

Last Name:  
First Name:  
Instructor:

Math 151  
Group Final (Spring 2009)

You are not allowed to use notes, books, calculators, personal stereos or cell phones.

You have exactly two hours.

Write clearly so that you can avoid mistakes and count on partial credits. Carry out the obvious simplifications so that you can display your answers in an easily readable manner.

The following list is for the recording of the points only. Do not write your answers on this page.

Points

- 1            /8
- 2            /8
- 3            /6
- 4            /10
- 5            /8
- 6            /6
- 7            /8
- 8            /8
- 9            /10
- 10           /6
- 11           /8
- 12           /6
- 13           /8

Total:                    /100

1 (8 pts.) Evaluate

$$\int \arcsin(2x) dx$$

2 (8 pts.) Evaluate

$$\int \frac{1}{4x^2 - 8x + 13} dx$$

Hint:  $4x^2 - 8x + 13$  is irreducible.

**3** (6 pts.) Evaluate

$$\int \sin^2\left(\frac{x}{3}\right) dx$$

**4**

a) (6 pts.) Evaluate

$$\int x e^{-x/4} dx$$

b) (4 pts.) Determine whether the improper integral

$$\int_1^{\infty} x e^{-x/4} dx$$

converge or diverges, and its value in case it converges.

**5.**

a) (2 pts.) Why is

$$\int_2^4 \frac{1}{(x-2)^{1/3}} dx$$

an improper integral?

b) (6 pts.) Determine whether

$$\int_2^4 \frac{1}{(x-2)^{1/3}} dx$$

converge or diverges, and its value in case it converges.

**6** (6 pts.) Determine the length of the graph of  $f(x) = \cosh(x)$  on the interval  $[0, 1]$ .

**7.**

a) (6 pts.) Use the technique of an integrating factor to find the general solution of the differential equation

$$\frac{dy}{dt} = -\frac{1}{t}y(t) + \cos(t^2)$$

(Assume that  $t > 0$ ).

b) (2 pts.) Find the solution of the initial value problem

$$\frac{dy}{dt} = -\frac{1}{t}y(t) + \cos(t^2), \quad y(\sqrt{\pi}) = 1$$

**8.**

a) (6 pts.) Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{y^2}{\sqrt{1-x^2}}$$

b) (2 pts.) Find the solution of the initial value problem

$$\frac{dy}{dx} = \frac{y^2}{\sqrt{1-x^2}}, \quad y(1/2) = 1$$

**9.** Let  $r$  and  $\theta$  be polar coordinates.

a) (4 pts.) Sketch the graph of  $r = \cos(2\theta)$  in the Cartesian  $\theta r$ -plane on the interval  $[0, 2\pi]$  as a polar equation in the  $xy$ -plane (i.e.,  $x = r \cos(\theta)$ ,  $y = r \sin(\theta)$ ). Indicate the values of  $\theta$  at which  $r = 0$ .

b) (6 pts.) Sketch the graph of  $r = \cos(2\theta)$  as a polar equation in the  $xy$ -plane (i.e.,  $x = r \cos(\theta)$ ,  $y = r \sin(\theta)$ ).

**10** (6 pts.) Determine whether the infinite series

$$\sum_{n=1}^{\infty} \frac{n^2 3^n}{n!}$$

converges or diverges.

**11** (8 pts.) Determine whether the infinite series

$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{\sqrt{n^2 - 1}}$$

converges absolutely, converges conditionally or diverges.

**12** (6 pts.) Determine the radius of convergence and the open interval of convergence of the power series

$$\sum_{n=0}^{\infty} (-1)^n \frac{n}{4^n} (x-1)^n$$

(You need not investigate the series at the endpoints of the interval).

**13.**

a) (4 pts.) Let

$$f(t) = \frac{1}{1-t^2}$$

Determine the Taylor series for  $f$  in powers of  $t$  (i.e. the Maclaurin series of  $f$ ). Display the first 4 nonzero terms and the term that involves  $t^{2n}$ .

Hint: Geometric series.

b) (4 pts.) Let

$$F(x) = \int_0^x \frac{1}{1-t^2} dt.$$

Determine the Taylor series for  $F$  in powers of  $x$  (i.e. the Maclaurin series of  $F$ ). Display the first 4 nonzero terms and the term that involves  $x^{2n+1}$ .