Effects of Early Field Experiences on the Mathematical Content Knowledge and Beliefs of Prospective Elementary School Teachers: An Experimental Study

Randolph A. Philipp
San Diego State University

Rebecca Ambrose
University of California, Davis

Lisa L. C. Lamb, Judith T. Sowder, Bonnie P. Schappelle, Larry Sowder
San Diego State University

Eva Thanheiser
Rutgers University

Jennifer Chauvot
University of Houston

In this experimental study, prospective elementary school teachers enrolled in a mathematics course were randomly assigned to (a) concurrently learn about children's mathematical thinking by watching children on video or working directly with children, (b) concurrently visit elementary school classrooms of conveniently located or specially selected teachers, or (c) a control group. Those who studied children's mathematical thinking while learning mathematics developed more sophisticated beliefs about mathematics, teaching, and learning and improved their mathematical content knowledge more than those who did not. Furthermore, beliefs of those who observed in conveniently located classrooms underwent less change than the beliefs of those in the other groups, including those in the control group. Implications for assessing teachers' beliefs and for providing appropriate experiences for prospective teachers are discussed.

Key words: Beliefs; Content knowledge; Field experience; Large-scale studies; Preservice teacher education; Teacher beliefs; Teacher knowledge

---

Elementary school children in the United States are not developing acceptable levels of mathematical proficiency (National Center for Education Statistics, 1999), perhaps because teachers lack the depth and flexibility of mathematical understanding and the corresponding beliefs they need to teach for proficiency (National Research Council [NRC], 2001). Few doubt that teachers' mathematical content knowledge plays a critical role in their instruction (Fennema & Franke, 1992; Hill, Sleep, Lewis, & Ball, 2007), but most realize, also, that teachers need more than content knowledge to be effective. In particular, teachers' beliefs about mathematics, teaching, and learning affect the ways they think about and teach mathematics (Philipp, 2007; A. G. Thompson, 1992). Historically, the development of the mathematical content knowledge of prospective teachers takes place in undergraduate courses, years before their beliefs are challenged by their considering how children think about and learn mathematics. The study reported here is based upon the assumption that the content knowledge and beliefs of prospective elementary school teachers (PSTs) will be enhanced if they are provided with opportunities to learn about children's mathematical thinking *while* they are learning the mathematics they will teach.

RATIONALE FOR OUR STUDY

The Importance of Addressing Beliefs

Developing deep understanding of the mathematics of elementary school is far more difficult than was once thought (Ball, 1990; Ma, 1999; Sowder, Philipp, Armstrong, & Schappelle, 1998). Furthermore, our experiences from teaching mathematics courses and talking to mathematics instructors indicate that even when PSTs attend a thoughtfully planned course designed to engage them in rich mathematical thinking, many react to the course in a perfunctory manner. We contend that most PSTs do not know what mathematics they need to know to be teachers and that many are not open to approaching the content anew in a deeper and more conceptual way than they experienced in elementary school because they hold a self-perpetuating belief that "If I, a college student, do *not* know something, then children would not be expected to know it, and if I *do* know something, I certainly don't need to learn it again." Because beliefs generally exist in relation to one another within a quasilogical structure (Green, 1971) referred to as a *belief system* (Green, 1971; A. G. Thompson, 1992), we conjecture that this derivative belief rests upon these students' belief that one either understands or does not understand and that simply knowing a procedure without knowing why it applies is understanding (Skemp, 1978, called this *instrumental understanding*).

The belief, held by many PSTs, that mathematics is a fixed set of rules and procedures together with their belief that children and adults learn mathematics by being shown how to solve problems in a prescribed, step-by-step fashion can clash with the more conceptual, meaning-making goals that many mathematics-course designers hold for PSTs. Our work is based on the assumption that by providing PSTs opportunities to develop more nuanced beliefs about mathematics, teaching, and learning

early in their undergraduate experiences, we might launch them on a different growth trajectory that may orient them toward learning mathematics from a relational or meaning-making, rather than an instrumental, perspective (Skemp, 1978).

Linking PSTs' Learning Mathematics With Children's Mathematical Thinking

In his influential essay "The Child and the Curriculum" (1902/1990), Dewey addressed an educational issue of major concern at that time (Phillips, 1998): whether the elementary school curriculum should be determined by focusing upon the structure of the content to be taught or the interests and capacities of children. He set out to resolve this issue by showing how this opposition can be reconceptualized if one views the child and the curriculum not as two distinct and dualistic choices between which educators must choose but rather as two limits that "define a single process" (Dewey, 1902/1990, p. 189). Dewey's educational contribution was to persuasively explain why an attempt to base a curriculum solely upon either the structure of the content or the needs of the child is misguided and, inevitably, deficient. In his classic style, Dewey, instead of choosing between two seemingly incompatible stances, reconceptualized the apparent duality and showed that the solution is to integrate the two positions.

A great challenge for teacher educators is to determine which issues can be resolved by rethinking apparent dualities. We believe that the approach of separating the mathematical preparation of PSTs from opportunities for them to see how children think about mathematics is one such duality. Regarding the relationship between the child and the curriculum, Dewey (1902/1990) asked, "Of what use, educationally speaking, is it to be able to see the end in the beginning? How does it assist us in dealing with the early stages of growth to be able to anticipate its later phases?" (p. 190). We contend that these same questions are important to ask about the relationship between teaching mathematics to prospective teachers and providing them opportunities to attend to children's mathematical thinking.

In the general approach, PSTs complete their mathematics courses separately from, and often long before, they study issues of teaching and learning in their mathematics methodology courses. This separation of learning mathematics from learning about teaching mathematics oversimplifies the learning of both critical components. We conjecture that until PSTs begin to learn about children's mathematical thinking so that some of their beliefs about mathematics, teaching, and learning change, they fail to recognize that their own mathematical understanding is insufficient. But if this recognition comes only after they have completed their mathematics courses, their interest in learning mathematics arises too late for them to derive maximum value and benefit from these courses. Instead of trying to interest PSTs in learning mathematics for mathematics sake, we believe that educators should begin by tapping into that aspect of teaching with which PSTs are fundamentally concerned: children.

Dewey (1902/1990) noted that every subject might be thought of as having two aspects, "one for the scientist as a scientist; the other for the teacher as a teacher"

(p. 351). He wrote, "[The teacher] is concerned, not with the subject-matter as such, but with the subject-matter as a related factor in a total and growing experience [of the child]. Thus to see it is to psychologize it" (p. 352). Dewey's claim is supported by recent research indicating that whereas many PSTs report having had negative experiences learning mathematics (Ball, 1990), they do care about children (Darling-Hammond & Sclan, 1996). Because many PSTs have little mathematical experience with children, they are initially able to project only their own, too often negative, mathematical experiences onto those of children, with the result that they avoid placing children in challenging situations (e.g., never asking children to solve a problem before they have been shown how to do so).

