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ON A RELATION BETWEEN ECONOMIC AND THERMODYNAMIC OPTIMA*

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We present necessary and sufficient conditions that a least-cost production system operates within ε of its thermodynamic limit. The theorem can be extended, with the knowledge of the production function, to fix the economically-efficient level of thermodynamic efficiency or effectiveness. The analysis is also extended to cases in which several factors of production have different thermodynamic limits. The theorems, while weak, appear to represent the first precise relationship between economic and thermodynamic extrema.

1. Introduction

Our subject, the establishment of a first and very limited bridge between economics and thermodynamics, is unconventional in its scope. In consequence, it merits careful introduction for practitioners in both of the relevant disciplines. Creating such a bridge has been an elusive but tantalizing goal for a long time [Klein (1971) and Soddy (1922)]. We refer here to the physical content of thermodynamics, not to its mathematics, especially the calculus of variations for functions of several variables, which has been well integrated into economics by Samuelson (1947), or to its relational structure, the analysis of equilibrium for systems with several variables and constraints, which has also been used occasionally in economics [Samuelson (1947), Georgescu-Roegen (1971), Boulding (1964)]. The motivation to make such connections grew with the need for a better understanding of the role of energy in the functioning of the economy, and of how the market allocates energy resources.

Scholars from economics and from the natural sciences have studied energy problems, of course. Both groups have brought their own criteria of effective use of energy and related resources. The economist usually defines an efficient allocation as a Pareto optimum: that allocation of resources such

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that no other allocation would make any one individual better off without making at least one other worse off. Efficiency in this sense is a property of the entire economic system, and therefore refers to relationships among all the variables of the system, including labor as well as physical variables. Bounds in thermodynamic analyses are the natural limits of heat and work determined by the First and Second laws of thermodynamics and by the constraints that define the process under study. These bounds are given by the change in a generalized potential. These potentials exist for all reversible and all quasistatic processes [see Callen (1960), Salamon et al. (1977) and Gibbs (1948)],[1] so that if the range of possible changes in a process or set of processes is well-defined, thermodynamics can tell us the natural lower bound on the work required to make the system operate, or the maximum work that can be extracted, or the maximum heat that can be exchanged. Such an extremum is a condition that applies to physical resources, and thus to only a subset of all the economic variables.

As with many other economic questions, one recognizes that at least two kinds of extrema could be attained. We naturally ask whether there are any relations between them. If there are, we can then ask whether these relationships give us any useful tools for analyzing the allocation of resources. Rather than try to establish a complete structure, we have set ourselves the much more modest task of working forward in small steps, bringing the subject to the scientific community as it evolves. We have been troubled by misunderstandings that have grown out of ardent but unfruitful attempts to meld the two approaches. The physical scientist is tantalized by the notion that an efficient economic system 'ought', in some sense, to use physical resources in the most efficient way possible – meaning 'efficient' according to one of the thermodynamic criteria of merit. The economist, on the other hand, sees the variables governed by thermodynamics as a subset of all the variables of the complex economic system, and often argues that there is no reason that the existence of a thermodynamic extremum should have anything to do with the much more complex economic optimum. The former position has led some people to propose an 'energy theory of value' [see, for example, Hannon (1973), Odum (1973), and Gilliland (1975)]. This simplistic view, predictably, has drawn bitter attacks from economists who call up the criticisms of the antique but analogous labor theory of value [Huettner (1976), and Harcourt and Massaro (1964)]. We hope that our analysis here will begin to clarify some of the confusion in this debate.

We ask the indulgence of both the economists and the natural scientists who may read this. We are quite aware that many things we say are well known in one field or the other. But from our own experience, we believe

[1]A quasistatic process is one in which the system is, within limits of detectability, at equilibrium at every point between its initial and final states. The theorem for existence of potentials is actually much stronger: they exist for any path containing at least a countable set of points at which the system is, within limits of detectability, everywhere locally in equilibrium.

that many of these things are not well known to people in the other field, and their role in logic impels us to repeat them here. Moreover the following discussion deals only with static, equilibrium relationships between thermodynamics and economics. There may well be richer possibilities for systems out of equilibrium, and we hope to explore these, particularly by exploiting general principles of non-equilibrium thermodynamics [Salamon et al. (1977)]. For the present, we restrict our discussion to the more modest confines of equilibrium.

