A CONTRAST BETWEEN THE PHYSICAL AND THE ENGINEERING APPROACHES TO FINITE-TIME THERMODYNAMIC MODELS AND OPTIMIZATIONS

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1. Introduction

Finite-time thermodynamics [1] is an extension of traditional thermodynamics that seeks to characterize in-principle limits to the performance of thermodynamic processes given the constraint that such processes take place in a finite time. This constraint forces any process that involves transport through a finite conductance to produce entropy and thus leads to strengthened versions of the second law and provides positive lower bounds on the associated entropy production.

The present paper differentiates between the physics and the engineering perspectives regarding the value of such studies and gives some of the highlights of what can be gained from physical (as opposed to engineering) studies of such processes. The difference can be characterized as follows: physical studies seek in-principle limits to what is possible while engineering studies seek to characterize the best possible process design given current technology and current prices.

2. Bounds and the role of optimization

One important reason for studying thermodynamic processes in finite time is to pursue the quest for understanding the limits of what can be achieved in such processes quite aside from any direct implications for engineering. The simplest analyses of this kind reveal how different sources of irreversibility individually influence the limits to power generation and entropy production. In such analyses there is no pretense of counting **all** sources of irreversibility -- one merely counts those sources one is focusing on and thereby obtains **bounds** on what can happen in real processes.

The underlying mathematical fact that allows this is a basic lemma of optimization theory: solving an optimization problem with fewer constraints (additional controls) bounds the value of the objective for the original problem. Analyzing the optimal performance of a system when we improve the control that we are able to exercise, can only improve the optimal value of our objective and thereby gives a bound on what is physically possible.

One important example of this lemma in finite-time thermodynamics is the theorem of Orlov and Berry [2] which is striking in its simplicity and power. They begin by writing down the full dynamical equations for the working fluid, the heat baths, and the power source coupled to a heat engine complete with partial differential equations for the distributed subsystems. The heat transfer to the working fluid is proportional to the temperature difference across a surface. They complete the careful formulation of their control problem by specifying an objective function f. They then switch to a problem in which the temperature at each position inside the working fluid and the heat exchangers is controllable. This discards the partial differential equation constraints describing the dynamics of the temperature in the regions *inside* the engine and heat exchangers, thereby changing these temperatures from state variables to control variables. The optimum in the simplified problem bounds the optimum in the real problem since the simplified problem results by effecting better controls. Ignoring a great number of the dynamical equations converts state variables to control variables and enlarges the set of allowed processes. Of course this must be done carefully respecting the laws of thermodynamics -- these are constraints that we must not ignore.

Physically it is easy to ensure that the laws of thermodynamics are not violated. We simply take the additional (fictitious) controls on our process to be reversible, i.e. effected by reversible processes using additional work and heat sources as needed. Thus we assume reversible control over all but a few of the degrees of freedom in our system. We then ask what is the least entropy production, or maximum power output, or the optimal value of various other possible objectives, when we control the rest of the universe in an ideal way -- namely reversibly. This focuses the analysis on the limitations to energy conversion inherent in that part of the system or device which we did **not** assume to be reversibly controllable. The approach is not limited to heat engines and the results bound the possible values of the objective in a more meaningful way than the bounds provided by classical thermodynamics. While the additional controls may be unrealistic from an engineering point of view, they do not violate physical laws and can help us extract information regarding the limitations to real processes imposed by specific modes of irreversibility which are the focus of a specific study. Since typical applications of the Orlov-Berry theorem require the controls to maintain uniform intensities inside a system, they can even be realized by real processes in which the transport coefficients inside a system are large compared to the transport coefficients across the boundary. This is usually referred to as endoreversibility which is often useful in simplifying the analysis of limitations to the behavior of thermodynamic systems[1]. Note that this is not always needed for our analysis but only when the dissipation inside a system is not part of the irreversible modes which are the focus of our study. Note further that the term "endo" is slightly misleading since this argument can be equally well applied to subsystems such as thermal reservoirs which form part of the "environment" and are thus more aptly labeled "exo".

