slow, well-considered responses are rewarded, this approach does not serve well for one required to provide quick answers on timed tests.

Walen and Williams (2002) reported case studies of Pat and Jo, two women enrolled during different semesters in the same mathematics course for elementary school teachers, and a case study of Em, a third grader. Pat, of Asian descent, and Jo, of American Indian descent, were nontraditional students returning to school after spending time as mothers and homemakers. Pat reported past experiences of failure that caused her to feel anxious and insecure about doing mathematics, but she also reported that she enjoyed mathematics. This seemingly contradictory comment left the instructor (the first author) unsure of Pat’s meaning. During problem-solving, small-group activities, the instructor came to see Pat as confident and articulate and not at all anxious or insecure. The instructor was convinced that Pat was learning the mathematics well and so was baffled when Pat failed the first 1-hour exam. As a matter of practice, the instructor informally interviewed each student after each test; during her interview, Pat explained that she had felt that she was progressing satisfactorily on the test until she realized that she had only 20 minutes left to complete the exam and panicked. She explained, “My heart pounded. I saw spots. For me, the test was over. I turned in the exam and left.” The instructor offered Pat the opportunity to take as much time as she needed on the subsequent test; Pat responded with great relief to the offer. On the next exam, Pat required an extra 30 minutes to complete the exam; the instructor noted, however, that she saw Pat begin to panic when she saw the first student turn in the exam. On the remaining exams, Pat needed no additional time, and she told the instructor, “When I have all the time I need, I don’t even need the whole hour” (p. 369).

Jo’s story is almost identical to Pat’s. She appeared to understand in class, but during the first exam, the instructor noted that Jo “paled, then flushed, then broke out with beads of sweat” (Walen & Williams, 2002, p. 369). Although Jo assured the instructor that she was okay, the instructor did not believe her. Jo failed the exam, and she reflected later on her experiences taking tests in elementary school: “It was those tests . . . those tests that we took in multiplication. I’m so slow. I just couldn’t do it. I understand, but I just can’t do it when there’s time” (p. 369). The instructor arranged for Jo to have extra time on subsequent tests, but Jo, like Pat, did not need it. When turning in the next exam, “she smiled and said, ‘I did it. I did it in the hour. I don’t need more time. Because I could have more, I didn’t need it. I didn’t even feel a bit scared’” (p. 370).

Em’s story provides a glimpse into the possible development of Pat’s and Jo’s feelings of anxiety and fear about mathematics tests. Em was described as a bright, articulate third-grade girl who was “particularly successful in mathematics” (Walen & Williams, 2002, p. 370). She was mathematically curious, and she seemed conceptually oriented. However, she had difficulty memorizing her multiplication facts and responding to the 100 facts in 5 minutes, and this frustration left her “dreading every single Friday” (p. 370).
She seemed aware of other children in the class who also struggled with these multiplication timed tests, and she distinguished between one student who was not very good in mathematics and other students who, like her, were. Her teacher had told Em that she was aware that Em knew her facts but that these timed tests had to be administered because the other classes were taking them. The culminating experience for Em was a Friday on which only the students who had mastered all their tables could attend a banana-split party. Em described feeling humiliated by the prospect of being excluded and stayed home to have her own banana-split party. She said that had she gone to school that day but not attended the party, the other students would think “that I wasn’t very good at mathematics” (p. 374).

In their analysis of these three case studies, Walen and Williams (2002) noted that these students experienced neither math anxiety nor even test anxiety; instead, their enemy was time. All three students were comfortable doing mathematics, and they had trouble with timed mathematics tests precisely because they were willing to view mathematics as we wanted them to—as personal, meaningful, and contextual. It was not the mathematics, but rather the imposition of a foreign temporal structure over their mathematical knowing, that led to their discomfort and eventual paralysis in mathematical testing situations. (p. 375)

This research is particularly important for thinking about teachers’ affect, not only because it provides two rich case studies of prospective teachers’ affect but also because the researchers draw connections between the anxiety experienced by a child doing mathematics and that experienced by adults. Pat and Jo were both studying to be teachers. I am left wondering how many adult students like Pat and Jo are teaching in the classroom now? Worse yet, how many of them feel constrained to perpetuate practices that they recognize as harmful to students because that is what they believe is expected of them?

Summary of Studies of Elementary School Teachers’ Mathematics Anxiety

Results of these studies indicate that mathematics anxiety among prospective elementary school teachers is of worldwide concern. Causes of mathematics anxiety have been identified as fear of failure, general test anxiety, and emphasis on right answers and right methods instead of on developing ways of reasoning about mathematics. Efforts to help prospective elementary school teachers reduce their mathematics anxiety are mixed, depending upon the approaches taken. Simply providing more time during test taking was sufficient for some prospective elementary school teachers to overcome serious anxiety experienced during exams. Coursework promoting mathematical sense making in less rigid environments supported reduced mathematics anxiety among prospective elementary school teachers, although in one case the changes that occurred supported the students’ sense of the usefulness of mathematics but not their deeply rooted anxieties toward mathematics.

Perhaps the most compelling finding is that the causes of negative affect toward mathematics or mathematics learning tend to go to prospective teachers’ experiences as learners of mathematics. The process of preparing mathematics teachers starts long before prospective teachers take college courses; it begins when students enter kindergarten and first begin to learn mathematics in school. Therefore, I turn now to several studies that addressed affect, but not teacher affect. I consider these studies important because understanding the affect of K–12 students may help us understand the affect of our prospective and practicing teachers. Further, these studies all extend Mandler’s widely used theory of affect.