2. Background of the problem

The economic system we describe could be presented in terms of production or of consumption. In the former case, we want to minimize a cost function $\mathscr{C}$ associated with production of a good whose output is Q, or equivalently, maximize Q, if $\mathscr{C}$ is regarded as fixed by a budget. In the latter case, we would maximize the consumer's utility $U(Q)$ associated with consumption of an amount Q of the good. For convenience, we shall use the language of the producer's example; the conclusions hold equally for both, when the conditions are met.

We suppose that the output Q is described by some function of input variables, some of which, say $\{x\}$, including the fuels and non-fuel material inputs, enter into a thermodynamic description of the process, and some of which, say $\{y\}$, such as labor (in the sense of time devoted to monitoring controls), lie outside the domain of thermodynamics. We suppose that the production process allows for some substitution between 'thermodynamic' and 'non-thermodynamic' variables as well as among the physical quantities. The allowed possibilities of substitution must, of course, be well specified in order for the notion of a best process to have meaning. These possibilities may, at one extreme, be the limits of technology on line; at the other extreme, they may be only the laws of conservation and of thermodynamics. We shall discuss both the technologically-restricted case and the case of no special technological constraints.

We suppose that at least one of the input variables to the production system is a fuel or other input acting as a source of work in its precise physical sense, the integral of a generalized force through a generalized displacement. It is common but imprecise to think of this input as identical with energy. The reason is of course that energy may be extracted from a fuel either as work (the quantity we presume to be the desired form of energy in the present discussion) or as heat. Even if heat, e.g. as steam, is a desired joint product with electric power, we can safely assume here that work is the most valuable immediate product from the use of a fuel or other energy source – i.e., work sets the opportunity cost of the fuel. (This is not strictly true; some rate structures make a domestic heat set the opportunity cost, an almost incredible anomaly from a thermodynamic viewpoint.) We assume that the production process can be carried out reversibly or quasistatically.[2] The reversible limit is the thermodynamic limit of efficiency for a process unconstrained with respect to time; we assume that for processes constrained to operate in real time the thermodynamic extremum corresponding to a minimum in work required, or maximum in work produced, will always correspond to a quasistatic process. In other words, we suppose that quasistatic processes are always at least as good at turning stored potential into work as nonquasistatic processes. Given the statement about reversible processes and the assumption about real-time processes, we can call upon this theorem: for every reversible process, or for every quasistatic process, there exists a potential $\mathscr{P}$ such that the maximum work obtainable from the system (or the minimum work required to drive the system) from state A to state B (and in the interval from time t_A until time t_B, for the quasistatic process) is equal to $\mathscr{P}(B)-\mathscr{P}(A)=\Delta\mathscr{P}_{AB}$ [Salamon et al. (1977)]. Hence there is a natural bound to the capacity to make any system, however flexible, carry out a process.

It is important to recognize that $\Delta\mathscr{P}_{AB}$ is not necessarily a bound on the direct fuel energy required to drive the system. Generalized potentials usually include terms such as the product TS of the absolute temperature and entropy of the system. A process may be driven by the decrease in energy, by the increase in entropy or by some combination of both occurring in the driving part of the entire system.

Thermodynamic extrema usually represent unattainable limits. It is useful, therefore, to introduce a term to describe the merit of a system that operates by consuming an amount of work ε greater than the ideal limit. The ratio of actual work derived from a system to the upper bound of what can be derived (or the ratio of the minimum work required for a task to the actual work consumed for doing the same task) is called the *effectiveness*[3] of a process. As a convenient abbreviation, we term a process ε-effective if it derives a fraction $(1-\varepsilon)$ of the maximum work W_{max}, from its fuels and any other sources of work, that is, if its effectiveness is equal to or greater than $1-\varepsilon$.