A second theorem due to A. Tsirlin [4] serves to simplify finite-time thermodynamic studies even further. This theorem states that provided the constraints and objective of a

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[1] Prior to the Orlov-Berry theorem, the argument required an assumption of the separability of the time scales for the transport processes within a system and across system boundaries[3].

problem depend only on net (or average) process quantities, i.e. on integrals over time, the optimal way to control the system is to take our controls to be piecewise constant in time. These two theorems make simple models interesting and able to give useful, rigorous bounds regarding what can be achieved during the control of real processes. The simplified problems often lead to solutions which tell us to run the process infinitely fast or infinitesimally slow. The interesting domain is in between: well posed problems which ignore enough irreversibility to be tractable and yet count enough irreversibilities to keep the solution finite. The value of the objective obtained in the simpler optimization bounds the value of the objective in real processes in a more realistic way than a reversible bound, although it is still often very far from what can be achieved in reality.

The results must be used with great care however. Other features of the optimal solution obtained cannot be directly compared with similar features of real processes. A simple and familiar example is provided by the well known equation of Curzon-Ahlborn-Novikov (CAN) [5,6] for the efficiency at maximum power

$$\eta = 1 - \sqrt{\frac{T_c}{T_h}}. \qquad (1)$$

The striking simplicity of this equation, as well as its similarity to the well known efficiency of Carnot, has brought it rapid acceptance. Nonetheless, as pointed out in [8], this efficiency is surpassed in real power plants! The explanation is that this equation does **not** represent the maximum efficiency but rather the efficiency at maximum power. The associated maximum power

$$P = \frac{\kappa}{4}\left(\sqrt{T_h} - \sqrt{T_c}\right)^2 \qquad (2)$$

must be more than the power delivered in any process which corresponds to the CAN calculation augmented by additional dynamical equations. This much is mathematical fact. Note that there is **no** implication regarding a comparison between the efficiency obtained at this power and efficiencies in real processes -- even when the real processes are also designed to maximize power, albeit with additional constraints!

3. Some misunderstandings

One of my goals in the present paper is to respond to certain criticisms of what I call the physics approach to the analysis of thermodynamic processes [7-9]. The discussion above regarding the CAN efficiency is one such issue. A second issue also concerns unwarranted conclusions based on the above formalism. Finite-time thermodynamics makes no claims to the effect that *every* process approaches reversible as the rate is slowed down. Thus Professor Gytopoulos's examples [8] concerning 1000 spark plugs and batteries attack a non-existent claim. Certainly, unless we are able to exercise a great deal of control over the process, slowing it down does not help. The strongest correct statement along these lines which one can make is as follows: Finite process time is a very real constraint and forces any process to produce entropy provided some transport is required and conductivities are finite. When the process time constraint is increased, it is *possible* to use more time although it is *not necessary* to do so unless we assume further restrictions on our controls. Thus we are free to use the same optimal control that was allowed for constrained time τ even after this is relaxed to a larger constrained time $\tau' > \tau$. It thus follows that increasing the allowed process time cannot make the optimal process perform worse by any measure since all we have done is allow additional possibilities. Thus the issue is whether one understands time as a constraint for a control problem or as a traditional physical variable.

Whether or not additional time actually helps improve a process depends on many factors. Transport processes through a finite conductance can proceed closer to reversibly, provided we have adequate control which can take advantage of the longer time. Systems which have an exergy leak, [8] such as electrochemical cells, certainly do not do well when operated very slowly. This is an essential feature of a finite-time thermodynamic analysis for these systems [10].

Another issue related to repairing the road of communications between science and engineering concerns the idea that there can only be one “freely variable reservoir”. Bejan, foremost among authors, has advanced this unfortunate point of view. I begin with a quote from his referee report on one of my papers [2].

