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Principles of control thermodynamics

P. Salamon [a,*], J.D. Nulton [b], G. Siragusa [c], T.R. Andersen [d], A. Limon [a]

[a] *Department of Mathematical and Computer Sciences, San Diego State University, 5300 Campanile Drive, San Diego, CA 92182, USA*

[b] *Department of Mathematics, San Diego City College, San Diego, CA 92101, USA*

[c] *Department of Chemistry, San Diego State University, 5300 Campanile Drive, San Diego, CA 92182, USA*

[d] *CAPEC, Department of Chemical Engineering, Technical University of Denmark, DK-2800 Lyngby, Denmark*

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Abstract

The article presents a partial synthesis of progress in control thermodynamics by laying out the main results as a sequence of principles. We state and discuss nine general principles (0–8) for finding bounds on the effectiveness of energy conversion in finite-time.

1. Introduction

This article presents a synthesis of progress using a particular approach to the meaning of time for thermodynamic processes. The approach captures one aspect of the flavor of traditional thermodynamics: that of providing bounds. Our aim is to understand the limiting role of time in a thermodynamic process. Specifically, our quest is to understand the limits to energy conversion processes in which the time evolution is only partially specified, i.e. the sequence of states traversed by some part of the system is given. We then ask the question: Of what total process might this given time evolution of our subsystem be a part? In general there are many possible answers to this question. One of the goals of the endeavors described below is to examine the mathematical structure of this set of possible co-evolutions of our subsystem and its sequence of environments. In particular, we look for extreme points in this set; notably ones that maximize work or minimize entropy production. As a simple example, consider the operation of a heat engine in which a gaseous working fluid traverses a given cycle as specified by a quasistatic locus in its (p,V) plane, i.e. by an indicator diagram. This example in various guises has resurfaced in

* Corresponding author.
E-mail address: salamon@math.sdsu.edu (P. Salamon).

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all approaches to what we classify here as control thermodynamics. The set of co-evolutions of this working fluid with an environment has been modeled and studied in various ways and with various possible thermal contacts with the co-evolving environment. Interesting general statements can be made about minimizing entropy production and maximizing power given the path traversed by the working fluid.

Our framework for attacking this problem of course builds on prior developments in thermodynamics and allied fields such as heat transfer and fluid mechanics.[1] These fields specify the dynamical equations for systems of interest. Generally, the framework we consider has some incompletely specified set of dynamical equations which leave some parameters available for control. Such parameters could represent, for example, the state of the environment. In the standard control problem formulation, this means that some of our variables appear in the dynamical equations but do not themselves have specified time derivatives. In this formulation the variables are divided into two classes: $x=(x_1,x_2,\ldots,x_n)$ and $u=(u_1,u_2,\ldots,u_n)$ where the xs are those variables for which we have a dynamical equation and the us are the rest. Thus the dynamical equations take the form

$$\dot{x}=f(x,u). \tag{1}$$

In the control theory literature, the xs are called the state variables and the us are called the control variables.[2] Specifying the values of the us results in a complete system of equations for the xs. One can then ask for the optimal controls $u^*(t)$.[3] A weaker, but much more generally tractable, question concerns finding bounds on the resultant optimal values of the objectives. These questions constitute the essence of control thermodynamics.

This finite-time control approach to thermodynamics has been pursued by engineers for a long time [3]. The present paper is concerned with the extraction of general principles which apply to studies using this approach.[4]

2. A little history

In the early 1970s, four “groups”[5] working independently developed some general principles governing optimal control of thermodynamic processes in finite time. These groups were Bejan (working alone as an undergraduate student and later as a graduate student) at MIT, Berry, Andresen, Salamon, and Nitzan in Chicago, Rozonoer and Tsirlin in the Soviet Union, and Curzon

[1] This dependence of our approach on its allied fields has been emphasized in Bejan’s textbooks where he depicts this new subject using a triangle with edges labeled by thermodynamics, heat transfer, and fluid mechanics [2].

[2] Note that this usage conflicts with the thermodynamic use of these words since at least some of the control variables in a problem may well coincide with a thermodynamic state variable of some system.