3. Statement of the problem

The question which the following section will answer is this: Given that a production process operates at an economic optimum, what are the necessary and sufficient conditions that the process also operates at a thermodynamic optimum? We can state this more precisely in terms of a cost function

[2] *Quasistatic* means that temperatures and pressures remain well-defined locally at all times for the materials being processed.

[3] Sometimes called the 'second-law efficiency' also. The concept was introduced by Gibbs (1948).

$\mathscr{C}$, starting with the conditions

$$\left(\frac{\partial\mathscr{C}}{\partial x_i}\right)_Q = \left(\frac{\partial\mathscr{C}}{\partial y_k}\right)_Q = 0, \qquad \left(\frac{\partial^2\mathscr{C}}{\partial x_i^2}\right)_Q \geqq 0, \qquad \left(\frac{\partial^2\mathscr{C}}{\partial y_i^2}\right)_Q \geqq 0,$$

for all x_i, y_k, what other conditions are (1) necessary and (2) sufficient to insure that the work $W(x)$ extracted from the thermodynamic inputs $\{x\}$ satisfies

$$W_{\max} \equiv \Delta\mathscr{P} \leqq W(x)/(1-\varepsilon)?$$

That is, what added conditions make economic optimality imply that a process is ε-effective in a thermodynamic sense? (If the question is stated in terms of a production function, the inequalities are in the opposite direction, of course.)

Note that we are *not*, at this juncture, trying to argue whether conditions satisfying this question are desirable goals for resource allocation. Rather, we take the view that we should not ask whether or when thermodynamics is generally useful for normative decisions until we have accertained the logical relationship between thermodynamics and economics. Only then can we analyze the implications of the answer.

4. The representation of the technology

To represent the productive process, we suppose that we can write Q and $\mathscr{C}$ as C^1 (or R^1) functions

$$Q = Q(\{x\}, \{y\}),$$
$$\mathscr{C} = \mathscr{C}(\{x\}, \{y\}).$$

From the set $\{x\}$ we select out the subset E that provide the work to carry out the process and serve to define the boundary potential $\mathscr{P}$. The term 'work' is used in its broad thermodynamic sense, and therefore includes chemical and electrical work. This set E might be a single fuel, a bundle of resources in fixed proportions, or a set of partially or completely substitutable resources. Each choice corresponds to a different set of technological constraints and to a different potential $\mathscr{P}$. If the set E is a bundle in fixed proportions, then we can use the amount of any one of them as a surrogate for the entire set, and immediately infer the total cost $\mathscr{C}(e')$ of using any amount e' of the set, from a knowledge of the amount e_i' of the ith member of the set. Thus the case of 'energy' resources in fixed proportions need not be distinguished from the single fuel case. We shall deal with the case of a substitutable set separately from the single-fuel case. To make our questions non-trivial, there must be some possibility of substitution for at least one factor in E.

4.1. Single-fuel case

We are concerned here with the set I of all combinations of a single fuel e and the other variables (which we may call y now) that yield an output of at least Q. The boundary of this set comprises all combinations of e and y that yield the amount Q,

$$\partial I = \{e, y \,|\, Q(e, y) = Q\}.$$

This set is conventionally called an isoquant in economics. We shall make the usual assumption that I is a convex set, meaning that if the amount of at least one input is held constant and the amounts of all others are increased by a proportion α, then Q may be constant or rise by a proportion $\leqq \alpha$.

The cost-minimizing or profit-maximizing system will find the lowest total cost, which we can express as a linear function of the amounts e and y_i, and their corresponding prices P_e and P_i. Thus, we wish to

$$\text{minimize } \mathscr{C}(e, y) = P_e e + \sum P_i y_i,$$
$$\text{subject to } Q(e, y) = Q_0.$$

The convexity conditions ensure that at any point where the first-order conditions are met, the second-order conditions are also satisfied. Note that we do not need to analyze all of the implication of economic optimality, but only those of cost minimization.

