Communications

Forms of formative assessment: Eliciting and using student thinking

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Prior learning provides the basis for new learning. Mathematics educators employ formative assessment to *elicit* and *use* student thinking as the foundation of their instruction. Yet, information can be elicited and used in a variety of ways, so not all formative assessment is equally "formative":

> Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited. (Black & Wiliam, 2009, p. 7)

This means that simply eliciting student thinking is insufficient; assessment must impact instruction to be considered formative.

Not all methods of eliciting student thinking are equally useful. For instance, when students provide single-word answers to initiate-response-evaluate (IRE) sequences, they provide less information than when they give deeper explanations (Cobb, McClain, Lamberg & Dean, 2003). Ultimately, how teachers access and use student thinking depends on their goals and their specialized knowledge as educators. Here we propose a framework to better describe nuances in formative practices.

The Reactive-Active-Proactive (RAP) framework

Reactive, active, and proactive refer to when information is elicited and used with respect to an instructional sequence. *Reactive* assessments take place *after* instruction, *active* assessments take place *during* instruction, and *proactive* assessments take place *before* instruction. When information is elicited constrains its use (see Figure 1). For instance, student thinking accessed after an instructional sequence is over (reactive) can support re-teaching or providing feedback, but not modifying instruction as it unfolds. Information gathered actively can be used immediately, but it requires a teacher to think quickly and respond on-the-fly. Information gathered proactively (before a lesson) is the most flexible, because it may be used before a lesson, during a lesson, or after a lesson. For instance, it supports lesson planning (proactively), that embeds opportunities to elicit and use student thinking (actively) into the lesson itself.

		Use Information		
		Reactive	Active	Proactive
Elicit Information	Reactive	Provide Feedback →		
	Active	-	Guide Activities →	
	Proactive	-	-	Plan

Figure 1. The RAP framework.

To illustrate the RAP framework, we draw from three projects: (1) reflection in introductory calculus (Reinholz, 2015), (2) the Mathematics Assessment Project (Herman *et al.,* 2014), and (3) Japanese lesson study (Fernandez & Yoshida, 2004).

Reactive assessment

Participants were introductory college calculus students (Reinholz, 2015), asked to complete a one-minute paper (Stead, 2005) after each lesson. These *reactive* assessments provided information to the instructor, Michelle, about topics she had taught. Responding to reflections on the definition of the derivative, Michelle modified the next day's lesson, noting:

> It looks like students are confused between the limit of a function and the limit of a difference quotient.

Michelle designed a lesson-opening activity that gave students three examples of power functions. Students had to find the limit as the input approached zero, the limit of the difference quotient, and explain what the limits meant. While this activity responded to student struggles, it did not uncover specific aspects of *what* students were struggling with.

Students first worked individually on these prompts and then Michelle led a discussion, in which she asked them to explain their reasoning using phrases such as "What does this limit mean?" and "What does this represent?":

Michelle We've got limits in different places, and they all mean different things. We don't have to be technical, we can keep it loose for now, but what does this limit mean? *[Points to limit as x approaches 0 of $f(x)$]*

Student A As x values approach 0, y values also approach 0.

Michelle Yeah, that's right. This limit ties to function values.

[…]

Michelle What does this represent? *[Points to the limit of the difference quotient.]*

Student B Instantaneous rate of change for any point on the graph.

Michelle Yeah. Other descriptors?

Student C Derivative, slope of tangent line.

Michelle had previously identified two types of limits that her students were struggling with, and her questions implicitly helped contrast the two types. Yet, it appears Michelle was playing "catch up" to repair student confusions, rather than following her students' thinking in depth. This is typical of reactive assessments. While teachers regularly *elicit* student thinking, it is difficult to *use* it meaningfully. For the most part this activity only directed students towards clarifying incorrect responses (*i.e.* providing feedback).

Active assessment

The Mathematics Assessment Project (MAP) has designed lesson plans imbued with opportunities for formative assessment (Herman *et al.*, 2014). Each lesson was tested and refined through multiple classroom trials to determine how prompts could best provoke student thinking and how teachers might tailor instruction to typical student responses (proactively). Yet, a teacher may not leverage these opportunities in enacting the lesson, as we describe.

Kevin was an eighth-grade teacher, participating in a professional development project. The lesson, *Generating Polynomials from Patterns,* required student groups to match card pairs. The first card set contained four rectangular dot arrangements ordered in either an arithmetic or geometric sequence. The second card set described these sequences algebraically; each card also contained some missing expressions to be completed (see Figure 2).

Students were asked to derive an expression for a given sequence of arrays eight times during whole class discussions. Each time, classroom instruction proceeded in a routine manner: (A) students worked individually, (B) a student was asked to share the expression they wrote, and (C) the teacher evaluated the expression for values of n to verify its truth. This mathematically valid routine allowed students to share answers, but it was not insightful in terms of the information elicited: (1) students were not asked to share reasoning for how they derived their answer, and (2) the method of verification did not leverage the geometric organization of the arrays.

Students showed a readiness for more sophisticated mathematical discussions. For instance, early in the lesson, a student recognized how the organization of dots informed the writing of an expression:

> Like if it was [the] 4[th configuration], you see there are four sets of four circles so it's four times four.

The student noted geometric regularity in a dot diagram and used it to write an expression, "four times four." In his post-lesson reflection, Kevin noted that students made visual connections even though he relied upon computation:

> I was impressed that students looked at the dots from more of the visual standpoint, instead of counting out the dots, as I did. I tried to show that more to other students and have that thinking shared with everyone.

Kevin recognized a discrepancy between his mathematical understanding and student thinking elicited during the lesson. While he tried to highlight this thinking, this more robust form of reasoning was not central to class discussion.