[3] Or, better still, the optimal *feedback* controls $u^*(x)$ where the values of the control variables are given in terms of the state variables rather than in terms of the time.

[4] We caution the reader that the authors whose works are discussed here do not necessarily see their work as “control thermodynamics”.

[5] Strictly speaking, the first of these “groups” (Adrian Bejan) was a single individual. Furthermore, his association with MIT was only as an undergraduate and a graduate student. Despite this, we refer to four groups for convenience.

and Ahlborn in Canada. While many of their results coincided, each group has made important contributions; a sample result from each group is presented later in this paper.

While the approach of Berry and co-workers [4–7] and of Curzon and Ahlborn [8] treated reciprocating operation of heat engines, the approach of Bejan [2,9] was based on steady-state operation of a distributed cycle. The difference in focus between these two approaches implies a real physical difference in the way the processes are conceptualized. In the case of a steady-state operation, we envision the working fluid as flowing continuously around the apparatus, with some portion of the fluid in each of the states along the quasistatic locus at each instant of time. Each point on this locus then corresponds to a particular physical location in the apparatus. In the case of reciprocating operation, we envision one homogeneous copy of the system in contact with one environment after another. Thus, in this case, the state of the entire working fluid is only one point on the quasistatic locus at any one time. A useful way to think of the difference between these two conceptualizations is as distributed in space or time, respectively. This idea of characterizing the process as a distribution is central in the approach of Rozonoer and Tsirlin [10]; it can then be applied to distributions in space or time with equal ease. The equivalence of these descriptions for most problems is implicit in most engineering books.

One of the early findings of the Berry group was the existence of a number of interesting objective functions for thermodynamic processes in finite time [11]. In particular, two interesting optimal strategies represented the polar extremes of the *go-all-out* strategy versus the *save-for-tomorrow* strategy. For heat engine processes, these represent the extremes of maximum power versus minimum entropy production. Where exactly the economic optimum lies depends on the relative prices of work and lost exergy. The general nature of these two extremes with intermediate economic optima is the first important lesson of finite-time thermodynamics.[6] Bejan used a different approach which missed the distinction between these two objectives because he begins his analyses by assuming that a certain amount of exergy has been consumed [12]. This assumption comes in the form of a given burn rate for the fuel and certainly represents a problem of interest. In this case, however, no save-for-tomorrow strategy is possible; once one commits to the consumption of a certain fixed amount of exergy (or rate of exergy consumption), maximizing work (or power) becomes equivalent to minimizing entropy production (or entropy production rate). Given that we burn a fixed amount of exergy, the less we waste the more we get [13]. When the specification of the process is less stringent in constraining the amount of exergy consumed and allows for the possibility of saving some of the exergy for later use, maximizing work (or power) is no longer equivalent to minimizing entropy production (or production rate).

3. Principle zero

Let us begin by focusing on a simple thermodynamic system[7] undergoing a process that we can represent as a sequence of states in its state space of thermodynamic variables. Thus we take

[6] See also Fig. 1 and the related discussion following Principle 7.

[7] Recall that Callen defines simple systems as "systems that are macroscopically homogeneous, isotropic, uncharged, chemically inert, that are sufficiently large that surface effects can be neglected, and that are not acted on by electric, magnetic, or gravitational fields." See [14], p. 8.

what most texts would call a quasistatic process of a simple system and ask what the constraint of a given time might mean for such a process.[8]

Our first result concerns something we choose to call

Principle 0 (Callen [14]): *If we are given infinite time, we can make the quasistatic process of our simple system reversible.*

The result appears implicitly in many works on thermodynamics. A fairly explicit discussion of this point was added to the second edition of the book by Callen [14].[9] More precisely, given a sufficiently large time, we can make the entropy production as small as desired.[10]

To see the validity of Principle 0, consider bringing the system along the quasistatic locus by successively placing it in contact with a very large copy of itself in a state corresponding to a point a little further along the quasistatic locus. This causes the system to equilibrate toward this next state and we then proceed by a sequence of (possibly incomplete) equilibrations. As we make these equilibrations smaller and smaller, the process produces less and less entropy although it slows down. To illustrate this point, we pause for a very simple yet concrete example which makes this basic idea explicit.