Noddings (1984) viewed caring as a "displacement of interest from my own reality to the reality of others" (p. 14). In our work, we have attempted to facilitate the PSTs' expanded interest in the child by providing them with opportunities to better understand children. We presented a model (described in more detail in Philipp, Thanheiser, & Clement, 2002) that incorporated Noddings' description of *caring* to capture the way we have found that PSTs expand their interest from caring about children in general, to caring about children's mathematical thinking, to caring about mathematics (see Figure 1) We place children (rather than children's thinking, for example) at the center of caring because we believe that for most PSTs, the initial
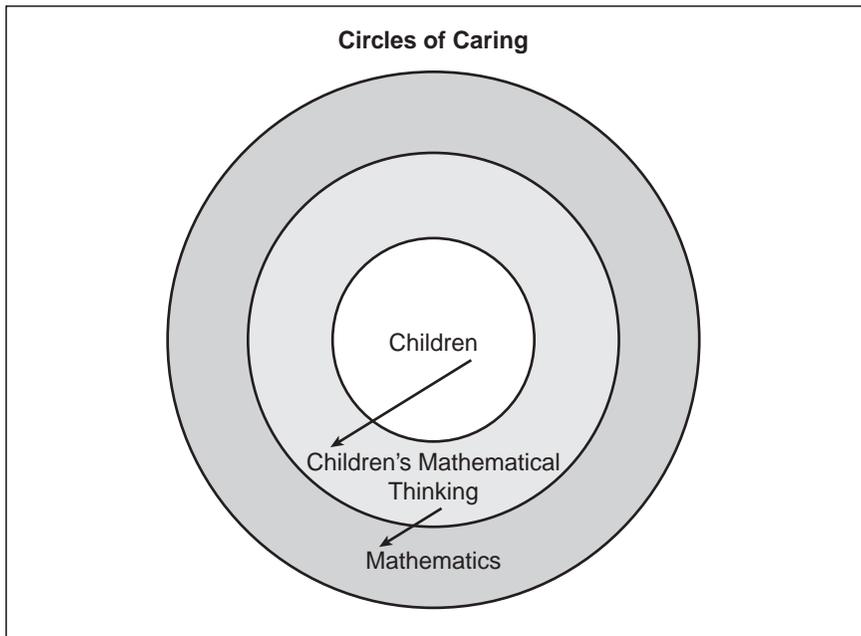

Figure 1. Circles of Caring. A model of growth, by way of children's mathematical thinking, from PSTs' caring about children to their caring about mathematics.

caring is a phenomenological act of concern for the whole child versus for a particular characteristic of the child. Tapping into PSTs' caring about children is the first step, but we hypothesize that when the PSTs engage children in mathematical problem solving, their circles of caring expand to include children's mathematical thinking, because it is in problem-solving settings that children's mathematical creativity and dispositions emerge. PSTs begin to see how children think about mathematics and come to recognize that children solve problems in varied, and sometimes mathematically powerful, ways; moreover, they see that many children are interested in problem solving and find it rewarding. We predict that at that time their circles of caring extend to mathematics, because they realize that to be prepared to understand the depth and variety in children's mathematical thinking, they must themselves grapple with the mathematics they will teach.

Research supports the idea that learning about children's mathematical thinking positively affects teachers. In their literature review, Wilson and Berne (1999) found that professional development based on children's thinking helped teachers create rich instructional environments that promoted mathematical inquiry and understanding, leading to documented improvement in student achievement. They highlighted as exemplary one particular program of research, Cognitively Guided Instruction (CGI), which has shown that helping teachers learn detailed research-based knowledge about children's mathematical thinking has led to significant changes in teachers' beliefs and practices that have, in turn, led to improvement in students' mathematical learning (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema et al., 1996), including in the setting of urban classrooms (Villaseñor & Kepner, 1993).

Although directly linking changes in teacher education programs to PSTs' current or subsequent teaching is nearly impossible, evidence shows that PSTs' beliefs can be affected by their learning about children's mathematical thinking. In a study of five preservice teachers enrolled in their senior year of the mathematics program at the Catholic University of São Paulo, Brazil, D'Ambrosio and Campos (1992) found that providing preservice teachers with opportunities to learn about children's mathematical thinking led them to reflect upon conflicting situations and to question normally accepted instructional practices. McDonough, Clarke, and Clarke (2002) found that PSTs who conducted one-on-one interviews with children came to appreciate the diversity of children's approaches and the importance of attending to these approaches in teaching. Vacc and Bright (1999) countered previous research that had indicated that PSTs' beliefs are resistant to change. Although they were unable to link the PSTs' beliefs changes directly to particular activities, they tentatively suggested that "intensity of experience and a focus on children's thinking in the mathematics methods course may be keys for helping preservice teachers change their view" (p. 108). In a study of PSTs growing out of our previous work, Ambrose (2004) found that those who focused on children's mathematical thinking underwent changes in their beliefs. Because Ambrose conducted an in-depth case study of 1 PST and then compared the emergent themes from this PST with those of 14 other PSTs, she was able to link the changes in PSTs' beliefs to opportunities to engage in focused experiences working with children, which she concluded

enabled the PSTs to make their initial undifferentiated beliefs salient to them: "Providing prospective teachers with intense experiences that involve them intimately with children poses a promising avenue for belief change" (p. 117).

These studies indicated that PSTs' beliefs may be affected by working with children. In our experimental study, we build upon this research by testing the theory that learning about children's mathematical thinking while concurrently learning mathematics offers PSTs advantages over other experiences. However, knowing *that* we want to integrate PSTs' learning of mathematics with their learning about children's mathematical thinking does not produce a blueprint for *how* to do so. Next we contrast two approaches used in teacher education.

The Apprenticeship Approach and the Laboratory Approach

In his essay "The Relation of Theory to Practice in the Education of Teachers" (1904/1964b), Dewey argued that both practical and theoretical work are required for the professional development of teachers. Teacher educators' responsibilities are, on one hand, to prepare teachers to manage the practical aspects of teaching that arise on a daily basis and, on the other hand, to prepare teachers to grapple with the deeper questions of the relationship between subject-matter knowledge and educational principles and theory. Dewey referred to a focus on preparation for the practical aspects of a job as the *apprenticeship* approach, a traditional approach in which past performance serves as a model for future performance. He referred to a focus on the more theoretical aspects of a job as the *laboratory* approach, a forward-looking approach that is "local, particular, situated" (Shulman, 1998, p. 512).

The apprenticeship or practical approach to teacher education helps prospective teachers learn how to do that which is currently being done. Examples of the apprenticeship approach are prospective teachers' engagement in early field experiences or student teaching in traditional classrooms, because in both cases they take current performance as their model for teaching. Examples of the laboratory approach are prospective teachers' analyzing students' understanding of mathematics before being taught how to teach a lesson. In this laboratory environment, the prospective teachers learn to attend to *how* children perceive their mathematical worlds, so that when they later take on the role of teacher, they can connect what they are learning about teaching with what they already know about students' mathematical understanding.

As important as the laboratory approach can be for preparing teachers, many forces act against this approach and in favor of the apprenticeship model that continues to dominate teacher preparation (McIntyre, Byrd, & Foxx, 1996). It is easier and less expensive to prepare teachers to teach in the traditional manner in which they had been taught than to challenge their beliefs and expectations through the laboratory model. Dewey (1929) noted that most people associate teaching ability with the use of procedures that lead directly to success, and PSTs approach their teacher preparation with expectations for learning successful procedures: "Put baldly, they [PSTs] want recipes" (p. 15). Furthermore, Dewey noted that when

forced to choose, the novice teacher will focus on classroom management and discipline instead of on ways to make subject matter more accessible to children. And forced to choose they are, because "*the mind of a student cannot give equal attention to both at the same time*" (Dewey, 1904/1964b, p. 318).

We believe that for the laboratory approach to influence prospective teacher education while PSTs are engaged in student teaching, a radical restructuring of teacher education in the United States will have to occur. However, little has changed in the 100 years since Dewey recognized that beginning teachers are trained to manage classes before they develop ways to think about linking students to content (Dewey, 1904/1964b). Introducing the laboratory model while students are student teaching may be impossible, but educators do have opportunities to introduce earlier into teacher education a culture of experimentation, invention, and discovery.

