Histories of modal engagement with mathematical concepts: A theory memo

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In our view, there is nothing about the extended body or covert sensorimotor simulation for conceptual understanding that rules out interactive, cultural, or historical aspects of embodiment. As more research accumulates in this area of interdisciplinary study, this more inclusive view is gaining traction (Ciolek & Kendon, 1980, Gallagher, 2005, Gallagher, 2008, Wilson, 2008, Gallese, 2009). Specifically in studies of mathematical activity, we are now able to ask how people and their bodies (plural) are engaged to use drawing surfaces persuasively, or to build and manipulate physical models that are understood and appreciated as explanations. But as we move beyond the skin to consider tool use in multi-party activity, how can we choose units and levels of analysis that are productive for studies of mathematical activity and learning?

Within the diverse set of proposals reviewed above, three common themes may help us approach embodied mathematical cognition. These are the central role of specific modalities of experience for understanding and using concepts, the constitutive role of acting bodies in mathematical thinking, and possibilities for extending the body (and modalities of experience) associated with making and using representational tools. We propose a framework for thinking about mathematical concepts, how they are created, learned and used as histories of modal engagement. Modalities, in this framework, are more complex than a simple view of the five human senses, such as touch and vision, for the following reasons:

1. Senses are types of body-environment couplings that can be instantiated in many ways (e.g. visual experiences can be generated with sensors acting on the skin, (Bach-y-Rita, Collins, Saunders, White & Scadden, 1969).
2. Perception and motor activity merge at all levels (e.g. vision cannot be understood apart from body motion, (Noe, 2006)).
3. Multisensoriality is pervasive (e.g. talking with someone simultaneously involves hearing, vision, body motion, etc.; (Alac, 2005, Stivers & Sidnell, 2005).
4. Senses are shaped by immersion in culture and tool-use (Goodwin, 1994).

Instead of treating modalities as separate perceptual capacities that might support having or understanding mathematical concepts, we propose a focus on "modal engagements." Modal engagements are a way of participating in activity, with others, tools, and symbols.

We propose that modal engagement can be productively analyzed along four dimensions of the body:

1. Acting Body—Place, spatiality, and body-based scale
2. Encultured Body—Horizon of availability/tool- and symbol-use
3. Expressive Body—Interactive stances
4. Imaginary Body—Temporality

By focusing on dimensions of modal engagement, we propose a new approach to the content and meaning of mathematical concepts under development and in use, attending to how these concepts are enacted with whole bodies, in interaction between people, and with cultural artifacts that carry a rich history for thinking and action. We start from the now substantial body of research available concerning distributed and situated cognition, previewed evocatively in the opening paragraph of Jean Lave's (Lave & Wenger, 1991) widely influential analysis of cognition in practice:

> The point is not so much that arrangements of knowledge in the head correspond in a complicated way to the social world outside the head, but that they are socially organized in such a fashion as to be indivisible. "Cognition" observed in everyday practice is distributed—stretched over, not divided among—mind, body, activity and culturally organized settings (which include other actors). (p. 1)

While there is a large body of work on how cultural settings are organized for thinking and learning, the whole body in action is seldom at the center of analysis. Indeed, even in our own studies (some published in this journal) it has been difficult to bring the body forward in attempts to explain learning, teaching or understanding mathematics.

For example, in Hall (1996), two pre-service teachers used their own, moving bodies as a model for demonstrating why to add rates together when teaching how to solve an algebra story problem in which cars were driven towards each other. Central in that analysis, and important for our arguments here, the first-person perspective of participants in the modeling activity appeared to be critical for establishing the meaning of symbolic expressions for adding rates, and linking these two different experiences—walking through a collision and writing the term structure of a symbolic equation—for arriving at a convincing explanation. Viewed as an ensemble performance with acting bodies moving as cars, participants re-purposed the surrounding instructional space, and their interaction *was* the model for explanatory purposes. As the enacted model progressed, there was also clear evidence of changes in expressive stance for each teacher. Though initially skeptical, the teacher positioned to receive the explanation changed his prosody and gait as the purpose of the activity became clear, and both teachers expressed genuine surprise (perhaps even discomfort) when their bodies collided more quickly than they anticipated (i.e., the sum of rates was much faster; “WOAH! We’re goin faster! RIGHT!” (Hall, 1996, p. 220). Similarly, in Noble, Nemirovsky, Wright and Tierney (2001), the meaning of different representations of the mathematics of change depended on how 5th graders took up or inhabited the teaching environments provided for and constructed through their activity with these representations. Understanding the mathematics of change, for these students, meant being able to inhabit different representational environments and experience a family resemblance among these different conceptual spaces.