Example: We consider the example of heating a cup of coffee. We do it in the manner suggested above by placing it in contact with a sequence of slightly hotter heat baths. Assuming that our coffee cup has a constant heat capacity, C, and that we wish to heat the cup from T_0 to T_{f}, we ask for the sequence of K temperatures $T_1, T_2, \ldots, T_K = T_{\mathrm{f}}$ such that the K complete equilibrations of the cup starting from T_{j-1} and ending at T_j, $j=1,\ldots,K$, minimize the total entropy production.

The entropy production for step j is

$$(\mathrm{d}S_u)_j = \mathrm{d}S_{\mathrm{cup}} + \mathrm{d}S_{\mathrm{room}} = C\left(\frac{1}{T} - \frac{1}{T_j}\right)\mathrm{d}T. \tag{2}$$

$$(\Delta S_u)_j = \int_{T_{j-1}}^{T_j} \mathrm{d}S_u = \int_{T_{j-1}}^{T_j} C\left(\frac{1}{T} - \frac{1}{T_j}\right)\mathrm{d}T. \tag{3}$$

The total entropy production ΔS_u is then

$$\Delta S_u = \sum_{j=1}^{K} \Delta(S_u)_j = C\left(\ln\frac{T_{\mathrm{f}}}{T_0} - K + \sum_{j=1}^{K} \frac{T_{j-1}}{T_j}\right) \tag{4}$$

[8] Note that we need internal time scales fast compared to the time evolution of the process or the system is not in internal equilibrium states during the process. For this reason some authors [14] only think of the point as appearing infinitely often along the path, i.e. on a countable subset, rather than following the path as a continuum.

[9] See Sections 4.2 and 4.3. Note that this does not mean that any process can be made reversible in infinite time. The principle only applies to the traversal of a sequence of states in a simple system. See, for example, [3] for a discussion of several examples which cannot be made reversible in infinite time.

[10] Given any $\epsilon > 0$, there exists a time τ_0 such that for times τ larger than τ_0 it is possible to control the system along the given quasistatic locus with entropy production less than ϵ.

which is minimized by the sequence of temperatures

$$T_j = T_0\left(\frac{T_K}{T_0}\right)^{\frac{j}{K}} \tag{5}$$

with the minimum entropy production given by

$$(\Delta S_u)_{\mathrm{opt}} = \frac{C}{2K}\left(\ln\frac{T_{\mathrm{f}}}{T_0}\right)^2. \tag{6}$$

We will have more to say about this example in connection with Principle 5 below. For now, we note only that near equilibration with slightly hotter baths would produce slightly more entropy but would assure us of reaching the next temperature stop in the sequence in a finite time. The larger the time allotted, the less this extra temperature has to be in each case. We note also that the minimum entropy production goes to zero as $1/K$.

What Principle 0 tells us is that unless we limit the time allotted to the process, the answer for minimum entropy production turns out to be the reversible process in which the environment is always in equilibrium with the specified subsystem. We emphasize that Principle 0 does not say that any process approaches reversibility as we slow it down. It does say that a simple system can be made to traverse any quasistatic locus in its state space reversibly in infinite time. This leads to general principles by considering the case of very slow processes; for large times, asymptotic analysis can give general answers. This was the reason why the Berry group published their work under the rubric of finite-time thermodynamics. As a criticism of this name, it has been pointed out that a reversible operation can also be achieved in the limit of infinite thermal conductance for the coffee-cup example above or, more generally, infinite resources of some sort. For this reason, Michel Feidt advocated calling this body of work finite-resource thermodynamics.[11] While this argument has some validity, we believe that the role of time is special and deserves to be singled out. Nevertheless, we adopt the name control thermodynamics to describe these results.