Research Statement

In our study, we applied the laboratory approach in two of our treatments during which we supported PSTs in learning about children's thinking concurrently with learning mathematics. For purposes of comparison, we additionally applied the apprenticeship approach in two treatments in which PSTs visited classrooms while learning mathematics themselves. Thus, for PSTs enrolled in a mathematics course for elementary school teachers, we investigated whether differences could be found in their content-knowledge growth or beliefs change depending upon whether they concurrently experienced one of the following four early field experiences:

1. Learn about children's mathematical thinking by watching videos (laboratory approach),
2. Learn about children's mathematical thinking by watching videos and working with children (laboratory approach),
3. Visit typical elementary school mathematics classes (apprenticeship approach), or
4. Visit specially selected mathematics classes (apprenticeship approach).

METHOD

All 159 PSTs in our study were enrolled in the first of four mathematics content courses for prospective elementary school teachers. The course content focused on whole-number and rational-number concepts and operations. The instructional materials were designed to support PSTs' conceptual development of the mathematics of the elementary school curriculum and included examples of children's ways of solving problems (Sowder, Sowder, Thompson, & Thompson, 1999).[1] The

---

[1] Although all four treatments and the control group addressed children's mathematical thinking, only in the CMTEs was there a sustained focus on children's mathematical thinking. We estimate that the PSTs in the CMTEs focused upon children's mathematical thinking almost 100% of the time, whereas the other PSTs focused on children's mathematical thinking only about 5% of the time.

course instructors were mathematics graduate students who received instructional support from a senior mathematics educator. Approximately 30 students were enrolled in each of the 12 sections of this mathematics course taught during the fall semester of 2001; PSTs were recruited from all 12 sections to participate in the study.

Treatment Groups

Each student was assigned to one of four treatment groups or to the control group (see Figure 2). Those in two of the treatment groups were provided with opportunities to study children's thinking in "Children's Mathematical Thinking Experiences" (CMTE) treatments. Participants in the other two treatment groups observed mathematics classes at elementary schools in "Mathematical Observation and Reflection Experiences" (MORE) treatments. We collected data on a control group to determine the extent to which PSTs' beliefs and content knowledge developed as a result of the mathematics course. The students assigned to the control group were enrolled in the mathematics content course for prospective teachers and completed the instruments administered to all PSTs in the study but did not engage in a field experience. We employed a modified random assignment, constrained by the students' personal class and work schedules and the times scheduled for school visits; most students were available for at least two treatments.

Laboratory Models	
Children's Mathematical Thinking Experience (CMTE)	
CMTE-Live (CMTE-L)—PSTs watch and analyze video of children solving problems, and PSTs conduct six problem-solving experiences with individual children. $n = 50$	CMTE-Video (CMTE-V)—PSTs watch and analyze video of children solving problems. $n = 27$
Apprenticeship Models	
Mathematical Observation and Reflection Experience (MORE)	
MORE-Select (MORE-S)—PSTs visit selected teachers identified as reform oriented. $n = 23$	MORE-Convenient (MORE-C)—PSTs visit teachers with classrooms close to campus. $n = 25$
Control	
No field experience	
$n = 34$	

Note. Two CMTE-L courses were offered, each taught by a different instructor. During our data analysis we initially compared the student data of the two sections, found similar change scores, and so classified the data of students in both CMTE-L courses as from a single treatment.

Figure 2. Models and treatment groups.

The CMTEs

The CMTE-L and CMTE-V, which are two-unit courses cross-listed between the Mathematics Department and the School of Teacher Education, met for 14 2–2.5 hour sessions over the semester. Both courses might be thought of as concurrent labs for the mathematics course described above, although the instructors (the first and third authors) of the CMTE-L and CMTE-V courses did not collaborate with the mathematics course instructors.[2] Video clips were created for both courses to highlight children's mathematical strengths (ability to invent strategies, agility with numbers, reasoning that is sometimes difficult to follow) and weaknesses (mistakes and misconceptions) (see Philipp, Cabral, & Schappelle, 2005).

The CMTE-L PSTs worked with children at a local elementary school on six occasions, forming pairs of PSTs who together interviewed and tutored[3] individual children (see Ambrose, 2004, and Philipp et al., 2002, for more extensive descriptions of the CMTE-L).[4] The CMTE-L differed from a mathematics course in that the mathematics studied was not an end in itself but instead generally arose from the PSTs' work with children; it differed from a mathematics methodology course because we did not attempt to help students learn to teach a group of students. For example, we did not discuss lesson or unit planning, textbooks, testing, or classroom management. By working with only one child, PSTs seldom encountered discipline issues that arise in whole-class lessons; they could focus solely on the child's mathematical thinking. Although the complexities associated with managing a group of students were reduced, PSTs faced the challenges of grappling with that individual child's understanding and finding ways to support the child, challenges possibly avoided by a classroom teacher who, noticing that one child is confused, moves the lesson along by "fishing" for another student to give a correct answer (cf. P. W. Thompson & A. G. Thompson, 1994).

We considered our PSTs' circles of caring (see Figure 1) when we initiated them into the interview process. Most had yet to consider how a child's point of view toward mathematics differed from their own; they had thought little about the mathematics under consideration, and they certainly had not considered the intersection of the two areas. We initially constrained the role of the PSTs so that they assessed children's understanding of concepts by using carefully selected tasks. One belief we wanted to address was the belief that primary-grade children come to school with knowledge and strategies that they were not taught in school but that

---

[2] Although we believe that PSTs would benefit if explicit connections were drawn between their mathematics courses and their experiences working with children, we chose not to link the mathematics course with the CMTEs in this study because we recognize that most mathematics courses taught to PSTs are offered independently of other educationally related experiences.

[3] We distinguish interviewing from tutoring by focusing upon the primary intent of the interaction. If the PST's goal is to assess the child, we refer to it as interviewing; if the goal is to teach the child, we refer to it as tutoring.

[4] We provide more details about the CMTE-L than the other treatments because it was the treatment of most interest to us and it differed most from typical early field experiences.

might be invoked through the use of relevant situations, and so we wanted the PSTs to see that many primary-grade children understand mathematics they *have not* been formally taught. Another belief we chose to address was that traditional school practices have led to many students' failing to develop mathematical understanding, so we positioned the PSTs to see that many intermediate-grade children do *not* understand mathematics they *have* been taught. We designed experiences to expose the PSTs to the variety of appropriate, creative, and mathematically powerful ways that children can think about mathematics. We hoped that they would be challenged in coming to understand some approaches that children used to solve problems and that their interest in children would stimulate their desire to understand the children's thinking about the problems.

In the CMTE-L, PSTs (in pairs) worked directly with children in almost half the sessions; when not interviewing children, they analyzed previous sessions with children, planned subsequent sessions, or considered more general issues related to children's thinking or mathematics. After each interview, the instructor led the PSTs in a discussion supported by clips from one interview videotaped that day. In the first part of the course, the PSTs examined the mathematical thinking exhibited in young children's solution strategies for various types of mathematics problems (Carpenter, Fennema, Franke, Levi, & Empson, 1999) and conducted one interview of a first-grade child. In the second part of the course, the PSTs investigated third-grade students' place-value understanding and interviewed the same third-grade child on two occasions. During the remainder of the course, the PSTs investigated students' rational-number reasoning; they interviewed and tutored the same fifth-grade student on three occasions to delve into the students' thinking about rational numbers and to teach the child. PSTs tape-recorded their interactions with children, and for homework, they listened to the tape and answered questions about the interview.