We consider the simple case of a single variable in the set y, with y completely independent of thermodynamic considerations. Fig. 1 is a standard representation of production with inputs y and e, and isoquants Q_1, Q_2, Q_3.

The one piece of thermodynamic information put into this figure is the existence of the *asymptotes* $\mathscr{P}_1$, $\mathscr{P}_2$, $\mathscr{P}_3$ for the corresponding isoquants. The form of the isoquants is conventional, apart from this characteristic which reflects the existence of a lower bound to the work required to produce any given output. We can safely assume that the family of asymptotes constitutes a functional linear in Q, over most or all of the range of Q of any interest to us here, though the density of asymptotes could, in the general case, increase or decrease, corresponding to positive or negative returns to scale, in the thermodynamic limit.

Clearly all points on Q_1 to the left of the vertical line denoted by $e = e_1^* = \mathscr{P}_1(1+\varepsilon)$ are ε-effective. Hence whenever the economic system operates to

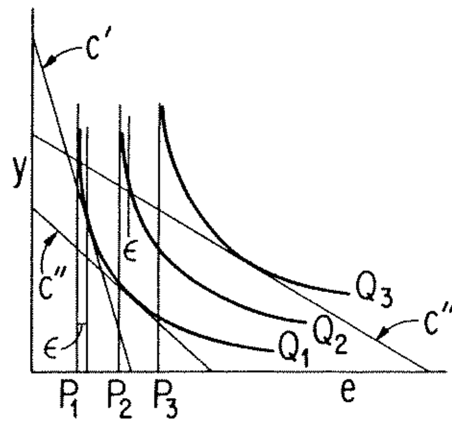

Fig. 1. Production isoquants Q_1, Q_2, Q_3 and lines of constant expenditure $\mathscr{C}'$, $\mathscr{C}''$ and $\mathscr{C}'''$, for production of the good Q from the input factors y and e. The vertical lines P_1, P_2 and P_3 are the thermodynamically-determined limits of the corresponding isoquants, with respect to use of the energy resource e. The vertical lines corresponding to levels of consumption of e that are ε greater than the limits P_1 and P_2 are also shown.

produce the amount Q_1 by using e_1^* of e and the corresponding amount of y as defined by $Q_1 = Q_1(e, y)$, the production system is ε-effective. This much is tautology and makes no use of the condition of cost minimization.

The condition of cost minimization for fixed output or output maximization for fixed costs is represented as follows. We take one input as the price numeraire or normalizer and measure all costs with respect to the cost of that factor. If cost is a linear function of all amounts, the surfaces of constant cost are hyperplanes in general or lines in the 2-factor case of fig. 1, as shown. (Linearity here is assumed for convenience; it is not necessary to the argument.) The condition of cost minimization is met when the output curve Q and the cost line or plane are tangent. The prices are the slopes of the cost lines, measured with respect to the numeraire – in fig. 1, the price of e in terms of y is $(\partial\mathscr{C}/\partial e)_y/(\partial\mathscr{C}/\partial y)_e = -\mathrm{d}y/\mathrm{d}e$.

The condition we seek is now immediately clear – when the price P_e (relative to the price P_y) is at least as great as the slope of the isoquant at a value of e situated at $e^* = \mathscr{P}(1+\varepsilon)$, the system operates with ε-effectiveness if it is optimized:

$$\left(\frac{\partial y}{\partial e}\right)_Q \leqq P_e^*(Q, e^*) \quad \text{and} \quad \left(\frac{\partial \mathscr{C}}{\partial e}\right)_Q = \left(\frac{\partial \mathscr{C}}{\partial y}\right)_Q = 0$$

imply

$$e \leqq \mathscr{P}(1+\varepsilon) \qquad \text{and} \quad y(Q, e) \geqq y(Q, e^*).$$

Whatever the price of e may be, the cost-minimized system must be ε-effective, if and only if the price is at least as great as $P^*(Q, e^*)$.