Frameworks for Considering Affect: Extending Mandler’s Framework

Hannula (2002) extended the work of Mandler (1989) by incorporating the influence of less intensive emotional states not addressed in Mandler’s theory. Whereas Mandler focused upon interruptions that occur only while students are in the process of engaging in mathematics, Hannula argued that he considered students’ reactions when they are thinking about mathematics, both generally and specifically. He presented a framework in which the mathematics-related emotions students experience were classified into four evaluative processes: (a) associations when generally thinking about the concept mathematics, (b) expectations when thinking about engaging in mathematics, (c) situational emotions when actually doing mathematics, and (d) values when considering the role of mathematics in relationship to other goals. For Hannula, each is a process that produces an expression of an evaluation of mathematics. The first evaluation is a nearly automatic association students make when, for example, they are asked how they feel about mathematics; it depends solely upon their prior experiences. Because it is not related to a particular context, it is an evaluation commonly assessed using questionnaires. The second evaluation is the most cognitive of the four; it goes deeper than the first because whereas for the first, students react generally to their image of
mathematics, for the second, students imagine a specific mathematical situation and the consequences involving some emotions that would follow. The third evaluation occurs when a student, while engaged in doing mathematics, engages in a continuous, often unconscious, evaluation of the situation with respect to her or his personal goals. This evaluation is represented as an emotion. The fourth evaluation is based on, often unconscious, cognitive analysis in which a student evaluates his or her life and the value given to different goals in it.

In a study of Finnish students in Grades 7–9, Hannula (2002) applied his framework to the case of Rita. At the beginning of the year, Rita held negative associations about mathematics in general and about word problems in particular. In spite of these negative associations and negative expectations, Rita exerted much effort when studying mathematics, a fact Hannula interpreted as a reflection of Rita’s values related to mathematics. Hannula described experiences Rita had while learning mathematics in a group of three students; two goals she held during the first task were the cognitive goal of understanding the solution of the task and the social goal of effectively interacting with her two peers. Rita experienced frustration with the mathematics and with her feelings of being left out of the group discussions. Later when Rita began to think that she understood the mathematics, her attitude toward mathematics improved. Hannula found that Rita’s evaluation of mathematics changed for the better “regardless which of the four evaluations (emotion, expectation, association, values) we consider” (p. 41), and he concluded that the proposed framework provided a useful way to describe attitudes and their change. For Hannula, this case study indicated that attitudes can change over a relatively short period of time and, finally, that Rita’s negative attitude toward mathematics provided a “successful defence strategy of a positive self-concept” (p. 42).

Gómez-Chacón (2000) presented a model for the study of the interaction between cognition and affect in mathematics. She subdivided affect related to mathematics into two constructs, local affect, which referred to the states of change of feelings or emotional reactions one experiences while engaged in mathematical activity, and global affect, which is the concept of self and the beliefs about mathematics and learning one holds. The global affect results from “routes followed in the local affect” (p. 151); that is, global affect might be thought of as the beliefs and identity that one develops as a result of the feelings and emotional reactions one experiences while engaged in mathematics within the individual’s sociocultural context. Fear of mathematics is an example of global affect. Gómez-Chacón developed instruments for measuring the cognition-affect interaction, and she applied those instruments in a study of 23 cabinetmaking students in their late teens enrolled in a job-training workshop in Spain.

One instrument Gómez-Chacón (2000) developed was a problem mood map, in which codes are used by students to reflect their emotional reactions while engaged in mathematics. At the end of each mathematical activity, the students were asked to select from a list of emotions they might have experienced while working on the activity. The 14 emotions and descriptors offered, selected as a consensus of the most relevant emotional reactions the students in this study exhibited in the classrooms, were curiosity, cheerful, despair, calm, hurry, boredom, bewildering/confused/puzzlement, brain teaser, liking, indifference, amusement, confidence, blocked, and De aburrido, which is a Spanish term meaning “just great.” Gómez-Chacón contended that students’ coding of their moods while engaged in mathematics could foster awareness of their emotional reactions, and, as a result, they might feel more in control of their emotions; also, teachers could gather information about their students’ affective reactions at different stages of a lesson.

In applying her framework to a student named Adrián, Gómez-Chacón (2000) concluded that using her instrument to assess both local and global affect was a valuable way to describe Adrián’s affect. For example, although Adrián exhibited the global affective trait of fear toward mathematics, he exhibited a variety of local affective traits, including anxiety, satisfaction, and surprise, while solving problems. She also found that Adrián’s negative emotions were associated with a lack of understanding of the mathematics. Gómez-Chacón concluded that although the model for affect put forth by Mandler (1989), whereby students experience affect because of interruptions in problem-solving situations, was valid, the model needed to be enhanced to account for both local and global affect.

Hannula (2002) and Gómez-Chacón (2000), like Mandler (1989), considered affective factors experienced by students while engaged in mathematical problem solving. However, they both believed that Mandler’s framework needed to be expanded to consider additional factors. Hannula extended Mandler’s framework by considering associations, expectations, and values one holds toward mathematics, and Gómez-Chacón extended the framework by considering the global affective characteristics. Both also found in their case studies that a student’s affect was inversely correlated with the student’s mathematical understanding.
In her dissertation, Malmivuori (2001) presented a theoretical analysis centered on affect but integrated with cognition and beliefs around particular social environments. She addressed, in addition to affect, beliefs, metacognition, self-regulation and self-perceptions, motivation, and the influence of context on all these factors; she built her framework upon constructivist and sociocognitive theories, and I contend that she attempts to synthesize constructs in such a way as to address teacher identity. Although considering all these elements simultaneously is difficult, rather than isolate these constructs, she consistently drew connections between and among them in an extraordinarily ambitious review, analysis, and reconceptualization based upon 600 references! She wrote that the main point of her study was to constitute the theoretical and dynamic linkages between the often applied constructs and educational research results, as well as of the mathematics education results with affect, that would also apply to and clarify self-regulated learning processes or the dynamic interplay of affect and cognition more generally. (p. 299)

Envisioning a study that utilizes her rich theoretical framework is difficult; still, her work is important for those rethinking affect, and Malmivuori’s work is an important contribution to the future of research on affect.