The CMTE-V met on the university campus, and to more tightly control one variable of interest (work with children), we overly constrained the CMTE-V treatment by assigning no interviews of children as homework. CMTE-V students engaged in many of the same group activities as the PSTs in the CMTE-L, including viewing a video developed for the CMTE courses, analyzing problem types, and anticipating children's solution strategies. In addition, on six occasions, the CMTE-V PSTs received unedited videotapes of entire interviews conducted the previous day by CMTE-L PSTs working with children. To provide privacy for the interviewers, we filmed only the child, without showing the PST interviewer in these videos. The PSTs watched and answered questions about these videos for homework. Because the PSTs in the CMTE-V spent no class time planning for interviews or working directly with children, they spent more class time than the CMTE-L PSTs discussing children's mathematical thinking, in essence, having more time-on-task than the CMTE-L PSTs had. Experiences of the PSTs in the two groups differed because those in the CMTE-V did not have to "think on their feet" and develop a response to a child, nor did their experience have an interactive, affective component. We considered, on the one hand, that because the CMTE-V was less personally intense

than the CMTE-L, its effect might be diminished or, on the other hand, that because the PSTs in the CMTE-V spent additional time discussing children's mathematical thinking, the effect might be increased for them.

The MOREs

PSTs assigned to the MOREs made 14 weekly visits to local elementary schools. Pairs of PSTs were assigned to classrooms, according to the PSTs' schedules and transportation capabilities and the teachers' mathematics class time. For consistency in the time commitments in all treatments, the weekly visits were 90 minutes, although in some cases the mathematics lessons did not last 90 minutes and the PSTs observed other lessons. No specific arrangements were made for the students to meet and debrief with the cooperating teachers. At midsemester, PSTs were assigned new classrooms and grade levels, so that over the course of the semester they made 7 visits in primary grades and 7 visits in intermediate grades. Each week the PSTs in the MOREs wrote a one- to two-page reflection paper about the visit. The PSTs also wrote mid- and end-of-semester reflections about the experience.

We considered the MORE to be akin to Dewey's apprenticeship model, in which PSTs learn by observing practicing teachers. We hoped that PSTs would make connections between their university courses and the world of teaching by visiting schools and classrooms, observing students of diverse backgrounds, and gaining a sense of mathematics curricula and mathematics instruction typically used in schools today. The teachers visited had latitude in determining how the PSTs spent their time in class, and some arranged for PSTs to help children with their mathematics work. Because we wondered whether specially selecting teachers for field placements would significantly affect PSTs' experiences, we created two MORE groups. The MORE-Select (MORE-S) group observed in classrooms of teachers recommended by our colleagues as having been enthusiastic participants in reform-based professional development efforts. The MORE-Convenient (MORE-C) group visited teachers selected simply because their schools were close to the university. We included this treatment because we considered the MORE-C experience to be typical of commonly offered early field experiences, in which PSTs find their own placements (often chosen on the basis of convenience) and university faculty have little or no control over the quality or type of teaching a PST observes. PSTs assigned to the MORE-S visited two select teachers, and PSTs assigned to the MORE-C visited two conveniently located teachers. We expected that teachers in the MORE-S group would tend to use reform-based practices, and we expected that although teachers in the MORE-C group would be varied in their approaches, most would use more traditional approaches than the MORE-S teachers used. We expected greater positive changes in the beliefs we were assessing for PSTs in the MORE-S than for those in the MORE-C because of a higher probability that the MORE-S PSTs would observe teachers committed to reform-oriented practice, including a focus on mathematical concepts instead of procedures, a problem-solving perspective, and a commitment to students' sharing their reasoning.

Remuneration and Attrition

Although generally students received remuneration based on the time required for their participation in our study, to entice enough students to participate fully, we paid students assigned to the CMTEs or MOREs up to $600 (which included a bonus, in addition to the hourly rate, if they completed all semester activities and all pre- and postinstruments) for their semester's work. Control students were paid only the hourly rate to complete the surveys, on average about $100.

Attrition rates for the study were 6% for the CMTE-L, 10% for the CMTE-V, 16% for the MORE-C, 18% for the MORE-S, and 42% for the control group. We speculate that the control group's attrition rate was high because they earned no monetary bonus for completing all instruments and had no instructor with whom to form a connection to a treatment. We speculate that the attrition rate was slightly higher for the MOREs than for the CMTEs because PSTs in the MOREs received no course credit.

Data Sources

Our experimental study required the use of instruments to measure the beliefs and content knowledge of large numbers of students. Although beliefs and content knowledge are interconnected, we treated them separately in an effort to effectively measure each. To investigate the treatments' effects on PSTs' knowledge of mathematics and beliefs about mathematics, teaching, and learning, we had all participants complete a beliefs survey and a content assessment as pretests and posttests. We then examined change scores from pretest to posttest on both instruments. To add texture and depth to the quantitative results, we applied descriptive statistics to look for patterns in the data; we conducted individual interviews with students from each treatment and group interviews with students across treatments; and we examined written reflections of students in the MOREs, written end-of-course surveys of students in the CMTEs, and responses to open-ended items on the beliefs instrument.

Instruments

The IMAP Web-Based Beliefs Survey

A major obstacle in undertaking a study of changes in beliefs about mathematics, mathematics teaching, and mathematics learning is the difficulty in measuring change. A strength of our measure-of-beliefs instrument is that it presents contexts to which subjects respond; another strength is that we have developed rubrics that can be used to standardize the scoring of the survey.

Rationale for the way we measured beliefs. "For the purposes of investigation, beliefs must be inferred" (Pajares, 1992, p. 315) because individuals can be unaware of beliefs that shape their actions. Mathematics education researchers have typically

used case-study methodology to infer teachers' beliefs related to mathematics teaching and learning (e.g., Clarke, 1997; Cooney, Shealy, & Arvold, 1998; Raymond, 1997). Using this approach, researchers provide rich descriptions of the beliefs of a small number of prospective teachers by relying on rich data sets that include multiple observations, interviews, and surveys collected over a long period of time. Findings from such research provide details of the conceptions of small numbers of teachers, with conclusions supported by multiple data points. These rich reports are important for theory building, but theory testing often requires tools for studying larger groups of individuals. Given the nature of our work, we faced two problems in assessing beliefs: We needed to assess the beliefs of PSTs years before they were in the classroom, and we needed an assessment that could be administered to more than 150 prospective teachers.

We thus began our work to create a suitable beliefs survey by identifying characteristics of beliefs that account for the critical role they play in teaching and learning and, thus, are important for the approach we use to measure the beliefs. First, beliefs influence perception (Pajares, 1992). That is, beliefs serve to filter enough complexity of a situation to make it comprehensible, shaping individuals' interpretations of events (Grant, Hiebert, & Wearne, 1998). Teachers and students are constantly faced with uncertain situations requiring interpretations. In our survey, we provide respondents with complex situations to interpret.

Second, beliefs might be thought of as dispositions toward action, having a motivational force (Cooney et al., 1998; Rokeach, 1968). When faced with challenging decisions, which often have to be made spontaneously, teachers are often compelled by their beliefs to act in particular ways. In measuring beliefs, we provide respondents with scenarios in which they are called upon to make teaching decisions. Their dispositions to act in these situations provide us with evidence from which to infer their beliefs.

Third, beliefs are not all-or-nothing entities; they are, instead, held with differing intensities (Pajares, 1992, citing Rokeach, 1968). To address this characteristic in our survey, we provided tasks with multiple interpretation points. In devising the scoring rubrics, to allow for the differing intensities with which individuals hold beliefs, we used four categories, differentiating among *strong evidence, evidence, weak evidence,* and *no evidence* for a respondent's holding a belief. We do not claim that an individual lacks a particular belief but instead state that we found no evidence for the belief in the responses the individual provided.

Fourth, beliefs tend to be context specific, arising in situations with specific features (Cooney et al., 1998). Hence, we situated survey segments in contexts and inferred a respondent's belief on the basis of his or her interpretation of the context.