Our proposal reflects a growing convergence of ideas at the intersection between research on multimodal human activity in diverse social and technical settings, on the one hand, and cognitive neuroscience research concerning the nature of concepts and conceptual understanding, on the other. We link these ideas to our proposal for four dimensions of modal engagement in the remainder of this section.

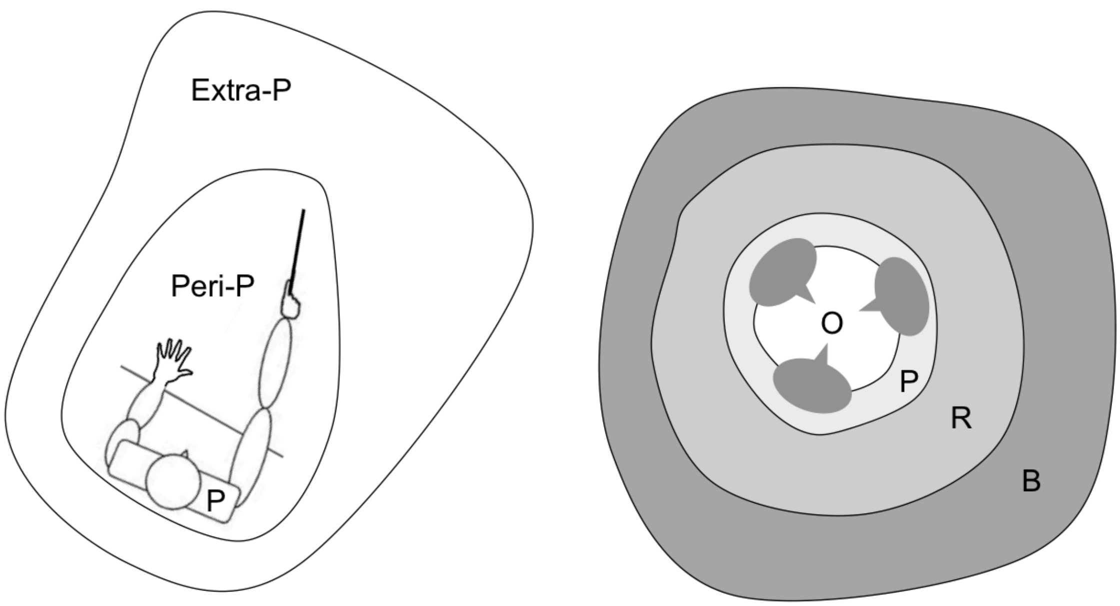

Figure 1. Modal engagement through tools and interaction with others. (Left) Space is experienced at different body scales and can be extended by cultural tools (e.g., a walking stick or a hand-drawn map giving directions; (Farne, Iriki & Ladavas, 2005). (Right) Space is formed in interaction with others, as a place for focused attention and meaningful action that is constantly monitored, and expanded or contracted to support shared activity (Ciolek & Kendon, 1980).

Acting Body—Place, spatiality, and body-based scale. This dimension in our analysis focuses on the experience of space, realized as being in place and at the scale of the body. There is experimental evidence that our experience of space for action is not isotropic, scaled uniformly in all directions (Figure 1, Left). Instead, tools extend one's sense of the body (personal space) to include what is reachable or graspable (peri-personal space). What can be attended to, including actions of others with tools, extends the experience of space still further (extra-personal space). As a result, one's experience of current action, purpose, and affordances for "what comes next" are extensible through engagement with tools and other people in the proximal environment. If cultural tools like maps or given directions are considered, the experience of one's body in space may be extended still further into reachable or navigable space (Schegloff, 1972, Psathas, 1979, Davies & Uttal, 2007).