Mathematical Intimacy, Integrity, and Meta-Affect

Goldin (2002a) expressed concern that research in mathematics education has focused primarily on cognition and far less on affect; he attributed this situation, in part, to the popular myth that mathematics is a purely intellectual endeavor in which emotion plays little role. Yet, he noted that, contrary to this myth, negative emotion is widespread in mathematicians, and he recommended that further attention “be devoted to the psychology of developing effective affect in students” (Goldin, 1998, p. 154). According to Goldin, “When individuals are doing mathematics, the affective system is not merely auxiliary to cognition—it is central” (2002a, p. 60). He, like McLeod, subcategorized affect as emotions, attitudes, and beliefs, but he included a fourth category comprised of values, ethics, and morals. Emotions are rapidly changing states of feeling usually embedded in context. Attitudes are moderately stable predispositions toward ways of feeling in classes of situations that involve a balance of affect and cognition. Beliefs are internal representations to which the holder attributes truth, validity, or applicability, and these representations are usually stable, highly cognitive, and highly structured. Also like McLeod, Goldin (2002a) viewed emotions as rapidly changing and highly affective; he viewed attitudes as more stable and cognitive than emotions, and he viewed beliefs as the most stable and cognitive of the three. The values, ethics, and morals comprising his fourth category are deeply held preferences that might be characterized as personal truths, are stable, may be highly structured, and are highly affective and highly cognitive.

Goldin, working with DeBellis, identified several other affective constructs related to mathematics; two of these constructs are mathematical intimacy and mathematical integrity. Mathematical intimacy is used to describe a psychological relationship between an individual and mathematics that connects with the individual’s sense of and value of self. DeBellis and Goldin (1999) provided the example of a child, Jerome, experiencing difficulty solving a problem in which he moved 10 jelly beans from Jar 1 containing 100 green jelly beans into Jar 2 containing 100 orange jelly beans, and then, after mixing up the jelly beans, he moved 10 jelly beans from Jar 2 (now containing 100 orange and 10 green jelly beans) back to Jar 1. The child was puzzled to discover that there were the same number of the orange jelly beans in Jar 1 as green jelly beans in Jar 2, and this relationship stayed constant regardless of how many jelly beans he moved from Jar 1 if he moved the same number back to Jar 1. The mathematical intimacy of the child was reflected in the child’s concern about, and involvement in, the problem.

Jerome’s interactions, from which we infer intimate engagement, include his close proximity to the jelly beans when performing the experiments, his raised voice, his deep breaths, the gesture of brushing his hand through his hair, his shrugging of shoulders, his smiling, and the silent pauses. He sits back in his chair as if to push himself away from the experiment, to distance himself when the outcome contradicts his expectation. (p. 253)

Mathematical intimacy might be thought of as the extent to which a person working on a problem is engaged with the problem.

Mathematical integrity “describes an individual’s affective psychological posture in relation to when mathematics is ‘right,’ when a problem is solved satisfactorily, when the learner’s understanding is sufficient, or when mathematical achievement is deserving of respect or commendation” (DeBellis & Goldin, 1999, p. 253). Mathematical integrity is associated with a learner’s desire to want to understand, with one’s dis-
comfort toward inconsistencies, and with a focus on justifying one’s reasoning. Often students confronted with tests or other evaluations are encouraged, by being awarded partial credit for giving responses even though they know that they do not understand, to bluff. This behavior, though it may lead to a higher test score, compromises the student’s mathematical integrity because instead of focusing on whether the submitted work makes sense, the student focuses on the grade that results from bluffing.

DeBellis and Goldin (1999) contended that a problem solver who possesses both mathematical intimacy and mathematical integrity will experience more mathematical power and perseverance than one who lacks these attributes. They noted that a relationship exists between mathematical intimacy and mathematical integrity. Absence of integrity is a major obstacle to intimacy, because the experience of pretending or bluffing one’s way through a problem not only blocks the individual’s understanding of the problem but also prevents one from experiencing intimacy in relation to the mathematics. Conversely, if one lacks mathematical intimacy, the need for mathematical integrity is reduced.

Goldin (2002a) also proposed the construct of meta-affect to refer to “affect about affect, affect about and within cognition that may again be about affect, the monitoring of affect, and affect itself as monitoring” (p. 62). Goldin suggested that most affective experiences exist in a context that contributes to the interpretation of the affective experience and provided the example of a person riding a roller coaster because he or she seeks the experience of fear, which, in that context, is pleasurable. “The cognition that the person is ‘really safe’ on the roller coaster permits the fear to occur in a meta-affective context of excitement and joy” (p. 62). However, the experience can change entirely if, for example, the rider hears an unfamiliar noise and thinks that the roller coaster may be malfunctioning. The change in the rider’s meta-affect creates a different experience for the rider, whether or not anything has actually changed. Goldin wrote, “It might seem that the ‘cognitive’ belief, that the ride is in fact safe, is the main essential to the joyful meta-affect. In this sense, the belief stabilizes the meta-affect” (p. 62). He explained that other beliefs and values play stabilizing roles for meta-affect. Applying this notion to mathematics might help explain how a particular experience, such as difficulty solving a mathematics problem, might be interpreted in different ways depending upon the beliefs and values held by the problem solver. Whereas one student might interpret the difficulty as an indication that he or she is a failure, another might view the difficulty with anticipation for a feeling of satisfaction at the expected success. The ways students view the context surrounding the task, together with the beliefs or values held by the student, are integrally related to the interpretation of the experience. “Powerful affective representation that fosters mathematical success inheres not so much in the surface-level affect, as it does in the meta-affect” (p. 63).

Summary of Teachers’ Affect

Because so little literature focuses on the intersection between teachers and affect, I have included studies of student affect. Goldin and DeBellis introduced the constructs of mathematics intimacy, mathematical integrity (DeBellis & Goldin, 1999), and meta-affect (Goldin, 2002a), and Hannula (2002) and Gómez-Chacón (2000) extended Mandler’s theory by including less intensive states and global states, respectively. Malmivuo (2001) presented a framework centered on affect but integrated with cognition, metacognition, beliefs, self-regulation and self-perceptions, motivation, and the influence of context on all these factors.

Research indicates that attitudes are less amenable to change among students in high school (Ma & Kishor, 1997) and college (Hembree, 1990) than among those in elementary and middle grades. One might, therefore, reasonably assume that prospective and practicing teachers’ mathematics affect or anxiety may have been formed when the teachers were precollege students, and the research supports this assumption. Although the view that the affect of students provides insight into the affect of teachers is a conjecture, evidence shows that similarities exist between adults’ and children’s feelings about mathematics.