If there is more than one variable in the set of non-thermodynamic factors y, the argument can be extended quite simply. The asymptotes $\mathscr{P}$ become planes or hyperplanes of constant e, perpendicular to the e-axis, the isoquants become concave hypersurfaces and the lines of constant cost become planes or hyperplanes that cut all the axes. The points of minimum cost for a chosen level of production are the points at which the constant-cost planes are tangent to the isoquants. The condition of ε-effectiveness is met if the point of tangency of the isoquant surface and isocost plane lies at a point where $e \leqq e^* = (1+\varepsilon)\mathscr{P}$.

Fig. 1 puts quite graphically the difference between the viewpoints of the believer in a pure thermodynamic theory of value[4] (in relatively sophisticated form, not in its most naive way) and that of the conventional economist. Consider the constant-cost line $\mathscr{C}'''$, which we assume now is the locus of the highest-cost combination the buyer can afford. The standard economic arguments put the optimum mix of e and y at the point of tangency of $\mathscr{C}'''$ and Q_3, the highest isoquant with which it shares a common point. One sort of thermodynamic theory of value would decide on its ε and then have the system operate at the isoquant Q_2 that crosses at the e-value just ε greater than its asymptote. No higher isoquant can be reached within ε of its asymptote along $\mathscr{C}'''$, and Q_3 is the highest isoquant for which ε-effectiveness can be maintained. Clearly, the two approaches coincide if the price condition $(\mathrm{d}y/\mathrm{d}e)_e = (\mathrm{d}y/\mathrm{d}e)_Q$ is met at $(1+\varepsilon)\mathscr{P}$ or e^*.

A second kind of thermodynamic theory of value (suggested by an anonymous referee, to whom we owe thanks) formally more sophisticated than the first maximizes $(1-\varepsilon)$ subject to requirements that total costs $\mathscr{C}$ be less than or equal to a maximum $\mathscr{C}'''$, and that output be greater than or equal to a minimum Q_2. This model also has a simple solution, as the referee pointed out, if we assume that $Q(e, y)$ shows constant returns to scale. With this assumption, each straight line through the origin corresponds to a line of constant ε. The intersection of the isoquant $Q_2(e, y)$ with $\mathscr{C}'''$ consists of two points, in general, corresponding to two constant-ε rays and thus to two values of ε. The point to the left, with lower ε and higher y/e, is the optimum by this criterion.

Clearly, neither of these thermodynamic theories of value offers a solution compatible with usual criteria for choice. The first provides the maximum product subject to a fixed budget and an exogenous lower bound on thermodynamic effectiveness. The second provides the minimum acceptable product in order to maximize the thermodynamic function $1-\varepsilon$. If one wished to ascribe special value to saving energy, it appears from our analysis that it would be better to try to evaluate externalized costs and add them to

[4]See, for example, Hannon (1973), Odum (1973), and Gilliland (1975).

the direct cost of energy, thereby tilting the budget lines more steeply, than to introduce *ad hoc* criteria such as those in our two model thermodynamic theories of value. Energy theories of value have been criticized previously [Huettner (1976)], and, as we said, are subject to most of the same vulnerabilities as labor theories of value [see, for example, Harcourt and Massaro (1964), for a discussion of the labor theory].

The growing number of empirical studies of production systems justifies adding a simple corollary to the foregoing argument. Suppose our assumption about the shape of the isoquants is narrowed, by making the optimistic supposition that the isoquant contours Q_j are actually known, e.g. from engineering studies. Under this condition, the tangent point of the chosen budget line $C^{(i)}$ with the highest attainable isoquant $Q_{\max}$ is a known point for a real system. Hence the distance along the e-direction between this operating point and the corresponding thermodynamic asymptote $\mathscr{P}_{\max}$ is completely determined. This distance is the amount of e expended beyond the thermodynamic lower bound. In other words, if the isoquants are known, specification of the budget line is sufficient to specify the economically optimal amount of thermodynamic waste!