Survey development. We set out to create a survey to assess beliefs that might affect PSTs' subsequent learning of mathematics: beliefs about mathematics and about mathematics understanding and learning. The beliefs of importance in this project were those that could promote the PSTs' mathematical learning. We were interested in beliefs that might be called *generative* (Franke, Carpenter, Levi, & Fennema,

2001), because prospective teachers who develop them will continue to grow in their learning of mathematics and will continue to develop beliefs that will help them to implement the reforms articulated in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) once they begin to teach. We identified eight such beliefs but could reliably and validly measure only seven. (Although the beliefs were not stated in the beliefs survey, we identified each belief with a statement that described what we intended to measure with our survey. Our statement of the seven beliefs measured is listed in Appendix A.)

We created a survey that could be used to (a) derive a common metric for measuring change in individuals and for comparing individuals to one another and (b) obtain qualitative data that could be used for more holistic analysis. To capture the characteristics of beliefs we deemed relevant, we developed a survey in which prospective teachers constructed responses, providing more authentic answers than are available from multiple-choice instruments. We consulted with six mathematics education researchers with expertise in teachers' beliefs and six mathematics education graduate students who completed our survey and attested to the validity of the items as measures of the specified beliefs and the rubrics we applied to score the data. Using pilot data, we developed 17 rubrics (each of the 7 beliefs was assessed using 2 or 3 rubrics) for quantifying these constructed responses. This rubric-development process was lengthy, taking approximately 72 person-hours per rubric (4 weeks × 6 hours per week × an average of 3 people per team). Respondents' written responses provided insights into their beliefs and interpretations, and the numerical scores were used to statistically analyze differences among groups in different treatments. (For detailed information on survey and rubric development, see Ambrose, Clement, Philipp, & Chauvot, 2004.)

Survey features. The Web-based survey includes video and written teaching episodes about which PSTs constructed responses.[5] The survey consists of seven segments, each of which includes several questions about a particular situation. Four segments are in the domain of whole number, two are in the domain of fractions, and one is a general teaching segment. The chosen domains were the domains of focus for our experimental treatments and were important topics in the mathematics course in which the PSTs were enrolled. Two segments include video clips of individual children solving mathematics problems with an interviewer. Each segment is associated with two or three beliefs, and each belief is assessed using a separate rubric for each of two or three segments.

Responses to open-ended questions enabled us to discern which issues affected respondents' interpretations. For example, we used the segment in Figure 3 as one of three segments designed to assess a belief about the relationship between procedural and conceptual knowledge: *One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts.*

---

[5]A browse version of the survey and a manual describing the survey and the scoring rubrics (as well as the other instruments used in the study) are available at http://www.sci.sdsu.edu /CRMSE/IMAP/pubs.html.

Teachers often ask children to share their problem-solving strategies with the class. Consider the following students' strategies.

Carlos	Elliott	Sarah
149 + 286	149 + 286	149 + 286
Written on paper	Written on paper	

Carlos
149 + 286
Written on paper

$$\overset{1\ 1}{149} \\ +286 \\ \overline{435}$$

Elliott
149 + 286
Written on paper

$$149 \\ +286 \\ \overline{300} \\ 120 \\ 15 \\ \overline{435}$$

Sarah
149 + 286

Sarah says, "I know that 149 is only 1 away from 150, so 150 and 200 is 350, and 80 more is 430, and 6 more is 436. Then I have to subtract the 1, so it is 435."

(a) Do you think that Carlos could make sense of and explain Elliott's strategy? Why or why not?

(b) Do you think that Carlos could make sense of and explain Sarah's strategy? Why or why not?

Figure 3. One item used to assess a belief about the relationship between procedural and conceptual knowledge.

That is, students or adults may know a procedure they do not understand. (PSTs were not provided the statements of the beliefs at any time.) The highest score on this belief for this item went to respondents who recognized that Carlos may not be able to use Elliott's or Sarah's strategies because Carlos may have only procedural knowledge, whereas the other two strategies require deeper understanding of the underlying place-value concepts. For example, the following response to Part (a) received the highest score: "Maybe. It would depend on Carlos' level of understanding. He might only know how to do the problem in a rote fashion and not have much understand [*sic*] for what he's doing, so he wouldn't understand Elliott's approach." In contrast, the lowest score went to respondents who stated that because Carlos could perform the standard algorithm, he could definitely make sense of and explain the other strategies. For example, the lowest score was assigned to one who said, "Yes, especially since Carlos is able to carry without difficulty. He already has a sense of place value."

IMAP Mathematics Content Assessment

We designed a paper-and-pencil content assessment to determine whether the treatments had measurable effects on the PSTs' performances on items that address the main content of the accompanying mathematics course: place value and rational numbers (both fractions and decimals). A few items were solely objective in nature (i.e., multiple choice or with *correct/incorrect* as choices), but many items also called for explanations. We attempted to assess conceptual understanding instead of

computational skill, and we situated several tasks in relevant school-based contexts. (Figure 4 shows an item from the content assessment.) The content assessment was administered as both a pretest and a posttest measure of PSTs' content knowledge, and although it was designed for completion in 1 hour, students were allowed to work as long as they needed. We piloted preliminary versions of the content assessment to refine the items and to develop scoring rubrics. The final version consisted of 27 items, some with several parts.

---

5. Antonio asks, "When I multiply [for example, 49×23, shown to the right], why do I have to put in the zero [points to the zero in 980]?"

    What would you say to Antonio?

$$\begin{array}{r} 49 \\ \times\, 23 \\ \hline 147 \\ 980 \\ \hline 1127 \end{array}$$

Figure 4. One content-assessment item.

Data Analysis

Graduate students external to the project (several from other universities) met at our research site to code our assessments. Our project researchers trained the coders but had no input on final codes assigned by the coders. The responses were blinded so that neither coders nor trainers could determine in which treatment the respondents were enrolled or whether the responses came from pre- or postassessments. A total of 20% of the responses were double-coded, and we achieved, on average, 84% interrater agreement in coding the beliefs survey. Differences, when they occurred, were resolved on those responses coded in common, and retraining took place if agreement was less than 80%. Most agreement percentages for the content-instrument items were greater than 90%.

Each of the seven beliefs was treated independently, and for each belief, all 10 pairwise comparisons among the four treatments and control were conducted, with the conservative Holm's procedure (Holm, 1979, cited in Hochberg, 1988) used to reduce the likelihood of a Type I error. The beliefs-survey data were treated as ordinal, not interval, data; we were, therefore, unable to aggregate the scores and perform distribution-dependent statistical tests, such as ANOVAs. Pairwise differences between groups were computed as follows:

1. Individual participants were assigned one of four scores for each belief (no evidence; weak evidence; evidence; strong evidence) on the presurvey and on the postsurvey; the change calculated between their presurvey and postsurvey scores was either no positive change, a small positive change (going up one level,

e.g., from weak evidence to evidence), or a large positive change (going up two or more levels, e.g., from weak evidence to strong evidence). Each participant received one change score for each of the seven beliefs.

2. The distribution of change scores was analyzed using a polychotomous log-linear odds ratio using the ordered logit procedure in the STATA software package (Long & Freese, 2001). An ordered logit procedure generates a log-linear regression and corresponding goodness-of-fit statistic to test the prediction that change scores vary by group assignment. (See Appendix B for further explanation of the odds ratio.)