I know of no research linking teachers’ affect about mathematics and mathematics learning to their classroom instructional decisions, but the prospect that the two may be linked is sufficient reason to think more carefully about what can be done to support prospective and practicing teachers in developing more positive affect toward mathematics and mathematics teaching and learning. In an age of accountability, can one reasonably assume that teachers who enjoy mathematics will teach more of it, or teach it with a deeper sense of intimacy and integrity (DeBellis & Goldin, 1999)? Will those who have more positive affect toward mathematics be more positive in their approaches to the teaching of mathematics? If teachers are able to share enthusiasm for mathematics, will their students develop more positive impressions of mathematics?

Consideration of the relationship between fear of failure and mathematical learning is important in
the preparation of mathematics teachers. In-service work with practicing teachers generally does not include evaluative components, and some in-service providers attribute their success to the nonevaluative aspect of the experience. But prospective elementary school teachers generally must complete mathematics courses at the university level, and these courses are difficult for some students. The tension between the gatekeeper role of such courses and the affect promoted as a result of the students’ concern seems to be worth additional consideration. For example, although we educators may believe that an important component of understanding mathematics is knowing when we do not understand something, students may feel compelled to hide their lack of understanding lest they receive an unsatisfactory grade and fail to meet the requirements for becoming teachers; students may learn that the best way to pass their courses is to hide their lack of understanding from the teacher, and possibly from themselves, thereby compromising the development of mathematical integrity (DeBellis & Goldin, 1999). Or, perhaps worse, if we help prospective teachers learn the mathematics that we consider important for them to learn but they feel that the experience was unpleasant and prefer to avoid mathematics, as practicing teachers, they may teach mathematics in a way that is consistent with their negative affect. Can a position be found between the evaluative, gatekeeper model generally applied with preservice teachers and the nonevaluative, collaborative model generally applied with practicing teachers, a position that would maintain the academic integrity required by universities but would also incorporate the long-range benefits valued by those engaged with in-service teachers’ professional development? Even though providing professional development for practicing teachers and preparing preservice teachers raise different sets of issues, turning to professional development collaboratives that take as their primary goal the teaching of mathematics to practicing teachers may be useful as a context for considering innovative approaches to teaching mathematics to prospective teachers in ways that also foster prospective teachers’ development of positive affect (see Sowder, this volume).

When we researchers in the field of mathematics education strive to make the field a scientific discipline, we draw heavily from other disciplines, such as psychology and sociology, gaining more theoretical perspectives in our field (Educational Studies in Mathematics [ESM], 2002). One result of a young and healthy discipline is that “it has become the norm rather than the exception for researchers to propose their own conceptual framework rather than adopt-

ing or refining an existing one in an explicit and disciplined way” (ESM, 2002 p. 253). The editors of Educational Studies in Mathematics (2002) and the Journal of Mathematics Teacher Education (Cooney, 2001) raised the possibility that the existence of these journals may have contributed to the proliferation of local theories that may not sufficiently push the field forward. This tendency to develop new conceptual frameworks seems strong in the area of research on affect in mathematics education; each study seems to introduce a new theory about affect. Mandler’s theory provided a much-needed starting point around which others could build, and most researchers studying affect in mathematics education at least reference Mandler. We should keep in mind Mandler’s (1989) caution that although a search for consensus on definitions of affect may be futile, we should take care in how we use the term. Defining our terms is certainly not a problem limited to the study of affect, as we saw in research on teachers’ beliefs.

**BEYOND BELIEF AND AFFECT?**

Some researchers contend that teachers’ beliefs should not be isolated but, instead, should be considered a component of more encompassing constructs. In this section, I look to such constructs, teachers’ orientations and teachers’ perspectives, and to a new area of research that holds possibilities for integrating research on teachers’ cognitions, beliefs, values, and affect: communities of practice and teachers’ identities.

**Teachers’ Orientations**

In her handbook chapter, A. Thompson (1992) argued for the position that researchers ought not attempt to separate the study of teachers’ beliefs from teachers’ knowledge, and she used the term conception to refer to both. Later, she and colleagues (A. G. Thompson et al., 1994) used the term orientation to refer to teachers’ views of mathematics and mathematics teaching. They introduced the terms conceptual orientation and calculational orientation to refer to the images teachers hold toward pedagogical tasks and the goals they served. Although teachers with conceptual and calculational orientations may agree upon the long-term goal of helping students develop problem-solving skills, their approaches differ in important ways. Actions of a teacher with a conceptual orientation are driven by an image of a system of ideas and ways of thinking she intends her students to develop; an image of how these ideas and ways of thinking can be
developed; ideas about features of materials, activities, and expositions and the students’ engagement with them that can orient students’ attention in productive ways; and an expectation and insistence that students be intellectually engaged in tasks and activities. Although a teacher with a calculational orientation may share the general view that solving problems is important, the actions of such a teacher are driven by a fundamental image of mathematics as the application of calculations and procedures for deriving numerical results. Associated with a calculational orientation is a tendency to speak exclusively in the language of number and numerical operations, a predisposition to cast solving a problem as producing a numerical solution, an emphasis on identifying and performing procedures, and a tendency to disregard context and to calculate upon any occasion to do so. Whereas a conceptually oriented teacher can identify the important concepts embedded in a problem, a calculationally oriented teacher tends to treat problem solving as flat; that is, nothing about the problem stands out, except that the answer is most important.

Conceptual and calculational orientations lead to differences in classroom discussions. Consider the problem “Susan drives 240 miles in 5 hours. What is the average speed of her trip?” The question “How did you solve this?” usually elicits from students the calculations they performed: “I divided 240 by 5, and the answer is 48.” A teacher with a calculational orientation may consider this an appropriate answer, but a teacher with a conceptual orientation might question further to direct students toward the underlying conceptual ideas, the quantities in this problem, and the relationships among those quantities: “When you divided 240 by 5 and got 48, what is 48 a number of? That is, to what does 48 refer in this situation?” Whereas a calculationally oriented teacher might ask, “What did you do?” and mean “What calculation did you perform?”, a conceptually oriented teacher might look for an explanation of the students’ reasoning, not for the students’ calculations, and ask, “What were you thinking?” or “What are you trying to find when you do this calculation (in the situation as you currently understand it)?”