This is a hallmark of *active* assessment. Kevin used questioning to elicit student thinking, but could not anticipate the thinking before it came out, and had to respond on-the-fly. Here Kevin's lack of mathematical sophistication inhibited his use of the opportunities to build on student thinking. Nevertheless, because Kevin was adept with general active assessment techniques, he was able to make desired lines of mathematical reasoning accessible to some students. Depending on his goals, Kevin was now positioned to develop a follow-up lesson (reactively) to focus more deeply on justification through geometric patterns.

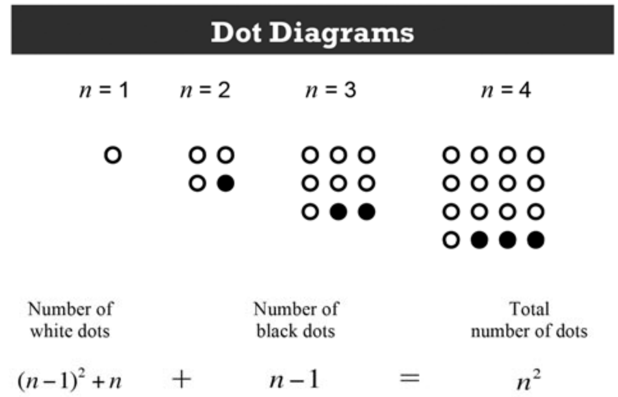

Figure 2. A sample match from Generating Polynomials from Patterns.

Proactive assessment

In lesson study, teachers collaboratively design, test, and refine a lesson over multiple iterations (Lewis, 2009). At Tsuta Elementary School in Japan, a five-member team designed a lesson on subtraction with regrouping, the introduction to a 12-lesson unit (Fernandez & Yoshida, 2004). The lesson plan was formatted with four columns: learning activities and questions, expected student reactions, teacher response to student reactions, and evaluation. Over multiple implementations, teachers add actual student thinking to the lesson plan (*proactively*), so they are prepared to respond *actively* to student thinking in productive ways.

The primary task for students was to subtract 7 from 12. The teachers created a custom manipulative with a blank space and two-coloured tiles that could be flipped over to count or subtract. The manipulative allowed students to express various conceptions of the number 12 (*e.g.,* lining up 12 tiles, having a group of 10 and a group of 2). The teachers anticipated four different student solution strategies.

In an enactment, three students presented their solutions, exhibiting three of the anticipated strategies. Yet, the lesson did not support students towards sharing coherent explanations; in fact, students' verbal explanations conflicted with their use of the manipulatives. Because the lesson did not *elicit* student thinking as desired, it limited how teachers could *use* it in discussion. In revising the lesson, teachers incorporated actual student ideas in the plan, and revised the manipulative: students were given a strip of 10 connected squares, a strip of 2 squares, and a pair of scissors. Thus, eliciting thinking *proactively* informed the revised lesson plan.

During the second iteration, four students presented their solutions, demonstrating only two strategies. In post-lesson discussions, Ms. Tsukuda stated her surprise that so many students used the same strategy. Once again, students' written explanations mismatched their physical demonstrations. Although the teachers did not develop a third version of the lesson plan, they had elicited a wealth of information.

Proactive assessment can be challenging. Even with decades of teaching experience and multiple opportunities to teach the same lesson, the teachers were surprised by what students actually did. The example also highlights opportunity. Given that this subtraction lesson was the first in a 12-unit lesson, the teachers now had deeper knowledge of student thinking that they could use in teaching future lessons. This would support lesson planning (proactively) and responding to student thinking when it was elicited (actively).

Discussion

The RAP framework distinguishes three forms of formative assessment: reactive, active, and proactive. Reactive assessment involves eliciting student thinking after a lesson and providing feedback or modifications to a future lesson. Opportunities for reactive assessment are frequent, and they allow teachers to provide feedback to students. Yet, if a teacher's formative practices are solely reactive, it is as if they are always playing "catch up," because they are not using student thinking as it emerges. As such, active assessments are useful because they elicit student thinking during a classroom episode. This allows for immediate teacher response, but it requires thinking on-the-fly. In our second example, even Kevin, a relatively skilled teacher, had difficulty optimally using student thinking that he had not anticipated.

Finally, proactive assessment provides the greatest opportunities for formative practice. By eliciting information about student thinking *before* it needs to be used, teachers can engage in thoughtful planning of lessons, such as in lesson study. Similarly, advance knowledge of student thinking can enhance the use of information when it is elicited, strengthening active assessments. Yet, given the amount of planning time required to enact proactive assessment, teachers need proper support, including materials, time to collaboratively plan with peers, and space to be creative.

The reactive, active, and proactive constructs, we believe, offer insight into how to cultivate a disposition towards and a practical approach for *learning to learn from teaching* (Hiebert, Morris, Berk & Jansen, 2007). Such practice makes assessment integral to instruction. Reflection grounded in assessments of student thinking and evidence of learning directs teaching more squarely towards being a learning profession. Moreover, we support research that helps teachers place student thinking at the forefront of decision making. Ultimately, we believe these three types of assessments each have a place in instruction, and that in distinguishing between them, teachers can be more intentional in how they engage student thinking.

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The early work of Brookover points to the importance of evaluation by others in the formation of self concept, in particular when the "other" is perceived by the individual concerned to have status and credibility (which presumably is the case when the "other" is a teacher). Brookover also found that self concept of ability was differentiated into specific self concepts corresponding to specific subject areas, a matter of interest in research focused on the mathematics classroom.

Celia Hoyles, Judith Armstrong, Rosalinde Scott-Hodgetts and Linsay Taylor (1984)
Towards an understanding of mathematics teachers and the way they teach,
For the Learning of Mathematics **4**(2) p. 27