4. General principles of simplification

Our first principle (Principle 1) says that if we are interested in bounds on process parameters, it is sufficient to consider lumped systems. The exact generality of this principle is not known; Orlov and Berry [15] published the proof for the special case of maximum power from a cycling working fluid with access to two constant temperature heat reservoirs. It is clear from their proof that the result is very general; it follows from a basic lemma of optimization theory—solving an optimization problem with fewer constraints (more controls) bounds the value of the objective for the original problem. The optimal performance of a system when we have more control over it can only improve the optimal value of our objective and thereby gives a bound on what is physically possible.

Orlov and Berry begin by writing down the full dynamical equations for the working fluid

[11] This suggestion was made during the discussions at Eurotherm98 in Nancy, France, June 1998.

complete with partial differential equations for the distributed subsystems: the heat baths, and the power source coupled to a heat engine. The heat transfer to the working fluid is proportional to the temperature difference across a surface. They complete the careful formulation of their control problem by specifying an objective function. They then switch to a problem in which the temperature is controllable at each position inside the working fluid and the heat exchangers. The switch is achieved by discarding the partial differential equation constraints describing the dynamics of the temperature in the regions inside the working fluid. They thereby change these temperatures from state variables to control variables. The optimum in the simplified problem bounds the optimum in the real problem since the simplified problem results by effecting better controls. Ignoring a great number of the dynamical equations converts state variables to control variables and enlarges the set of allowed processes. Thus it can only improve the objective.

Assuming the possibility of such fictitious controls to bound the possible behavior of the system, the optimal control for the intensities inside the working fluid turns out to keep the systems homogeneous, thereby making the process "endoreversible". This term, coined by Morton Rubin [16], was chosen to mean internally reversible since the result is to eliminate any internal irreversibilities in the system. This gives a tremendous simplification of the problem since now the number of degrees of freedom describing the system has been reduced from infinite to a small finite number.

Principle 1 (Orlov and Berry [15]): *Optimal control of a simple thermodynamic system in finite time is bounded by the optimal control of an endoreversible version of the process.*

By Principle 1, we can obtain bounds on what is possible by focusing only on lumped as opposed to distributed systems. We can conceptualize this in terms of time scales by envisioning that all the internal modes of our systems equilibrate instantaneously.

Principle 2 is due to Rozonoer and Tsirlin [10]. This principle is very general and also serves to restrict the number of variables required for the problem. The result is derived in a formalism that views the process as a distribution over the set of states and is of a rather technical nature. The conclusion severely limits the number of parameters needed in the optimization and is simple to state and understand.

Principle 2 (Rozonoer and Tsirlin [10]): *In optimal control problems where the constraints and the objective depend only on net (or average) process quantities, i.e. on integrals over time, the optimal way to control the system uses controls that are piecewise constant in time. The number of constant branches in these controls is never more than the number of constraints plus one.*

Principle 2 reduces most of the questions of control thermodynamics to problems in n-variable calculus. Combining Principles 1 and 2 shows that simple, analytically tractable models can be interesting and can give useful, rigorous bounds regarding what can be achieved during the control of real processes.

Example: Consider once again the case of a cycling working fluid in a heat engine. Using the temperature of the surroundings as the control parameter, the problem of extracting maximum power can be elegantly formulated with only a single constraint insuring that the net change in the entropy of the working fluid be zero [17]. This implies that the number of constant temperature branches in the optimal solution is at most two—in this case corresponding to the maximum and the minimum available temperatures.

5. Some principles related to minimum entropy production

Our next three principles are asymptotic in nature and concern minimum entropy production in the slow process limit. They continue the line of reasoning following Principle 0 which is exemplified by the coffee-cup example. This particular route to approaching a reversible process has become known as a horse–carrot process by way of an analogy in which a system (the horse) is coaxed along a sequence of states by controlling at each instant the state of its environment (the carrot). Heating the cup of coffee in the example above was one instance of a horse–carrot process. This principle gives a general bound of the sort discussed above—any other way to coax the system along the quasistatic locus will produce more entropy.

Principle 3 (Constant $\dot{S}_u$ Principle): *For a sufficiently slow thermodynamic process of a simple system, the total entropy production is bounded by the entropy production in a corresponding horse–carrot process which keeps the rate of entropy production constant.*

Principle 3 was announced almost simultaneously by Spirkl and Reiss [23] and by Diosi et al. [22]. A more general claim along these lines was published by Tondeur and Kvaalen [18] but its exact status remains unclear at this time.[12]

Closely related is the result of Salamon and Berry [19] which gives a general bound on the entropy produced.