Teachers who wish to become more conceptually oriented toward mathematics and mathematics teaching must reflect deeply about their goals for, and image of, mathematics and mathematics teaching. To shift their fundamental image from mathematics as doing things to mathematics as reasoning in particular ways is difficult. Furthermore, teachers who embark on such a journey frequently encounter their old patterns and goals, for example, in finishing a conceptual lesson by presenting the standard procedure instead of having students use notation to represent the conceptual methods they produced for solving one or several problems. Too little textbook support for conceptual teaching is provided, and far too little support is available in terms of state frameworks for mathematics instruction.

A. G. Thompson and P. W. Thompson (1994, 1996) applied the constructs of conceptual and calculational orientations in analyzing 2 one-on-one teaching experiments in which different instructors worked with the same child. The first instructor, a mathematics teacher named Bill who possessed a strong and elaborate understanding of the concept of rate but who experienced difficulty in teaching the concept of rate to one of the stronger students in his sixth-grade class, acted like other good quantitative reasoners who are unable to use arithmetic to reason quantitatively.

[Bill had] come to use arithmetic in two ways simultaneously—as a representational system and as a formulaic system to express an evaluation. What we did not foresee was the shortcoming of this development in regard to teaching. Bill’s quantitative conceptualizations appeared to be encapsulated in the language of numbers, operations, and procedures. He thus had no other means outside the language of mathematical symbolism and operations to express his conceptualizations. The language of arithmetic served him well as a personal representational system, or as a system for communicating with other competent quantitative reasoners. Yet, . . . that language served him poorly when trying to communicate with children who knew the tokens of his language but had not constructed the meanings and images that Bill had constructed to go along with them. (P. W. Thompson & Thompson, 1994, p. 300)

In the follow-up article, A. G. Thompson and Thompson (1996) described a subsequent lesson in which the same child worked with an instructor who had what the authors referred to as knowledge for conceptual teaching, comprised of “clear images of understanding a mathematical idea conceptually, how those images might be expressed in discourse, and what benefits might accrue to students by addressing the conceptual sources of their difficulties” (p. 3). In less than 30 minutes, the instructor had supported the child in her initial development of the concept of speed as the “mutual accrual of distance and time.” The teacher who had struggled with the child on the previous day observed the lesson but failed to notice that a key aspect of the successful lesson was helping the child attend to the proportionality of corresponding quantities, distance and time, by attending to the segments representing each of these two quantities—even though he reasoned proportionally.
to solve these problems himself. Teacher knowledge was not enough to support the child’s reasoning, and only the second instructor’s ability to coordinate his understanding of the content and the child’s thinking and to use language that oriented the child toward the important underlying concepts enabled the child to develop the concept of rate.

The construct *orientation* as used by A. G. Thompson et al. (1994) incorporated teachers’ knowledge, beliefs, and values about mathematics and mathematics teaching. In using the term, they described teachers’ images, views, intentions, goals, and tendencies, and they operationalized the construct by attending to teachers’ language and actions. Although they addressed more than beliefs, they did not explicitly address affective issues. In the next section I describe an attempt to expand the investigation of teachers’ beliefs to include affective factors.

**Teachers’ Perspectives**

When studying teachers, researchers apply their conceptual lenses to interpret teachers’ knowledge, beliefs, and practices. Simon and Tzur (1999) highlighted two approaches taken by researchers studying teachers. Some researchers apply their conceptual frameworks, which encompass the current knowledge from the field, to tell their own stories about teachers, providing, for example, deficit studies focused primarily on what teachers lack; the teachers’ perspectives, aspects of the teachers’ practices that are critical to the teacher, are generally missing from the story. In the other approach, the researcher tells the story from the teacher’s perspective, but the researcher’s conceptual framework may be missing, and consequently the story may not add to the theoretical development of the field. Simon and Tzur (1999), in their methodology for studying mathematics teacher development, focus upon the teacher’s approach by using their conceptual framework to provide accounts of the teacher’s perspective. A key principle guiding their work is that “every teacher’s approach is rational and coherent from his or her perspective” (p. 261) and that the job of the researcher is to understand the teacher’s underlying perspectives that might coherently account for the approach without separating aspects of that teacher’s perspectives into disparate components, such as beliefs, knowledge, and methods of questioning. They have contended that the term *teacher’s practice* includes not only everything the teacher does that contributes to his or her teaching, including planning, assessing, or interacting with students, but also the teacher’s values, skills, intuitions, and feelings about those aspects of the practice. These epistemological commitments are associated with particular methodological commitments, including the view that one cannot assess the relationships among teachers’ beliefs, knowledge, values, intuitions, feelings, and practices without gathering rich data. They refer to the unit of analysis for their qualitative work as a set, comprised of at least two consecutive related mathematics lessons and interviews before, between, and after the lessons.

Applying this methodological framework to study a group of teachers engaged in mathematics education reforms, they (Simon, Tzur, Heinz, Kinzel, & Smith, 2000; Tzur, Simon, Heinz, & Kinzel, 2001) considered these teachers in transition in terms of two perspectives commonly referred to in the literature: conception-based perspectives and traditional school-mathematics perspectives. The authors referred to *conception-based perspectives* as emergent and constructivist perspectives based on three assumptions: (a) Mathematics is created through human activity, and humans have no access to a mathematics that is independent of their ways of knowing; (b) individuals currently held conceptions constrain and afford what they see, understand, and learn; and (c) mathematical learning is a process of transforming one’s knowing and ways of acting. They contrasted this perspective with the *traditional school-mathematics perspective*, in which mathematics is viewed as existing independently from human experience and students are believed to passively receive mathematical knowledge by listening to and watching others. Teachers holding the perception-based perspective behave as if the mathematical relationships were properties of the objects being considered instead of as a function of the knowledge of the perceiver. Simon et al. (2000) provided a metaphor to capture the difference between the perception-based and conception-based perspectives:

We think of *perception* as looking through a lens. The lens represents what the perceivers bring to the situation, which structures their perception. Perceivers who hold a perception-based perspective do not consider that there are lenses affecting what they see. Rather, they assume that what they see is what is “out there,” that what is out there enters as it through their senses. . . . In contrast, perceivers who hold a conception-based perspective consider that they can see only that that results from looking through particular lenses and that they have no way to compare their perceptions (through their lenses) with lens-free perceptions. As a consequence, they make no claims as to what is out there, and they attempt to understand what is perceived as a contribution to understanding the perceiver. The analogy serves to emphasize that those who we infer hold a perception-based perspective are not rejecting a
The researchers found that neither perspective adequately characterized the views of these teachers in transition because the teachers did not view children as passively receiving mathematical knowledge, but, although they viewed mathematics as an interconnected and understandable body of knowledge, they also viewed mathematics as existing independently from human activity, and therefore “accessible as is to all learners” (Simon et al., 2000, p. 593).

**Communities of Practice and Teacher Identity**

When learning theorists evolved during the twentieth century beyond the stimulus-response of behaviorism to consider the contents of people’s minds, their research focus changed from teachers’ actions to understanding of teachers’ thought processes, including their planning, their interactive thoughts and decisions, and their theories and beliefs (Clark & Peterson, 1986). The preponderance of research on mathematics teachers’ beliefs is focused upon understanding teachers’ beliefs, investigating the relationship between teachers’ beliefs and practices, and changing teachers’ beliefs. However, even before the publication of the first NCTM *Handbook* (Grouws, 1992), researchers on learning questioned a focus solely upon the contents of a person’s mind without attending to the contexts in which learning occurred (Brown et al., 1989; Lave, 1988). By the early 1990s, during further evolution, some mathematics educators had adopted sociocultural perspectives, and a related theoretical framework that was to affect research on teachers’ professional development, *communities of practice* (see Sowder, this volume), emerged.

Lave and Wenger (1991) suggested that by shifting their focus from the individual learner to groups of learners, they could account for the learning that results from changing the way one participates with others who share common goals. Wenger (1998) further elaborated the relationship between a social community’s practices and individuals’ identity construction resulting from participating in these communities. Many researchers studying teacher professional development have adopted the *communities-of-practice* framework and the construct of *identity*; I provide three examples of research in which mathematics educators applied this theoretical lens to their work. Two examples are drawn from major professional development projects based around reform-oriented approaches to teaching and learning mathematics. In the CGI project (Carpenter et al., 1999) elementary school teachers learned about children’s mathematical thinking as a means for changing their beliefs and their practices; the QUASAR project (Silver & Stein, 1996) focused upon inner-city middle school reform through professional development of teachers. The third example is work of Forman and Ansell (2001) with a single teacher.

In work related to the CGI project, Kazemi and Franke (2004) applied a transformation-of-participation framework to describe the learning of a group of teachers engaged in professional development focused on children’s mathematical thinking. Building upon Wenger’s (1998) framework, they studied ways in which the participation of the group of teachers changed over time, because “shifts in participation do not merely mark changes in activity or behavior. Shifts in participation involved a transformation of roles and the crafting of new identities” (Kazemi & Franke, 2004, p. 205). Their contribution in writing about this work was “to provide an analytic frame for understanding teacher learning as shifts in participation” (p. 206). They met with 10 cross-grade teachers at an inner-city elementary school, after school once a month throughout the year, to discuss students’ mathematical thinking. Teachers adapted common problems for their students and brought student-work examples to discuss at each meeting. The study covered the first year of the teachers’ participation; data analyzed were seven workgroup-meeting transcripts from audio recordings, written teacher reflections, copies of student work shared by the teachers, and end-of-the-year teacher interviews. The authors analyzed their notes and the transcripts to understand how the teachers talked about student work, noting the mathematical and pedagogical issues that were raised, and identified other descriptive themes that consistently emerged across the year, such as teachers’ sharing successes from their practices or teachers’ generating questions about their practices.

Kazemi and Franke (2004) noted two major shifts in teachers’ workgroup participation. First, early in the year, teachers new at eliciting children’s thinking tended to underestimate the children’s abilities and reported students’ unsuccessful attempts to solve problems, but by the third session, they had begun to interact with their students about their strategies and expressed amazement at their students’ innovative strategies. The teachers helped one another by suggesting ways they might select students to share strategies so as to observe something identified as interesting.

Second, the teachers began to notice mathematical issues related to place value in the children’s
strategies; these issues led them to reconsider their mathematical goals for their students. Some teachers came to redefine place value less in terms of students’ memorizing the names of units (ones, tens, hundreds) and more related to their using groups of 10 to solve problems (e.g., to find the difference between 48 and 111, one teacher shared that two of her students reasoned that 48 + 60 is 108, and 3 more is 111, so the answer is 63.)

Kazemi and Franke (2004), although noting the importance of traditional components of studies of teacher change, explained that the framework they had applied enabled them to focus upon shifts in the participation of the study-group teachers:

We do not argue that examining individual teachers’ developing knowledge and beliefs is unimportant. In fact, these are key resources for a developing community. However, we believe that by attending to shifts in participation, we can understand the following aspects of teacher learning: (a) how teachers working together supported the development of each other’s thinking and the practices they used in their classrooms; (b) how and when teachers asked each other for help and contributed to discussions in the workgroup because of their own experimentation in the classroom; and (c) how teachers looked at the strategies students in other classrooms used and then used those as markers for what to expect from their students. (p. 231)