> “In my 1996 Am. J. Phys. paper I [Bejan] explained why the Curzon & Ahlborn constant is unrealistic, and why related claims are unfounded. The reason is that the optimization performed by Curzon & Ahlborn is based on the unrealistic assumption that the heat input (amount of fuel) to the engine is a true degree of freedom in the design, i.e., a quantity that varies as freely as the rejected heat. This assumption would hold only in a world with two environments, or two atmospheres ..., certainly not in today’s power plants on earth.
>
> My 1996 paper [9] was a challenge to modelers such as Curzon and Ahlborn to explain specifically (or graphically) where their free input comes from. This challenge was sidestepped by the present authors.”

Let me not sidestep the issue any further. As I hope was made clear in the discussion above, this challenge comes from a mistaken conception of the *physical* subject of finite-time thermodynamics which by its nature seeks bounds through the use of very limited counting. This is also the answer to Bejan’s similar challenge issued in this same 1996 paper [9]

> “Whether the entropy generation estimate of Salamon et al. accounts for all the irreversibilities of the power plant is a topic I would like to open for discussion.”

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[2] He chose to disclose his identity in the report.

Let me reiterate; there is no attempt to count *all* the irreversibility but simply to *bound* the performance by counting only *some* of the irreversibility in order to understand the intrinsic limitations set by the effects of these modes.

Bejan's idea is that we need to count the complete process from the burning of the fossil fuel to the equilibrated combustion gases at atmospheric temperature. According to Bejan "Any other 'entropy generation rate' calculation is incomplete, i.e. irrelevant and useless." I now argue that this statement is completely untenable from a physical, an engineering, and an economic point of view.

Let us begin with the physics. Here the debate is perhaps moot since Bejan has admitted in print that if one is talking about a heat engine operating between the atmospheres of earth and another planet, one can consider both reservoirs to be freely varying. Thus the idea of a single freely variable reservoir is terracentric at best. To focus our analysis on the effects of a particular mode of irreversible interaction, it helps to ignore the irreversibilities of the other processes. This is the essence of the program I outlined above. Thus we are free to treat the combustion process and the residual cooling of the exhaust gases as reversible. Is there something physically sacred about combustion and exhaust (as opposed to, say, the inside of the engine compartment) that we are allowed to take one as reversible and the other not? We can focus our irreversibility analyzer on any mode that we choose and there is no physical grounds for limiting what irreversibilities one chooses to count. Recall that the physics we are talking about understanding is used for the analysis of processes from the molecular to the cosmic scales. There are many more thermodynamic systems than the ones commonly met in mechanical engineering and thermodynamics applies to them all. In fact, some of the finest accomplishments of finite-time thermodynamics have been at the molecular level [11]. The idea of only one freely variable reservoir is irrelevant for a beam of molecules undergoing interactions with a buffer gas and a laser beam.

Are there good engineering grounds for insisting that one must count the irreversibilities in the combustion of the fossil fuels and the exergy thrown away with the exhaust gases? After all, both of these represent very significant exergy costs in most real power plants. In the analyses of these plants for engineering purposes I certainly agree that it is reasonable, nay even crucial, to count the irreversibility of the discarded exhaust. This is not the same as discounting calculations that do not count this irreversibility as "irrelevant and useless". This would discard the engineering study of solar and geothermal power plants not to mention devices such as James Senft's icewater powered heat engines. This is an unaffordably narrow view of engineering. Are we to be forced to count the fossil fuel burned in producing the ice for the icewater engine? Of what relevance is this for the *engineering* problem of how best to design an engine that is to be powered by the temperature difference between a glass of icewater and the atmosphere? The icewater in this engineering problem *should* be counted as a "freely variable" reservoir.