Because efforts were taken in scoring the content instrument to develop coding rubrics and weightings that allowed for assigning numeric scores in proportion to the understanding reflected in the responses, the content data were treated as interval data. Analysis of variance was employed to analyze the content data in 10 pairwise comparisons among the four treatments and the control group. Holm's procedure was used to maintain the Type I error rate at the .05 level. Because for some pairwise comparisons we could not justify a prediction in either direction, we used two-tailed tests in analyzing all pairwise comparisons.

RESULTS

The results are presented in two parts, the first part focusing on the beliefs data and the second on the content data. For each part, we first present the quantitative data subjected to statistical tests, then patterns in the quantitative data, and finally the qualitative data.

Beliefs Data

Statistical Tests on Quantitative Beliefs Data

Table 1 shows the distribution of beliefs-score changes for each treatment by belief. Because 10 comparisons were made for each of the seven beliefs, the Holm's procedure was used to maintain the Type I error rate at the .05 level. Although 31 of the 70 pairwise comparisons of beliefs-scores' changes resulted in a p value of less than .05, only 18 of these 31 tests were significant when the *nominal alpha level* was adjusted using the Holm's procedure. Each of the 18 significant differences resulting from applying the Holm's procedure was between a CMTE group and one of the other three groups, and Table 2 shows the odds ratios, the 95% confidence intervals, and the p values for comparisons between CMTE-L and CMTE-V groups with MORE-S, MORE-C, and control groups. As an example of how to interpret each odds ratio, consider the significant difference between the CMTE-L and the MORE-S group for Belief 5: The odds of having a small beliefs-score increase compared to no increase, or a large beliefs-score increase compared to a small increase for PSTs in the CMTE-L group are 9.2 times the corresponding odds for PSTs in the MORE-S group.

Table 1
Beliefs-Score-Change Percentages by Belief, Treatment, and Score-Change Category

Group	Large increase	Small increase	No change or decrease
Belief 1. Mathematics is a web of interrelated concepts and procedures (school mathematics should be too).			
CMTE-L (*n* = 50)	36%	46%	18%
CMTE-V (*n* = 27)	30%	41%	30%
MORE-S (*n* = 23)	22%	39%	39%
MORE-C (*n* = 25)	4%	52%	44%
Control (*n* = 34)	15%	26%	59%
Belief 2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts. That is, students or adults may know a procedure they do not understand.			
CMTE-L (*n* = 50)	48%	22%	30%
CMTE-V (*n* = 27)	41%	26%	33%
MORE-S (*n* = 23)	26%	26%	48%
MORE-C (*n* = 25)	16%	12%	72%
Control (*n* = 34)	12%	18%	71%
Belief 3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.			
CMTE-L (*n* = 50)	46%	22%	32%
CMTE-V (*n* = 27)	56%	22%	22%
MORE-S (*n* = 23)	35%	30%	35%
MORE-C (*n* = 25)	24%	16%	60%
Control (*n* = 34)	15%	21%	65%
Belief 4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.			
CMTE-L (*n* = 50)	32%	36%	32%
CMTE-V (*n* = 27)	37%	40%	22%
MORE-S (*n* = 23)	17%	26%	57%
MORE-C (*n* = 25)	0%	20%	80%
Control (*n* = 34)	12%	35%	53%
Belief 5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than their teachers, or even their parents, expect.			
CMTE-L (*n* = 50)	40%	38%	22%
CMTE-V (*n* = 27)	26%	33%	41%
MORE-S (*n* = 23)	4%	26%	70%
MORE-C (*n* = 25)	4%	36%	60%
Control (*n* = 34)	12%	38%	50%
Belief 6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.			
CMTE-L (*n* = 50)	38%	30%	32%
CMTE-V (*n* = 27)	44%	19%	37%
MORE-S (*n* = 23)	22%	26%	52%
MORE-C (*n* = 25)	0%	32%	68%
Control (*n* = 34)	15%	35%	50%

Table 1 (continued)

Group	Large increase	Small increase	No change or decrease
Belief 7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.			
CMTE-L ($n = 50$)	30%	16%	54%
CMTE-V ($n = 27$)	19%	48%	33%
MORE-S ($n = 23$)	4%	22%	74%
MORE-C ($n = 25$)	0%	20%	80%
Control ($n = 34$)	12%	41%	47%

Note. Because of rounding, some treatment totals by belief do not sum to 100%.

Table 2
*Beliefs Change-Score Differences, Using the Ordered Logit Procedure, of CMTE-L and CMTE-V PSTs Relative to Changes in Scores of PSTs in the MORE-S, MORE-C, and Control Group*s

	CMTE-L			CMTE-V		
Belief	Odds ratio	95% confidence level	p value	Odds ratio	95% confidence level	p value
		MORE-S				
1	2.5	1.0–6.3	.060	1.6	0.5–4.5	.399
2	2.3	0.9–5.9	.072	1.8	0.7–5.2	.243
3	1.4	0.5–3.3	.509	2.1	0.7–5.9	.167
4	2.8	1.1–7.3	.039	3.9	1.3–11.5	.013
5	9.2*	3.2–26.3	.000	4.0	1.3–12.5	.018
6	2.4	0.9–6.0	.075	2.5	0.8–7.3	.100
7	3.3	1.1–9.7	.030	4.6	1.5–14.5	.009
		MORE-C				
1	4.0*	1.6–9.9	.002	2.6	0.9–7.1	.069
2	6.0*	2.1–16.6	.001	4.7*	1.5–14.4	.007
3	3.1	1.2–8.1	.018	4.8*	1.6–14.1	.004
4	9.8*	3.2–29.8	.000	13.8*	4.1–46.6	.000
5	6.4*	2.5–16.7	.000	2.8	1.0–8.0	.060
6	5.5*	2.1–14.7	.001	5.8*	1.9–17.6	.002
7	4.8	1.6–14.7	.006	6.7*	2.0–21.9	.002
		Control				
1	5.4*	2.2–13.0	.000	3.4	1.3–9.3	.015
2	6.0*	2.4–14.7	.000	4.7*	1.7–12.9	.003
3	4.2*	1.8–10.1	.001	6.5*	2.4–17.8	.000
4	2.7	1.2–6.2	.019	3.8*	1.5–10.0	.006
5	4.1*	1.7–9.5	.001	1.8	0.7–4.6	.253
6	2.5	1.1–5.6	.028	2.6	1.0–6.9	.052
7	1.1	0.5–2.6	.750	1.6	0.6–4.0	.310

*p value significant after nominal alpha level is adjusted using Holm's procedure.

Beliefs scores increased significantly for a larger percentage of CMTE-L PSTs than for PSTs in the control group on four of the seven beliefs, than for PSTs in the MORE-C group on five of the seven beliefs, and than for PSTs in the MORE-S group on one of the seven beliefs. Beliefs scores increased for a larger percentage of CMTE-V PSTs than for PSTs in the control group on three of the seven beliefs

and than for PSTs in the MORE-C group on five of the seven beliefs. In summary, of the 18 significant differences in this study resulting with use of the Holm's procedure, 10 involved students in the MORE-C, 7 involved students in the control group, and 1 involved students in the MORE-S, with the students in these groups showing significantly less increase on beliefs scores than students in one of the two CMTE treatments.