This inference is *post hoc* not *ante hoc*, and therefore not very strong. The point of operation is selected by cost minimization or output maximization, which means that the prices and technology determine the optimal level of thermodynamic effectiveness, and not the reverse – at least within the confines of this little corollary.

The proposition just proved consists logically of four conditions:

(A) convexity of the production function,
(B) cost minimization,
(C) thermodynamic optimality, and
(D) the inequality condition on the slope of the production function.

The theorem says, in effect, that A and B imply C if and only if D. It might be tempting to turn the theorem around, to ask what conditions imply B. The well-known condition for this is the equality analogous to the inequality of D. But this equality, together with convexity, is enough to imply B, economic cost minimization, regardless of the thermodynamic condition C. Hence when conditions of economic cost minimization are stated in terms of prices and convex production functions, thermodynamic optimality adds no new insights.

4.2. *Partially or totally substitutable resources*

To amplify the picture, we turn to the case of substitutable thermodynamic factors. For example we may take e to be a fuel, as before, h as the amount of a high-grade ore and l as the amount of a low-grade ore. The low-grade

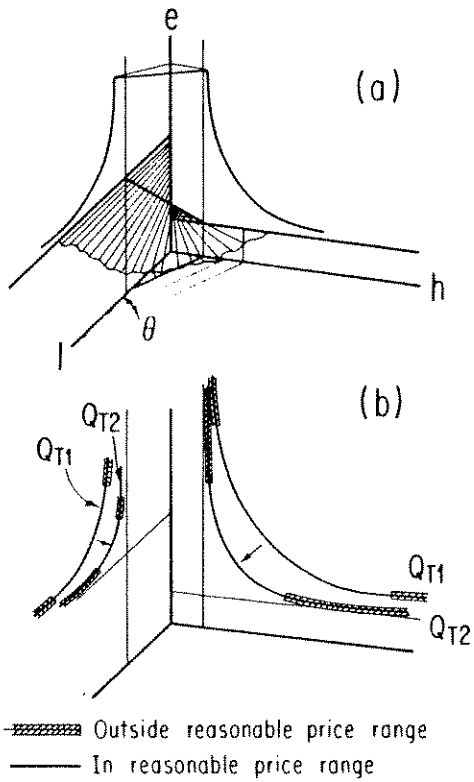

Fig. 2. The production space for a product using an energy resource e, and two kinds of ore, high-grade, h, and low-grade, l, all of which are substitutable factors. The thermodynamic limit is a surface, the ruled surface indicated in (a). In (b) are the isoquants Q_{T1} and Q_{T2} corresponding to two different levels of technology yielding the same amount of product. As technology improves, the isoquants are squeezed into the square corner defined by the thermodynamic and mass-conservation limits. If the producer were to achieve the physical extremum, he would find only a single point of operation for any fixed mix of h and l and a line of possible operating points, depicted in (a), if h and l remained substitutable. Note that if there are 'reasonable' upper bounds to the prices of the factors h, l and e, then the range of substitutability narrows as technology improves.

ore requires some energy e' per tonne of product for its processing; the high-grade ore requires $e'' < e'$ per tonne of product. In effect, the high-grade fuel supplies thermodynamic potential that replaces some of e'. We could also imagine treating fuel mixes in this context, or any other set of physical factors among which at least a limited substitutability is possible. The fuel-and-two-ore example will illustrate the essential relationships. We assume labor and capital are fixed or occur in proportions fixed to the three inputs in this model, in order to keep the picture relatively simple.

The isoquants in this space have natural thermodynamic bounds of the two kinds. One is determined by mass balance and defines a set of planes parallel to the e-axis, whose angle θ (fig. 2a) is determined by $\tan\theta =$(tonnes

h/tonne of iron)/(tonnes l/tonne of iron). The other kind of asymptote is the natural limit on the requisite work, as provided by the fuel or fuel mix e, to carry out the process within the constraints of technology selected for the model. Because the minimum energy input now depends on the mix of ores, these asymptotes form a family of *ruled surfaces* whose intersections with the mass-conservation asymptotes are straight lines. The isoquants are presumably convex, roughly hyperbolic curves in any plane containing the e-axis.