Bejan dismissed the geothermal power plant example by saying that "in the end the hot stream becomes a very expensive per-unit commodity, not a freely varying flow rate as assumed in ...". Let us therefore examine a possible economic basis to the idea of a single freely variable reservoir. Once we let prices become thermodynamic variables we enter the domain of what has been called thermoeconomics [1]. A careful examination then reveals that no reservoir is free. Pumping river water to cool process streams or building cooling towers is not free but has a very real price associated with amortizing the capital costs of the heat exchangers, pumps, etc.. Thus the atmosphere is not exactly freely varying either. Is it a useful approximation to take heat exchange with the atmosphere as free? Sometimes. In a more careful economic analysis, heat from each reservoir has a shadow price and these prices will be of both signs and none of them will equal zero.

4. Some classic results from finite time thermodynamics

In the domain of well posed problems there is a veritable plethora of possible optimizations and analyses by varying the system, the constraints, or the objective. This fact has brought some indignation from various editors flooded with papers working out these various and sundry cases. This argues all the more in favor of a physics which could serve to unify these sundry results. I now cite a few of the steps in the direction of such a physical theory.

Above, I have already cited the classic theorems of Orlov-Berry and Tsirlin. These two theorems enable us to reduce most analyses to the finite dimensional level where simple calculus suffices. Another classic result is a folk theorem due to Gouy and Stodola [12, 13] which was rediscovered several times by various authors [14]. This theorem guarantees that entropy production is always just the exergy loss divided by the atmospheric temperature; thus minimizing loss of exergy is always equivalent to minimizing entropy production. The atmosphere is certainly a unique heat bath in defining this constant of proportionality.

A similar result regarding the equivalence of minimum entropy production and maximum power also holds provided one specifies the initial and final states of all participating systems except the atmosphere and a work reservoir. Once one has committed to a certain change in the exergy of all the participating systems except one (note that any change in thermal energy of the atmosphere carries exergy zero), any uncaptured work is perforce released into the atmosphere and so maximizing power is equivalent to minimizing entropy production. If one includes the possibility of saving some of the exergy for later use, e.g., burn less fuel, use waste heat for cogeneration, etc., the final states can be allowed to vary and minimum entropy generation and maximum power become opposing objectives; minimum entropy production generally means opting for conservation while maximum power means opting for rapid consumption. For example, in a heat engine working between two heat reservoirs with a given entropy excursion[3], $\int |ds|$, where s is the entropy of the working fluid, there is a range of interesting optima with extremes at maximum power and at minimum entropy production [6]. There is interesting new physics at either end and thermoeconomics in between.

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[3] For a simple cycle, the entropy excursion of the working fluid is just twice the width of the cycle in a T,S diagram. See references [3].

For linear heat exchange, simple results abound. The minimum entropy production ΔS^u_{min} goes as the square of the entropy excursion of the working fluid, i.e., is given by

$$\Delta S^u_{\min} = \left(\int_0^\tau |ds| \right)^2 / \kappa\tau \tag{3}$$

where κ is the conductance to the working fluid and τ is the cycle time [3]. The maximum power from a working fluid in a periodically changing environment $T^{ex}(t)$ is given by

$$P_{\max} = \kappa \operatorname{var}\left(\sqrt{T_{ex}(t)} \right), \tag{4}$$

where var indicates the variance in time [15]. If the conductance to the working fluid can vary between the two heat exchange branches, the optimal allocation of area is inversely proportional to the square root of the conductivity [16].

These early results have been worked and reworked, extended and applied many times to many different systems [1, 17, 18]. So far the early results have served most importantly as a paradigm for how to calculate and reason. As argued above, the reasoning involved is delicate. For building a physical theory of such processes, reworking and reformulation is a necessary step toward coming up with a useful and general formalism. Such reworking brings the proper mathematical structure to the surface and allows us to give all of these problems a synthesis within one framework. Such a synthesis is hinted at in a number of recent efforts [19, 20].

