Physics 410 Quantum Mechanics

Homework #6

due **Friday, Oct 22, 2010**

Griffiths 2.5, 2.13, 2.23, 2.24 in Griffiths plus the following:

Johnson 6.1: The delta-function potential. Read section 2.5, pages 68-73 (up through equation 2.129). Eqn [2.115] gives the Hamiltonian, while [2.129] gives the bound state solution. Now consider this as a limiting case of the finite square well. Consider a square well from $-a$ to $+a$, of depth $\alpha/(2a)$, so that the total area of the well is α. Let $a \rightarrow 0$. What happens to z_0 (cf equation 2.155)? How many bound state solutions do you expect? What is the energy of the g.s. in this limit?

Johnson 6.2: For the bound state for the delta-function potential (see eqn [2.115] for Hamiltonian, [2.129] for solution]) compute $\langle x \rangle$, $\langle x^2 \rangle$.

Extra credit : compute $\langle p \rangle$, $\langle p^2 \rangle$. Hint: If you get $\langle p^2 \rangle < 0$ you haven't done it correctly. The hard part is the integral around x=0 (this is only for p, p^2, not for x, x^2); the book shows how to do this.

Johnson 6.3: Consider, in the infinite square well for $0 < x < L$, the wavefunction $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right) \exp(ibx)$. (a) Show this wavefunction is properly normalized. (b) Compute $\langle x \rangle$. (c) Compute $\langle p \rangle$.