The isocost surfaces in this system are planes in the positive octant. The condition that minimum cost assure thermodynamic 'optimization' in the sense of ε-effectiveness is, as previously, that the point of contact of the isocost plane and its tangent isoquant lie within e^* of the ruled-surface asymptote. A little care is required here, because one must specify whether the l/h mix is fixed, in which case the distance e^* must be taken parallel to the e-axis, or the l/h mix can be freely chosen, which means e^* can be taken as the e-component of the normal from the tangent point to the ruled-surface asymptote.

Just as a process will be ε-effective if the price of fuel is high enough, the process can be made comparably effective with respect to its use of mass. We call the mass effectiveness the ratio of actual amount of product per unit of mass used to the maximum amount of product that could be made within the constraints of the technology, with the same mass input. The process will be defined as μ-effective if the point of operation has a productivity of mass that is within μ of the ideal limit,

$$(\partial Q/\partial M)_{\mathrm{actual}} \geqq (1-\mu)(\partial Q/\partial M)_{\mathrm{ideal}}.$$

Let $M^* = (\partial M/\partial Q)_{\mathrm{ideal}}(1-\mu)^{-1}$, the absolute mass used, per unit output, analogous to e^*. As in all the foregoing arguments, if the price of ore is high enough to put the contact point of the isocost and isoquant surfaces within $M^* - M_{\mathrm{ideal}}$ of the mass asymptote, the process will be μ-effective.

We get a very interesting insight into the trade-offs in technological advance by examining the behaviour of a production system with physical substitutabilities and asymptotes. Fig. 2b suggests that it is very unlikely that a real system would be both ε-effective and μ-effective at the same time. If it were, the thermodynamic and mass conservation conditions would be acting simultaneously to constrain the system to operate within a very narrow range of substitutability, approaching the fixed-coefficients corner case as e^* and M^* are made smaller.

This narrowing is even more important when we conside what happens to the isoquants in the planes containing the e-axis, when technology is improved to make better (i.e., more nearly asymptotic) use of physical resources. In the region of the asymptotes, we cannot expect technological change to be Harrod-neutral, but Hicks-neutrality may or may not obtain. Whether the technological progress we envision is embodied or not is irrelevant to our model. However the kind of technological change that concerns us here is a sort that never decreases the average productivity of a physical resource, so that we can assume that the isoquants never move away from any asymptote when technology changes. For convenience, we shall refer to such technological change as resource-thrifty.

With the sketch of fig. 2b representing any plane containing the e-axis, we see immediately how resource-thrifty technological change acts: it *narrows* the range of substitutability from the wide arc of the 'old' isoquant Q_{T1} to the narrower arc on the isoquant Q_{T2}, associated with the new technology T2. This is the natural consequence of constraining the system to operate ever nearer to a point at which physical considerations determine all factor relationships, the unique mass-and-thermodynamic extremum.

It seems paradoxical that technological change could narrow the range of substitutabilities, rather than raise them, and of course the real change of substitutability never narrows because one could always go back to the old technology. However the practical consequence of resource-conserving technological change has been to move the isoquants as fig. 2b shows, and to be either labor-neutral or labor-augmenting, at least in the industries we have examined, coal mining and cement manufacture [Hebenstreit et al. (1976)]. In both these industries, the productivity of both labor and energy increased for long periods, as technology was improved. The paradox is only apparent, of course. Many processes do undergo technological change that augment substitutability. The narrowing range associated with resource-conserving change carries with it the implication that the change does not introduce possibilities of using heretofore-excluded inputs, or of new labor–resource substitutions that would allow for reducing labor. In this sense, the implication of narrowing substitutabilities suppose that the new technology is at least labor-neutral (and neutral with respect to any other non-thermodynamic variables), as well as that the isoquants move toward asymptotes.

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