Patterns in the Quantitative Beliefs Data

We found significant pairwise differences of beliefs only between CMTE groups and the MORE-C and control groups, with one exception of a difference involving the CMTE-L group and the MORE-S group. In investigating patterns within the data represented in Table 1, we note that the CMTEs had the greatest percentage of students with large increases on every belief. Although analysis of differences within specific beliefs is beyond the scope of this article, we contend that because the beliefs assessed by our survey are focused on a narrow band of mathematics education issues (beliefs about mathematics and mathematics understanding and learning for elementary school-aged children in the content areas of place value and rational number), looking across all the beliefs when drawing comparisons among the groups is useful. To compare treatments across beliefs, we computed an average percentage of students by treatment showing large increases, small increases, or no increase on the changes in their beliefs scores (see Table 3). Each percentage in Table 3 is the average of the seven corresponding values from Table 1. For example, the value 38.6% (the first entry) in Table 3 represents the average of the *percentages* of students in the CMTE-L group with large score increases for all beliefs and was the result of averaging the seven large-increase percentages, listed in Table 1, for each of the seven beliefs for the students in the CMTE-L group. Whereas each percentage in Table 1 represents the percentage of students in the corresponding treatment whose scores showed large (small, or no) increases on *the given* belief, each percentage in Table 3 is only an average of these percentages and should not be interpreted as representing a percentage of students. However, because each percentage in Table 3 is representative of the seven corresponding values in Table 1, the values in Table 3 are useful for our analysis.

Table 3
Average Percentages of Students in Each Beliefs Change-Score Category (With Ratio of Percentage With Large Change to Percentage With No Change), by Treatment

	CMTE-L	CMTE-V	MORE-S	MORE-C	Control
Large increase	38.6%	36.1%	18.6%	6.9%	13.3%
Small increase	30.0%	32.7%	27.9%	26.9%	30.3%
No increase	31.4%	31.1%	53.6%	66.3%	56.4%
Ratio large/No	1.23	1.16	0.35	0.10	0.24

Note. Because of rounding, some treatment totals do not sum to 100%.

The average (across beliefs) percentages of students whose beliefs scores increased a small amount are roughly equivalent across treatments. However, the average percentages of students whose beliefs scores increased a large amount or no amount showed much variation across treatments. We calculated, by treatment, a ratio of the average percentage of students whose beliefs scores showed large increase to the average percentage of students whose beliefs scores showed no increase, and those ratios are displayed in the bottom row of Table 3. A ratio of 1.00 for a given treatment would indicate that the average percentages of students in that treatment who showed large increases is the same as the average percentage of students in that treatment with no increase. The average percentage of students who showed large increases in each CMTE was greater than the average percentage showing no increase. Results for the other three groups reflect a different finding, with all three having a higher average percentage of students whose beliefs scores did not increase than of students whose beliefs scores increased a large amount. For the MORE-S group, the average percentage of students whose beliefs scores showed large increase was approximately one third (0.35) the average percentage of students whose beliefs scores showed no increase, and the ratio for the control group was approximately one fourth (.24). The lowest ratio for any group was for MORE-C, with the average percentage of students showing large belief change only one tenth (0.1) the average percentage of students showing no belief change. Note that not only no *significant* differences but also no *pattern-data* differences between the CMTE-L PSTs and the CMTE-V PSTs were found on any belief.

In summary, the results that are significant indicate that PSTs who focused on children's mathematical thinking (PSTs in the CMTEs) developed more sophisticated beliefs about mathematics and mathematics understanding and learning than those who did not focus on children's mathematical thinking. Two findings are of particular interest. First, students who visited conveniently located classes (MORE-C) tended to change little as a result of their semester's mathematics experiences. This finding indicates that visiting convenient classrooms had a dampening effect on the beliefs they might have developed as a result of attending the mathematics course. Second, no differences were found between the CMTE-L and the CMTE-V PSTs. These two findings were explored further with qualitative data.

Qualitative Beliefs Data

Benefits of learning about children's mathematical thinking. To understand how focusing on children's mathematical thinking might support the development of PSTs' beliefs about mathematics and mathematics understanding and learning, we consider written student responses from the beliefs survey for an item used to assess Belief 6 (*The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not*). Note that on this belief, scores for both the CMTE-L and CMTE-V students differed significantly from scores of MORE-C students and that data in Table 1 indicate a

pattern toward greater change on this belief for those who focused on children's thinking than for those who did not.

The student responses we consider are from one of three items used to measure Belief 6. This item, shown in Figure 5, was designed to assess whether respondents recognized that relevant real-world contexts often support children's mathematical thinking, whereas symbols (in this case, fraction symbols) are often confusing for children. A CMTE-V student ranked the word problem as most difficult to understand of the four items (rank of 4) in the presurvey but as easiest to understand (rank of 1) in the postsurvey. In the postsurvey, this PST wrote, "The problem involves sharing and it is put in a real world context, it is easier for children to see the problem in this way." Contrast this response with that of a MORE-C student, who ranked the word problem as difficult (rank of 3, equally as difficult as the symbolic comparison problem) in the presurvey then as most difficult (rank of 4) in the postsurvey. Additionally, in his postsurvey comments, the MORE-C student wrote, "Just due to the fact that I see more children [in the MORE-C placements] stumble on word problems because they don't know what info is important and what is not."

8.1 Place the following four problems in rank order of difficulty for children to understand, and explain your ordering (you may rank two or more items as being of equal difficulty). NOTE. Easiest = 1.

	Select rank	Please explain your rank
(a) Understand $\frac{1}{5} + \frac{1}{8}$	Select rank	Please explain your rank
(b) Understand $\frac{1}{5} \times \frac{1}{8}$	Select rank	Please explain your rank
(c) Which fraction is larger, $\frac{1}{5}$ or $\frac{1}{8}$, or are they same size?	Select rank	Please explain your rank
(d) Your friend Jake attends a birthday party at which five guests equally share a very large chocolate bar for dessert. You attend a different birthday party at which eight guests equally share a chocolate bar exactly the same size as the chocolate bar shared at the party Jake attended. Did Jake get more candy bar, did you get more candybar, or did you and Jake each get the same amount of candy bar?	Select rank	Please explain your rank

8.2 Which of these two items did you rank as easier for children?

_____ Item c is easier than Item d.   _____ Item d is easier than Item c.

_____ Items c and d are equally difficult.

Please explain your answer.

Figure 5. Segments 8.1 and 8.2.

In other words, whereas the CMTE-V student moved from viewing symbols as easier for children to understand than real-life contexts to seeing real-life contexts as easier for children to understand than symbols, the MORE-C student's belief as measured by this item moved in the opposite direction.

Why visiting traditional classes may have been detrimental for PSTs' beliefs change. The responses just discussed indicate that the beliefs of a MORE-C student and of a CMTE-V student changed in opposite directions. To better understand the effects of participation in the MORE-C on PSTs' beliefs, we analyzed PSTs' written reflections and found that many of the MORE-C PSTs tended not to observe children making sense of problems embedded in real-life contexts but instead saw instruction that was familiar to them and was centered on children's memorizing procedures. For example, one MORE-C PST concluded from his observations that because "several kids have tough times learning, they need structure, organization, and repetition, repetition, repetition." Instead of providing evidence that could challenge existing beliefs or form the basis of new beliefs, the MORE-C provided many of the PSTs with experiences that bolstered their initial beliefs. PSTs who studied children's mathematical thinking (those in the CMTEs) watched video clips of children making sense of problems situated in relevant contexts. They watched a video in which a child incorrectly solved 4 – 1/8 represented symbolically, then correctly solved the problem situated in the context of eating 1/8 of one of four cookies. The CMTE-V students also watched a video of a peer posing comparable problems for a fifth grader in the CMTE-L. These experiences provided CMTE PSTs with opportunities to draw distinctions between their own memories of struggling with word problems and their observations of children making sense of relevant contexts. Furthermore, whereas the PSTs in the CMTEs had opportunities to reflect upon their experiences in a community setting under the guidance of an instructor, the PSTs in the MORE-C did not have this experience, and so potentially negative perceptions about word problems may have remained intact or even have been strengthened. These results indicate that for students enrolled in a mathematics course for elementary school teachers, engaging in early field experiences in traditional classrooms may actually be more harmful for the development of their beliefs than participating in no early field experience.

