PHYSICS VERSUS ENGINEERING OF FINITE-TIME THERMODYNAMIC MODELS AND OPTIMIZATIONS

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1. Introduction

Finite-time thermodynamics [1] is an extension of traditional thermodynamics that seeks to characterize in-principle limits to the performance of thermodynamic processes given the constraint that such processes take place in a finite time. This constraint forces any process that involves transport through a finite conductance to produce entropy and thus leads to strengthened versions of the second law and provides positive lower bounds on the associated entropy production.

The present paper differentiates between the physics and the engineering perspectives regarding the value of such studies and gives some of the highlights of what can be gained from physical (as opposed to engineering) studies of such processes. The difference can be characterized as follows: physical studies seek in-principle limits to what is possible while engineering studies seek to characterize the best possible process design given current technology and current prices. Most of the papers at this conference describe the engineering perspective; the present paper describes the physics perspective.

2. Bounds and the Role of Optimization

One important reason for studying thermodynamic processes in finite time is to pursue the quest for understanding the limits of what can be achieved in such processes quite aside from any direct implications for engineering. The simplest analyses of this kind reveal how different sources of irreversibility individually influence the limits to power generation and entropy production. In such analyses there is no pretense of counting **all** sources of irreversibility -- one merely counts those sources one is focusing on and thereby obtains **bounds** on what can happen in real processes.

The underlying mathematical fact that allows this is a basic lemma of optimization theory: solving an optimization problem with fewer constraints (additional controls) bounds the value of the objective for the original problem. Analyzing the optimal performance of a system when we improve the control that we are able to exercise, can only improve the optimal value of our objective and thereby gives a bound on what is physically possible.

One important example of this lemma in finite-time thermodynamics is the theorem of Orlov and Berry [2] which is striking in its simplicity and power. They begin by writing down the full dynamical equations for the working fluid, the heat baths, and the power source coupled to a heat engine complete with partial differential equations for the distributed subsystems. The heat transfer to the working fluid is proportional to the temperature difference across a surface. They complete the careful formulation of their control problem by specifying an objective function. They then switch to a problem in which the temperature at each position inside the working fluid and the heat exchangers is controllable. This discards the partial differential equation constraints describing the dynamics of the temperature in the regions *inside* the engine and heat exchangers, thereby changing these temperatures from state variables to control variables. The optimum in the simplified problem bounds the optimum in the real problem since the simplified problem results by effecting better controls. Ignoring a great number of the dynamical equations converts state variables to control variables and enlarges the set of allowed processes. Of course this must be done carefully respecting the laws of thermodynamics -- these are constraints that we must not ignore.

Physically it is easy to ensure that the laws of thermodynamics are not violated. We simply take the additional (fictitious) controls on our process to be reversible, i.e. effected by reversible processes using additional work and heat sources as needed. Thus we assume reversible control over all but a few of the degrees of freedom in our system. We then ask what is the least entropy production, or maximum power output, or the optimal value of various other possible objectives, when we control the rest of the universe in an ideal way -- namely reversibly. This focuses the analysis on the limitations to energy conversion inherent in that part of the system or device which we did **not** assume to be reversibly controllable. The approach is not limited to heat engines and the results bound the possible values of the objective in a more meaningful way than the bounds provided by classical thermodynamics. While the additional controls may be unrealistic from an engineering point of view, they do not violate physical laws and can help us extract information regarding the limitations to real processes imposed by specific modes of irreversibility which are the focus of a specific study. Since typical applications of the Orlov-Berry theorem require the controls to maintain uniform intensities inside a system, they can even be realized by real processes in which the transport coefficients inside a system are large compared to the transport coefficients across the boundary. This is usually referred to as endoreversibility which is often useful in simplifying the analysis of limitations to the behavior of thermodynamic systems[1]. Note that this is not always needed for our analysis but only when the dissipation inside a system is not part of the irreversible modes which are the focus of our study. Note further that the term "endo" is slightly misleading since this argument can be equally well applied to subsystems such as thermal reservoirs which form part of the "environment" and are thus more aptly labeled "exo".