Stein and Brown (1997) drew upon sociocultural theories to explain teacher learning in the QUASAR project, suggesting that the “location of the phenomenon of learning” (p. 159) changed from being “located in the cognitive structures and mental representations of individual teachers” (p. 159) to being “situated in the fields of social interactions between and among individuals” (p. 159). Stein and Brown explained that when the unit of analysis shifted from the individual to the group, teacher change was defined with respect to the changing roles of the teachers as transformation of participation. For them, the communities-of-practice framework, found useful by many, lacked specificity as an analytic tool. To apply the framework in their study, Stein and Brown (1997) examined the range of the community’s work practices and determined that the breadth of teachers’ participation increased to include working on portfolio systems, planning curriculum, involvement with parents, making presentations to others, and working on articulation issues across other programs. Further they observed how teachers’ depth of participation, as measured by movement from peripheral to more central roles, changed. For example, participants moved from peripheral involve-
What is an identity? Collopy (2003) wrote, “A teacher’s identity is the constellation of interconnected beliefs and knowledge about subject matter, teaching, and learning as well as personal self-efficacy and orientation toward work and change” (p. 289). Van Zoest and Bohl (2005) also viewed teachers’ identities as embodying their knowledge, beliefs, commitments, and intentions, but they included, also, all the ways teachers have learned to think, act, and interact. For Van Zoest and Bohl, a teacher may hold many identities, but a given identity is not context specific—individuals carry it with them when they move from context to context. Wenger’s (1998) view of identity cannot be captured in a brief definition; he devoted one third of his book to discussing it, so I state what identity is not for Wenger instead of what it is:

[Identity] is not equivalent to a self-image; it is not, in its essence, discursive or reflective. We often think about our identities as self-images because we talk about ourselves and each other—and even think about ourselves and each other—in words. These words are important, no doubt, but they are not the full, lived experience of engagement in practice. (p. 151)

Sfard and Prusak (2005) took issue with Wenger’s notion that “there is something beyond one’s actions that stays the same when the actions occur, and also that there is a thing beyond discourse that remains unchanged, whoever is talking about it” (p. 16). They sought a definition of identity that is “operational, immune to undesirable connotations, and in tune with the claim about identities as man-made and collectively shaped rather than given” (p. 16). They thought of identity as narratives that are told by an author, about an identified person, to a recipient; the author, identified person, and recipient may be different people or the same person. They distinguished between actual identities, consisting of stories about the actual state of affairs (e.g., “I am an army officer” or “I am a good driver”), and designated identities, consisting of narratives presenting an anticipated state of affairs (e.g., “I want to be a doctor” or “I have to be a better person”). Sfard and Prusak argued that identities are crucial to learning because they may act as self-fulfilling prophecies, and they suggested that this perspective on identity might serve as the missing link between learning and sociocultural context.

Summary

Researchers have considered constructs related to beliefs, with orientation (A. G. Thompson et al., 1994), perspective (Simon & Tzur, 1999), and identity (Wenger, 1998) serving as three such terms. An important issue for the future of research on teachers’ beliefs is how researchers negotiate between constructivist and sociocultural perspectives. For example, Rogoff (1997) argued that the assumptions underlying the theoretical perspectives that explain how individuals acquire knowledge are fundamentally different from the assumptions underlying the transformation-of-participation perspective associated with focusing upon groups of people, and she argued that because no individual exists in isolation or out of cultural context, all learning, even reading a book, is social and should be seen from this perspective. Cobb (1995) presented the position that psychological and sociological perspectives should be viewed together:

It is tempting to respond to the conflicting assumptions of the sociocultural and constructivist perspectives by claiming that one side or the other has got things right. However, I will instead argue that the two perspectives evolved to address different problems and issues and they are complementary in several respects. (p. 379)

P. Thompson (in P. Thompson & Cobb, 1998) agreed that psychological and social perspectives depend upon each other, but he chose to view the psychological perspective as more fundamental because “it aligns more explicitly with what I take as our fundamental goal of making a positive, lasting difference in students’ lives after they leave our classrooms” (p. 19).

Lerman (1998, cited in Van Zoest & Bohl, 2005) suggested that in choosing a unit of analysis for educational research, one must be able to zoom in and out, changing focus to account for the full spectrum of locations of cognitive development. P. Thompson and Cobb (1998) applied the same metaphor, arguing, “To achieve a unification of psychological and social perspectives would mean that we become able
Researchers who take a psychological approach by focusing on an individual teacher’s beliefs will continue to inform the field as will those who take a sociocultural approach, by, for example, focusing on communities of practice. We in the field must balance these approaches to gather data useful for answering the types of questions driving our work. Furthermore, when we become comfortable with both approaches, the questions we consider asking will change.

### SOME THEMES AND FINAL THOUGHTS

**Changing Beliefs, Infusing Affect**

One noteworthy difference between research on teachers’ beliefs and affect is that whereas research on teachers’ beliefs has been extensive and subsumed into almost all areas of research on mathematics teaching and learning, the study of teachers’ affect has not. Mathematics educators generally agree on what beliefs are; we now face a greater challenge than defining beliefs: how to change teachers’ beliefs. If beliefs are lenses through which we humans view the world, then the beliefs we hold filter what we see; yet what we see also affects our beliefs—creating a quandary: How do mathematics educators change teachers’ beliefs by providing practice-based evidence if teachers cannot see what they do not already believe? The essential ingredient for solving this conundrum is reflection upon practice. When practicing teachers have opportunities to reflect upon innovative reform-oriented curricula they are using, upon their own students’ mathematical thinking, or upon other aspects of their practices, their beliefs and practices change. Furthermore, although prospective teachers seldom are embedded in practice-based environments, when they are provided opportunities to learn about students’ mathematical thinking and reflect upon the experiences, their beliefs change.

Researchers have found that for some teachers, beliefs change before practice, whereas for others, changes in practice precede changes in belief. I suspect that ultimately research will show that the most meaningful changes take place when teachers’ beliefs and practices change together, but additional research in this area will lead to a better understanding of the relationship between teachers’ changing beliefs and practices.