Principle 4 (Horse–Carrot Inequality [19]): *In a sufficiently slow process in which a system traverses a given quasistatic locus, the entropy production is bounded below by*

$$\Delta S_u \geq \frac{L^2 \bar{\epsilon}}{\tau} \tag{7}$$

where L is the thermodynamic length of the quasistatic locus, τ is the duration of the process and $\bar{\epsilon}$ is a mean relaxation time.

The thermodynamic length L appearing in this and the following principle is measured by using the second derivative matrix of the entropy of the system as a Riemannian metric on the set of equilibrium states [20]. The weighted average that goes into the mean relaxation time $\bar{\epsilon}$ weights the different values of the relaxation time ϵ by the thermodynamic distance traversed, i.e.

$$\bar{\epsilon} = \frac{\int \epsilon \, \mathrm{d}L}{L}. \tag{8}$$

This way of averaging ϵ prevents this bound from being useful in determining the optimal control for the process although it does not compromise its usefulness as a bound. A “discrete steps” version of this principle due to Nulton et al. [21] does not share this weakness. By a discrete version, we mean that the environment cannot be continuously controlled but is allowed only K distinct states. For this case, the principle leads directly to a prescription for what control will achieve the minimum entropy production.

Principle 5 (Discrete Horse–Carrot Inequality [21]): *In a process of K near equilibrations of*

[12] See also the discussion following Principle 6.

a simple system to states along a given quasistatic locus, $K\gg 1$, the entropy production is bounded below by

$$\Delta S_u \geq \frac{L^2}{2K}. \tag{9}$$

The minimum entropy production control satisfies Eq. (9) with equality. Such control requires that the nearly achieved equilibrium states along the quasistatic locus be equidistant as measured by thermodynamic length. Furthermore, for large time, the optimal allocation of time to the process is to spend the same number of relaxation times on each of the K (partial) equilibrations.

The example in Section 3 regarding the cup of coffee is a special case of this principle. More interesting examples are steady-state industrial processes which proceed by stages, notably distillation.

Our final principle regarding minimum entropy production concerns a constrained total rate of transport with the controls being the forces across which these transports are to take place.

Principle 6 (Equidistribution of Thermodynamic Force Principle [18,24]): *When a certain total flow must be maintained across any of n resistances $R_1,\ldots,R_n$, and these flows may be rearranged by controlling the thermodynamic forces at each resistance, then the control that minimizes the total entropy production rate is to keep the thermodynamic forces across all the resistances equal.*

Example [25,26]: Consider the simple circuit shown in Fig. 1. Two resistances R_A and R_B are connected in parallel to ground at point C. The voltages applied at the other ends of the resistors, points A and B in the figure, are the controls for the problem. The constraint is to pass a total current

$$I_{Tot}=V_A/R_A+V_B/R_B \tag{10}$$

to point C while minimizing the total entropy production

$$\dot{S}_u=V_A^2/R_A+V_B^2/R_B. \tag{11}$$

To minimize the total entropy production rate, we need to keep $V_A=V_B$. Note that this makes the entropy production rate different at the two resistors.

This principle was introduced by Sauar et al. [24]. Their reasoning followed closely that of Tondeur and Kvaalen [18] who formulated the problem of minimum entropy production by con-

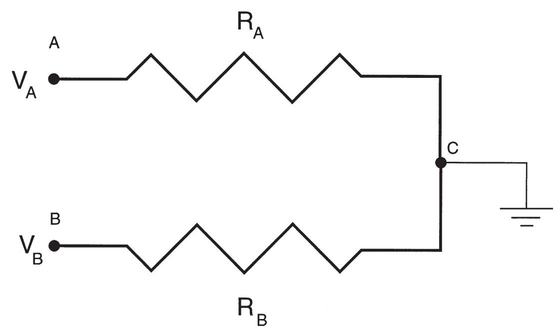

Fig. 1. The figure shows the simple example of a parallel circuit with two resistors R_A and R_B. The example concerns minimizing the entropy production by controlling V_A and V_B while maintaining a given total current.

sidering only the constraints of net flows. Their derivation established both an equidistribution of entropy production principle (a stronger form of Principle 5) and the principle of equidistribution of forces. Indeed, for simple examples, the two coincide. The exact appropriate generality for Principle 5 and Tondeur and Kvaalen's strengthened form of Principle 5 remains open, but see [25,26] for recent discussions.