A second theorem due to A. Tsirlin [4] serves to simplify finite-time thermodynamic studies even further. This theorem states that provided the constraints and objective of a problem depend only on net (or average) process quantities, i.e. on integrals over time, the optimal way to control the system is to take our controls to be piecewise constant in time. These two theorems make simple models interesting and able to give useful, rigorous bounds regarding what can be achieved during the control of real processes. The simplified problems often lead to solutions which tell us to run the process infinitely fast

[1] Prior to the Orlov-Berry theorem the argument required an assumption of the separability of the time scales for the transport processes within a system and across system boundaries [3].

or infinitesimally slow. The interesting domain is in between: well posed problems which ignore enough irreversibility to be tractable and yet count enough irreversibilities to keep the solution finite. The value of the objective obtained in the simpler optimization bounds the value of the objective in real processes in a more realistic way than a reversible bound, although it is still often very far from what can be achieved in reality.

The results must be used with great care however. Other features of the optimal solution obtained cannot be directly compared with similar features of real processes. A simple and familiar example is provided by the well known equation of Curzon-Ahlborn-Novikov (CAN) [5, 6] for the efficiency at maximum power

$$\eta = 1 - (T_c/T_h)^{1/2}. \quad (1)$$

The striking simplicity of this equation, as well as its similarity to the well known efficiency of Carnot, has brought it rapid acceptance. Nonetheless, as pointed out in [7], this efficiency is surpassed in real power plants! The explanation is that this equation does **not** represent the maximum efficiency but rather the efficiency at maximum power. The associated maximum power

$$P = \kappa \, (T_h^{1/2} - T_c^{1/2})^2 / 4 \quad (2)$$

must be more than the power delivered in any process which corresponds to the CAN calculation augmented by additional dynamical equations. This much is mathematical fact. Note that there is **no** implication regarding a comparison between the efficiency obtained at this power and efficiencies in real processes -- even when the real processes are also designed to maximize power, albeit with additional constraints!

3. Mathematical Structure

As the above discussion illustrates, understanding the physics is different from understanding the engineering. Physicists are not directly interested in how to design or operate a specific device or process but rather in understanding what limitations are set for the process by the thermodynamic laws. In mathematical physics, such understanding usually comes by studying the mathematical structure of the set of possibilities. This is not a problem that engineers have been concerned with, even when very similar optimizations were performed for very similar systems. It is a problem worth studying.

As an example of the type of insight that can be gained from an examination of the mathematical structure, we cite a result that has become known as the horse-carrot theorem [8, 9]. This theorem quantifies the minimum entropy production associated with bringing a system along a specific sequence of states, i.e. with making it traverse a certain path in its state space. The theorem concerns a Riemannian metric structure on this state space[2] and says that the minimum entropy produced in the process is the square of the length of the process divided by twice the number of relaxations. A recent application of the theorem says something of interest to engineering by specifying the optimal temperature profile for a distillation column [10].

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[2] The Riemannian metric is given by the second derivative matrix of the entropy of the system as a function of the extensive variables.

4. Difficulties of Communication

There exist nontrivial barriers to communication between physicists and engineers working in this area. To make this point, I cite an anecdote regarding the use of the phrase "industrial example". In theorist-speak, this phrase means that *some* industry might care about it *some* day. In a referee report, the esteemed organizer of our conference, Adrian Bejan, dismissed geothermal power plants as "hardly an industrial example" since "its share of the global energy industry is meaningless."[3]

As a second example I consider the word "applied". In a mathematics department, this word is used to mean that the arguments and results relate to something outside the sphere of mathematics, even when this "use" is extremely esoteric, e.g. characterizing certain cross sections of a fiber bundle over a seventeen dimensional space-time. Note that this use is very far from what "applied" typically means in a physics department which is still dramatically different from what "applied" means in an engineering department.

Other examples of words that mean very different things to engineers, physicists, and mathematicians include "realistic," "general," "optimal," and even "understanding."

5. Conclusions

In our modern multicultural and multidisciplinary world it is important to adopt a wider perspective. While the type of physical studies outlined above may be of interest to engineering design only in its earliest stages, there is a long tradition of physics *ultimately* leading to principles important for engineering. It is important to accept the fact that our field is interdisciplinary and to try to move beyond the perspectives offered by our own, narrowly defined disciplines.

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[3] We remark that in Iceland, the overwhelming majority of the energy industry is geothermal. Thus the perspectives offered by different cultural and ethnic affiliations also create barriers to communications.