It is important to emphasize here that the tool-extensible, acting human body is not only made up of the individual experience with cultural tools. As Kendon's studies of "facing formations" in postural arrangements for multi-party interaction demonstrate (Figure 1, Right), people experience each other as forming a place for concerted activity and mutual attention. These formations arrange whole bodies in the proximal spatial environment, experienced and maintained (e.g., against intrusions) as a place for joint action. People focus on what is "in play" together (O-space), manipulate supporting materials without disrupting that action (P-space), and are able to anticipate the arrival of new participants (or disruptions, R-space) and the reachable location of possible resources for current action, even when these are not visible (B-space). These aspects of the acting body in relation to other bodies have received little attention in studies of learning or teaching (for an exception see Leander, 2002). In contrast, the real-time organization of multi-party talk has been much more extensively studied as an environment for learning and teaching, where the sequential organization of talk-in-interaction (Schegloff, 1991), involving both speakers and hearers, provides structuring resources for accomplishing and making sense of joint activity. The meaning we intend for the acting body is that of being "in" or of "making" a place (following Noble, Nemirovsky, Wright & Tierney, 2001, a "lived in space") that includes what is graspable, understandable from a first-person perspective in action, but always sensitive and shaped by others in the proximal environment.

Encultured Body—Horizon of availability/tool- and symbol-use. The simple pointer shown in Figure 1 (Left) stands in, we propose, for a wide array of cultural tools, including symbolic notation and other representational media for doing and learning mathematics. Tools serve as meditational means for purposeful action in use, yet they have little or no meaning or functional consequences alone. In use, tools both extend the body into the world (the Acting body, above), and bring the world into the scale of the body. Possibilities for action with tools depend upon and extend the Encultured body's history of participation in specific cultural activity systems. Tools also structure activity with others, and so provide a durable, historical substrate for "co-membership" (Erickson, 2004, p. 172) in collaborative technical activities, including mathematics. In this sense, cultural tools are frozen aspects of past activity that are taken up in purposeful present activity, and through this engagement shape the course of that activity.

Modal engagement with cultural tools extends the Encultured Body into particular futures, and it is here that the "stretched over" (Lave, 1988, p. 1) metaphor for how cognition is distributed over situations is particularly helpful. Tools offer structuring resources for the Encultured body, and when engaged in use, provide a horizon of available, meaningful action for the person (see also (Hutchins, 2005, Hutchins, 2010). This horizon of availability is experienced by individuals, but is always formed in relation to ongoing cultural activity, either with co-present others or in the frozen, purposeful activity of tool designers. Understanding these relations as forms of modal engagement is critical for a theory of embodied cognition that turns away from texts in the mind as an explanation for mathematical thinking and understanding.

Expressive Body—Interactive stances. By focusing on the Expressive Body, we mean to capture emotional aspects of modal engagement (fear, desire, expectation), as experienced in and produced through a person's stance in ongoing activity, typically with cultural tools and others. Here again, the sense of "with-ness" in joint action—engaging with tools and others in concerted activity—is important, but the Expressive Body draws our attention to how one's stance while participating in activity is experienced and expressed (Goodwin, 2007). A person's stance can express pulling/moving towards or pushing/moving away from ongoing activity, and this can shift as the person negotiates and finds a position from which to participate in (or resist) that activity. This dimension of modal engagement has received relatively little attention in studies of teaching and learning (for an exception see Roth, 2007), and it is here that studies cognitive neuroscience and multimodal analysis may converge in productive ways.