5. The Horse-Carrot process and related theorems

One particularly important and general problem concerns controlling a system to follow a prespecified sequence of states. This has been termed a horse-carrot process by analogy since it requires coaxing a system (the horse) to move toward equilibrium with a controlled sequence of baths (the carrots). Here the coupling between the system and its environment is the mode of irreversibility considered -- the environment with which the system interacts is assumed reversibly controllable. For the process in a (sufficiently large) constrained time, the optimal solution is to control the system so the rate of entropy production is constant [21, 22]. The resulting entropy production is bounded by

$$\Delta S^u \geq L^2 \varepsilon / \tau \tag{5}$$

where L is the thermodynamic distance traversed in the state space of the system and ε is a mean relaxation time [23].

The theorem is perhaps more interesting for a staged process in which the system proceeds by a sequence of equilibrium stages. For this case, the horse-carrot inequality takes the form

$$\Delta S^u \geq L^2 / 2k \qquad (6)$$

where k is the number of stages. For minimum entropy production, these stages should be equally spaced in thermodynamic distance $\Delta L = L/k$=constant [24].

The version of the theorem for staged processes has been extended to steady flow processes including fractional distillation [25]. In this analysis, the irreversibility comes from the equilibration of the material that moves to the next tray with the material on that next tray, the latter acting as the bath. This result has very real implications for column design: *Distribute the heating so the temperature profile along the column is in steps of equal thermodynamic distance.* Economic use of the design requires a heat pump which can work efficiently between many temperature stages.

6. Mathematical structure

As the above discussion illustrates, understanding the physics is different from understanding the engineering. Physicists are not directly interested in how to design or operate a specific device or process but rather in understanding what limitations are set for the process by the thermodynamic laws. In mathematical physics, such understanding usually comes by studying the mathematical structure of the set of possibilities. This is not a problem that engineers have been concerned with, even when very similar optimizations were performed for very similar systems. It is a problem worth studying.

Gibbs' reformulation of thermodynamics focusing on the geometric structure of state space is a prime example of the type of mathematical structure I have in mind here. So is the Horse-Carrot theorem described above. The theorem has served to elucidate the Riemannian geometric structure on the state space[4] of a thermodynamic system -- a structure which turns out to extend from the quantum mechanical to the macroscopic level and which has given us an improved fluctuation theory accurate to larger fluctuations [26].

7. Further difficulties of communication

There exist nontrivial barriers to communication between physicists and engineers working in this area even at the level of how we use the English language. To make this point, I cite an anecdote regarding the use of the phrase "industrial example". In theorist-speak, this phrase means that *some* industry might care about it *some* day. In the referee report cited above, Adrian Bejan, dismissed geothermal power plants as "hardly an industrial example" since their "share of the global energy industry is meaningless."[5]

As a second example I consider the word "applied". In a mathematics department, this word is used to mean that the arguments and results relate to something outside the

[4] The Riemannian metric is given by the second derivative matrix of the entropy of the system as a function of the extensive variables.
[5] We remark that in Iceland, the overwhelming majority of the energy industry is geothermal. Thus the perspectives offered by different cultural and ethnic affiliations also create barriers to communications.

sphere of mathematics, even when this "use" is extremely esoteric, e.g. characterizing certain cross sections of a fiber bundle over a seventeen dimensional space-time. Note that this use is very far from what "applied" typically means in a physics department which is still dramatically different from what "applied" means in an engineering department.

Other examples of words that mean very different things to engineers, physicists, and mathematicians include "realistic", "general", "optimal" and even "understanding".

8. Conclusions

In our modern multicultural and multidisciplinary world it is important to adopt a wider perspective. While the type of physical studies outlined above may be of interest to engineering design only in its earliest stages, there is a long tradition of physics *ultimately* leading to principles important for engineering. It is important to accept the fact that our field is interdisciplinary and to try to move beyond the perspectives offered by our own, narrowly defined disciplines.

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