Judging by the attention to teachers’ affect in the mathematics education literature, one might conclude that teachers’ affect is not nearly as important as teachers’ beliefs. Yet teachers’ affect is critically important! If prospective or practicing teachers are to develop deeper content knowledge and richer beliefs about mathematics, teaching, and learning, then positive affect must be considered. I have asked several groups of prospective secondary mathematics teachers, the majority of whom had previously completed their undergraduate mathematics major, to define mathematical proficiency. Generally they talk about learning procedures, understanding concepts, and problem solving; a smaller number mention proofs or reasoning. Few students mention affective issues. The definition of mathematical proficiency presented in the consensus document *Adding It Up* (NRC, 2001) includes five interrelated strands that, together, comprise proficiency. The first four are conceptual understanding, procedural fluency, strategic competence (the ability to formulate, represent, and solve mathematical problems), and adaptive reasoning (the capacity to think logically and to informally and formally justify one’s reasoning). The fifth strand, productive disposition, is “the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (NRC, 2001, p. 131). The inclusion of confidence in the ability to learn mathematics as a component of mathematical proficiency is important as a statement that proficiency in mathematics has affective aspects. Teachers make important decisions about the manner in which they teach mathematics, and elementary school teachers often decide how much time to devote each day to mathematics. If the mathematics courses we educators offer to prospective teachers address the first four strands of mathematical proficiency without addressing mathematical disposition, I suspect that we will continue to produce many teachers who lack the positive dispositions associated with creating positive mathematics-learning experiences for students. Although secondary school teachers generally have more positive affect than elementary school teachers toward mathematics, we must help secondary school teachers to consider not only what mathematics they are teaching but also the experiences they create for their students.

We can support prospective and practicing teachers by helping them attend to their own affect toward mathematics. Do they suffer from mathematics anxiety? Are they affected by timed tests? When they experience what Mandler referred to as interrupted plans, how do they interpret the experiences? Do they have experiences of mathematical intimacy? Do they strive for mathematical integrity, that is, the ability to
know whether they understand—or have years of being falsely assured that they understand mathematics undermined their abilities to view their own mathematical understanding objectively? When they publicly reflect upon their own experiences, they may also reflect upon the experiences of their colleagues, and in so doing, they may begin to understand the wide range of experience represented by their colleagues and, thus, by the students they one day will teach.

**Constructs, Measurement, and to the Future**

As researchers, we do not study beliefs or affect in general; we study them in context. Researchers studying teachers’ beliefs or affect are generally careful to make explicit what beliefs or affect they are studying, and this practice has facilitated convergence upon common definitions in the field. Many researchers view knowledge as belief with certainty and values as deeply held, even cherished, beliefs. Researchers studying affect tend to include beliefs as a component of affect, but the majority of research on teachers’ beliefs has not included affect.

Ma (1999) found more definitive results in a meta-analysis about mathematics anxiety than he and a colleague found in a meta-analysis about attitudes toward mathematics (Ma & Kishor, 1997), and he attributed the difference to the fact that researchers have more effectively operationalized instruments for measuring mathematics anxiety than for measuring the general construct of attitude (Ma, 1999). Clearly, a field of research is extended by researchers’ applying careful definitions of their constructs. But so too is a field extended when researchers consider emergent constructs that encapsulate new ways to consider old constructs. The epistemological commitments entailed by considering teachers’ conceptions, perspectives, orientations, identities, or other general constructs that subsume beliefs or affect are associated with particular methodological commitments. For example, Simon and Tzur (1999) argued that studying teachers’ perspectives involves the investigation of the relationships among teachers’ beliefs, knowledge, values, intuitions, feelings, and practices, and to gather such rich data required a unit of analysis referred to as a set, comprised of observation of at least two consecutive related mathematics lessons and interviews conducted before, between, and after the lessons. In-depth qualitative research, in which data sources are triangulated using a variety of tools, is important for theory building, but for theory testing, researchers need methodologies that can be used with large numbers of subjects. Self-report data are widely used for measuring affect and beliefs of prospective and practicing teachers because such measures are easy to administer and score. However, because people may, at times, be unaware of their beliefs (Furinghetti & Pehkonen, 2002) and may offer opinions on matters about which they have given little thought (McGuire, 1969), Likert scales are of limited use. One promising alternative for assessing beliefs and affect is to employ internet technologies, such as video streaming and data downloading, in developing surveys that require open-ended responses to provide data richer than those gathered in Likert-scale instruments (Integrating Mathematics and Pedagogy, 2003).

We mathematics education researchers must continue to pursue fundamental theoretical questions that move our field forward, often through qualitative research. However, education is a political endeavor, and the past 10 years have found the mathematics education community embroiled in a political struggle for relevance. Until we produce more quantitative studies designed to test theories, too many people in positions of power will continue to ignore our research.

I accepted the task of reviewing the literature on teachers’ beliefs and affect for this volume, and although teachers’ beliefs and affect are closely related, I was unable to integrate the review of these two areas. Perhaps in the future, teachers’ beliefs and affect will converge in such a way as to make an integration more natural. To review all the research on teachers’ beliefs and affect since the publication of the last NCTM Handbook (Grouws, 1992) is beyond the scope of a single chapter, and with the increases in the size of the field and the number of publication outlets, I expect that for a third handbook, the amount of research to summarize will be far greater than the current body. What would such a volume contain—separate chapters on teachers’ knowledge, teachers’ beliefs and affect, teacher education and professional development, and teachers and teaching? Might all these chapters be integrated into one chapter under a common theme, say, teacher identity?

Teachers know; teachers believe; teachers feel; teachers participate; teachers belong. The choice researchers make about which constructs to apply in their work is one of the most important research decisions made. With new constructs come new questions and new methodologies. One way to think about how new constructs emerge and fit into existing constructs is to turn to Bernstein (1999, cited in Lerman, 2002), who distinguished vertical knowledge structures from horizontal knowledge structures. Science provides an example of a vertical knowledge structure in which each new theory can subsume its predecessors, whereas mathematics education is an example of a horizontal knowledge structure because new theories sit alongside
their predecessors. New theories come with their own language and epistemological commitments, and, as such, they cannot be used to refute other theories, but instead they introduce different questions and new methodologies for answering these questions. The research on teachers’ beliefs and affect has grown from psychological theories of learning that focused upon the contents of the minds of individuals, and the field has now adopted sociocultural theories through which researchers look at the world anew. I expect that the newer theories will neither replace nor subsume the old theories but, instead, will sit alongside them. Researchers will continue to study emerging sociocultural constructs, such as teacher identity, that include teachers’ knowledge, beliefs, affect, and more. But researchers will also continue to study these components in isolation, and I think that this work will continue to be important.

REFERENCES


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