The way that the four principles in this section count savings in entropy production represents an area of controversy and merits some comments. In these principles, entropy production is counted as due to transport into or out of our simple system. Thus entropy deposited in a slightly higher temperature environment is counted as better than depositing that same entropy into a lower temperature environment, i.e. these principles count entropy production as though a perfect heat transformer were available. Some authors, notably Bejan [12], consider such "savings" bogus since the heat deposited at a higher temperature could well end up degraded to atmospheric temperature, without providing anything useful. How the exergy savings obtained by the optimal control described in these principles is to turn into something useful is best considered as a separate problem. Certainly this gives a way to bound the amount of entropy produced. Whether or not the savings represented by the optimal control are "real" is a separate question depending upon other choices open to optimization. Although no perfect heat transformers are available, it is typically the case that when the optimal control for the simple system is combined with the problem of how to harvest the deposited entropy, the global optimum respects this separation of the problem into two parts. In fact, generally, when the second part of the problem is solved by a heat exchange network using pinch analysis, the problem separates and the global optimum follows Principle 5.

Example: One interesting application of Principle 5 is to a fractional distillation column [28]. The large K domain is reached when the number of plates is about twice the number in a conventional industrial-sized column. For K beyond this number, the equal thermodynamic distance heat integration implied by Principle 5 has been shown to be optimal [27]. One way to achieve the heat integration required (the second part of the problem) is in what is called a pressure-swing column [29]. In this type of column, the rectification portion—the part that needs heat removed—is kept at a higher pressure than the stripping portion, which needs heat added. The problem of minimizing external resources to such a column separates into the problem of how to place the stages in temperature—a problem neatly answered by Principle 5—and how to arrange the heat conductors between the trays.

6. Some principles related to maximum power

Many results exist about the maximum power operation of a heat engine [30]. Since these results are about extracting work as fast as possible, they go in a different direction than the asymptotic approach of the previous section. These results only need to assume that the processes involved are slow enough that the variables remain well defined. The generality of these results is limited, however, by the need for an additional assumption regarding the thermal coupling between the working fluid and its surroundings. For the case of linear heat exchange, in which the heat flow is proportional to the temperature difference, many simple and powerful results have been derived. The two listed below have been selected for their simplicity and robustness.

Principle 7 (Bejan [2,9]): *The maximum power that can be delivered by a heat engine coupled to its surroundings through a constant heat conductor is half the reversible power, i.e. half the work corresponding to the quasistatic locus traversed by the working fluid divided by the cycle time.*

This powerful theorem is very easy to apply and has found application in such diverse examples as thermoacoustic engines [31]. Its exact generality is not known but transcends the linear heat conduction regime for which it has been derived.

Principle 8 (Novikov, Curzon and Ahlborn [8]): *The efficiency at maximum power of a heat engine coupled to its surroundings through a constant heat conductor is*[13]

$$\eta_{\mathrm{MaxP}}=1-\sqrt{\frac{T_{\mathrm{cold}}}{T_{\mathrm{hot}}}} \tag{12}$$

The striking simplicity of this equation, as well as its similarity to the well-known efficiency of Carnot, has brought it rapid acceptance. Unfortunately, it is often misinterpreted; η_{MaxP} does *not* represent the maximum efficiency. In fact, by the general argument we gave in Section 2 regarding the economic optima being a compromise between the go-all-out and the save-for-tomorrow strategies, we always want to operate at efficiencies higher than η_{MaxP}. This is illustrated nicely in Fig. 2 showing power as a function of efficiency. The robustness of this principle is significantly more than the conservative statement above would indicate. For example, Kosloff [33] has shown that η_{MaxP} is given by Eq. (12) even for a quantum mechanical engine composed