From a perspective of cognitive neuroscience, understanding the intentional states of others ("mind reading," according to Gallese (2007) arises directly, without deliberative inference. Gallese proposes that the mirror neuron system performs embodied simulations providing the basis for conceptual understanding, even if the person merely observes the actions of others without directly contributing to the activity. How the Expressive Body creates and engages with interactive stance is available to our inquiry primarily through multimodal analysis of human activity (Streeck, Goodwin & LeBaron, 2011). This includes analysis of talk-in-interaction with others and cultural tools (as with our analysis of the Encultured Body, above), as well as detailed analysis of attention management and gesture. Specifically, gaze allocation and gaze following are critical aspects of attention, particularly as people build, maintain, or repair trouble with shared intentionality in coordinated activity. Similarly, analysis of representational gesture (Alibali & Nathan, 2007, Gerofsky, 2010) provides evidence for interactive stance that may not be available in talk, alone. For our purposes, gesture production is not a stand-alone form of modal engagement, but depends upon the whole body and its stance in the proximal environment (Ciolek & Kendon, 1980, Goffman, 1983, Streeck, 2002, Streeck, 2009). In this sense, gesture and its representational content or possible meanings are environmentally coupled (Goodwin, 2007), both for speakers and for hearers.

Imaginary Body—Temporality. Finally we draw attention to how imagination for what comes next, as well as reflection on the relation between present and past activity, depends upon the fluid experience of time. This fluidity of time, as experienced and imagined is nicely captured in Erickson's (2004) distinction between metric time (kronos) and time as experienced through modal engagement (kairos):

> *Kairos* is the time of tactical appropriateness, of shifting priorities and objects of attention from one qualitatively differing moment to the next. This is time as humanly experience; "in the fullness of time," the emergent "not quite yet," the "now" that once arrived feels right. It is a brief strip of *right time*, marked at its beginning and end by turning points. It is not simply a particular duration in clock time. Yet every *kairos* strip of time has a location in *kronos* time. (p. 7, italics in original)

In Erickson's (2004) analysis, doing things together (again, with-ness) produces and uses tensions associated with what is to come next. Action-relevant moments in the sequential, real-time organization of human activity are experienced as upcoming slots (i.e., turn-relevant moments), as tensions concerning the direction and outcome of joint activity. Participation in joint, technical activity, in this sense is opportunistic and adaptive, fitting and editing the Acting Body to present circumstances in a way that arrives at the Imaginary Body.

These ideas, developed in careful studies of talk-in-interaction as realized in particular historical contexts, are consistent with proposals about embodied cognition coming from evolutionary psychology and cognitive neuroscience. At the broadest human timescale, the Imaginary Body has developed "for action" (Glenberg, 1997) over a phylogenetic timescale. As a result, conceptual content and modality are shaped by how human bodies fit within and move through varied physical environments, even for seemingly abstract domains like mathematics (Núñez, 2005). At a fine-grained timescale, using or understanding a concept is grounded in embodied simulation over traces of modality-specific, prior experience (Barsalou, 2008). In this sense, to understand a "cup" or a "circle" is to anticipate taking action with that concept (e.g., grasping a cup, or inscribing a circle around a radius and center point). The imaginary body is first a human body, engaged with material and social environments in ways that reflect our endowments, but also a biographical body, drawing upon, renewing, and sometimes changing a history of modal engagement with particular cultural activities. In this sense, the Imaginary Body in mathematical activity both reflects what has come before (e.g., mutually sensible use of conventional representational forms) and opens onto the possibilities for innovation and change.

In summary, for the learning sciences and for new theories of embodied mathematical cognition, there are signs of convergence between studies of multi-modal interaction in diverse sociotechnical practices, including mathematics (Stevens & Hall, 1998, Núñez, 2005, Nemirovsky & Ferrara, 2009, Radford, 2009), and research findings from cognitive neuroscience concerning modality-specific processes of simulation, mind reading, and body extensibility with tools (Farne, Iriki & Ladavas, 2005, Gallese, 2009, Barsalou, 2010). The nature of modality at this intersection of theoretical proposals, however, needs further development. We propose that four dimensions of embodiment—the Acting Body, Encultured Body, Expressive Body, and Imaginary Body—will be useful in this further theoretical development.

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