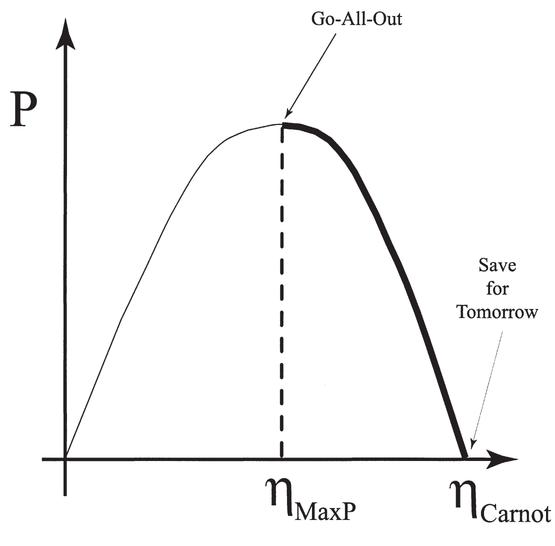

Fig. 2. The plot shows power as a function of efficiency for a simple Curzon–Ahlborn-type heat engine. For any fixed power, it is better to operate on the right branch shown with a thicker line. Note that this implies that the efficiency is always greater than η_{MaxP}.

[13] Bejan has pointed out that formula (12) occurs in several works that precede Curzon and Ahlborn's paper. For a thorough discussion, see [32].

of a three-level system coupled radiatively to two heat baths and producing work in the form of laser light.

We conclude this discussion by mentioning a certain controversy regarding this last principle [3,12]. Bejan believes it is problematic on the grounds the it has no practical counterpart. We believe there is no shortage of practical counterparts. He believes that the rate of heat input to a heat engine cannot be varied (as the derivation of the principle assumes) but must be set as a design consideration. We are not aware of any heat engines that have been built without some equivalent of a throttle to control the rate of heat input. For further details, the interested reader is referred to another manuscript [13].

7. Conclusion

The above survey gave a qualitative overview of control thermodynamics. Our thesis in this work was to show that a coherent subject is starting to emerge. The exact choice of the results we relegated to the status of principles was somewhat arbitrary and several more principles could have been added. Our criterion for inclusion was first and foremost generality. For example, we have avoided results which explicitly depend on the thermal conductance κ.

The principles presented naturally fall into three categories:

- principles of problem simplification;
- principles related to minimum entropy production;
- principles related to maximum power.

The latter two categories represent the extremes of the *go-all-out* and the *save-for-tomorrow* strategies. Economic optima always choose a strategy somewhere between these two extremes. Exactly what tradeoffs are used by the best economic strategy depends on prices. The strategy corresponds on the process level to what economists at the consumer level call the marginal propensity to consume; it measures the tradeoff between foregone future production (resp. consumption) and extra current production (resp. consumption).

We emphasize that while Principles 3–5 regarding minimum entropy production do not need additional assumptions regarding the dynamics of heat exchange, they are asymptotic in nature and do require the assumption of a slow process. Since in the slow process limit the forces must be small, this, in a sense, is already enough to imply linear heat conduction. Another difference between the principles in Sections 5 and 6 concerns their applicability. While principles relating to maximum power require a cycle, Principles 3–5 relating to minimum entropy production apply to any path.

The mathematical structure of the set of processes that correspond to a quasistatic locus in the set of states of a simple thermodynamic system is interesting and only partially understood. As the foremost precedent, we note that J.W. Gibbs' great contribution to the subject came after the realization and characterization of the mathematical structure of the set of states as a surface which he represented as the graph of the entropy of the system as a function of the extensive variables. Today we interpret this as saying that the mathematical structure on the set of states is what geometers call a manifold [34]. Principles 4 and 5 establish that there is a Riemannian structure on this manifold and that such a structure is closely related to minimum entropy production processes. This is the mathematical structure that arises out of the asymptotic considerations in Section 5. The mathematical structure that arises from the maximum power problems is not as well understood. The question of what structure underlies maximum power control is open and important.

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