Working with children may be important for PSTs. At the outset of the study, we were uncertain as to what differences we might find between those PSTs who focused on children's mathematical thinking by watching and discussing video but not working directly with children and those who focused on children's mathematical thinking through a combination of working directly with children and watching and discussing video. On one hand, we thought that the experience working with children might provide a powerful personal experience, leading to deeper changes for the CMTE-L participants. On the other hand, we recognized that because the CMTE-L students spent more than half their class time preparing for, conducting, and reflecting upon individual interviews, they might benefit less than students in the CMTE-V group, who had more time to discuss and reflect upon care-

fully selected video clips of children's mathematical thinking under the direction of the instructor. Our quantitative beliefs data showed no detectable differences between the two CMTE groups.

We looked to interview data and found that the students who worked with children (those in the CMTE-L group) spoke of important changes we did not measure with the beliefs instrument. PSTs in the CMTE-L reported that the experience of working directly with children was the most compelling aspect of the course. In an open-ended survey completed at the end of the semester, 94% of these PSTs indicated that the CMTE-L had affected their experiences in the content course; 55% of the PSTs attributed the effect to *their interactions with children*, and 40% suggested that the CMTE-L provided a rationale for learning the content in the mathematics course. Pat's[6] response is an example: "Taking the [CMTE-L] has enabled me to make a practical application to what I learned inside my [mathematics content course] and use it and see what function it serves in the elementary classroom."

We believe that the experience of working with children enabled PSTs not only to observe and investigate children's thinking in general but also to grapple with understanding and supporting a particular child who was trying to make sense of mathematics. Consider the comment of a CMTE-L student (cited in Ambrose, 2004): "Working with children is a very valuable experience. It is really easy to say or think what you are going to do in a situation, but sometimes in reality it doesn't work out or you think of something better. Working with children early helps you get comfortable and prepares you for what's to come." The PSTs in the MORE who worked with children also valued the experience. Many wrote that working with children helped them to better appreciate what they were learning in the mathematics course. Gail, a student in the MORE-C, wrote about the benefits that she experienced from visiting a fourth-grade classroom:

> Math is not my strong point and I was really worried that I would be asked to do something that I couldn't, like answer a question or something like that. However I learned so much about myself really. I didn't have any problems helping the kids with the work they were doing. I actually had a lot fun helping these kids and teaching them how to do problems and solve the different things they had to do. It was almost as if the things I saw and learned in my math class [at the university] came to life in the classroom I was observing. I was amazed at the different learning types and styles of the kids. . . . I had a blast with each and every one of these kids and I actually didn't want to leave them on the last day.

We note that despite Gail's enthusiasm for her experience, her beliefs scores changed little. She had a small change on only two beliefs. She claimed that the MORE-C helped to make what she "learned in the math class come to life," but her scores on the beliefs instrument reveal that she failed to adopt the children's-thinking-oriented beliefs the class was promoting.

In summary, whereas many PSTs in the CMTEs experienced disconfirming evidence for their initial beliefs, PSTs in the MORE-C may have experienced

---

[6] All student names are pseudonyms.

evidence in support of their initial beliefs, explaining, in part, why their beliefs appeared to have changed even less than beliefs of PSTs who did not visit classrooms. Also, although no differences were identified in the beliefs of those who focused on children's mathematical thinking by working with children and those who only viewed videos, those who worked with children may have experienced other benefits.

Content Data

Quantitative Content Data

Quantitative Content Data Subjected to Statistical Tests

Table 4 shows the content test (pretest, posttest, and change) scores by treatment. Although the average change scores were higher for the CMTE groups than for the other groups, no pairwise comparisons were statistically significant.

Table 4
Content-Test Results, by Treatment

Treatment	Pretest average	Posttest average	Change score	
			Average	Standard deviation
CMTE-L ($n = 50$)	36.8	51.5	14.7	7.64
CMTE-V ($n = 27$)	39.1	52.6	13.5	9.29
MORE-S ($n = 23$)	38.7	49.9	11.2	7.98
MORE-C ($n = 25$)	32.3	44.6	12.3	7.69
Control ($n = 34$)	37.6	50.4	12.8	8.08

Note. Points possible = 82.

Patterns in the Quantitative Content Data

Although no significant differences were found on the content-test change scores when treatments were compared in a pairwise fashion, we noted patterns in these data. Table 5 shows the percentages of students, by treatment, whose scores increased at least the stipulated number of points between the pretest and the posttest. Looking at the outliers may not be interesting, because every class has special students on each end of the spectrum. For example, 5% of the students, including at least one student from each treatment, scored lower on the mathematics content posttest than on the pretest for reasons that could not be explained by a pretest ceiling effect. Evidently, every treatment had students who were not poised to benefit from their mathematics class for reasons that had little to do with the treatments to which they were assigned. Note another difficulty with the outliers: The CMTE-V group had both the highest percentage of students whose scores increased by at least 25 points and the highest percentage of students whose scores decreased. Therefore, for this analysis we focus on the bulk of the students

Table 5
Percentages, by Treatment and for All Participants, of Students Whose Scores Increased at Least the Stipulated Numbers of Points From Content Pretest to Posttest

Group	Average change in content-test score from pretest to posttest							
	< 0	≥ 0	≥ 5	≥ 10	≥ 15	≥ 20	≥ 25	≥ 30
MORE-S	4.3%	95.7%	87.0%	47.8%	21.7%	8.7%	4.3%	4.3%
MORE-C	4.0%	96.0%	88.0%	52.0%	28.0%	24.0%	8.0%	0.0%
CMTE-V	11.1%	88.9%	81.5%	59.3%	51.9%	29.6%	11.1%	0.0%
CMTE-L	2.0%	98.0%	88.0%	68.0%	50.0%	28.0%	6.0%	0.0%
Control	5.9%	94.1%	88.2%	52.9%	38.2%	23.5%	2.9%	2.9%
All	5.03%	94.97%	86.79%	57.86%	40.25%	23.89%	6.27%	1.24%

in the mid range, that is, those whose scores increased more than 10 points, more than 15 points, and more than 20 points.

In each of these three intermediate categories, the highest percentages of students were the CMTE-L and the CMTE-V students. For example, two thirds of the CMTE-L and three fifths of the CMTE-V students' scores, more than in the other treatments, increased at least 10 points. Consider those who increased at least 15 points: Half the CMTE students' scores (51.9% for CMTE-V students and 50% for CMTE-L students) increased at least 15 points, contrasted with scores of 38.2% of the control students, 28% of the MORE-C students, and 21.7% of the MORE-S students. These percentages drop for increases greater than 20 points, but again the CMTEs had the highest percentage of students with such score increases.

A Secondary Analysis Comparing Content Change

Although no pairwise content comparisons among groups were significant, the means for those who studied children's thinking were higher than for those who did not. We conducted a secondary analysis to determine whether change scores for those in the CMTE-L and CMTE-V groups differed from the scores of those in the MORE-C, MORE-S, and control groups. Because we expected that those who studied the mathematics embedded in children's mathematical thinking would develop richer mathematical understanding than those who did not, this comparison was treated as a one-tailed test.

Table 6 shows the pretest, posttest, and change scores by the two groups pooled according to whether they focused on children's mathematical thinking. Results of a *t* test showed that these differences were significant at the .05 level. The effect size was 0.26 (Cohen's $d = 0.2645$), signifying that the mean change on the content test for PSTs in the CMTEs was about one-fourth standard deviation higher than the mean change for PSTs in the other groups.

In summary, the differences were modest, and we were disappointed in the overall performances of all of the students on the content posttest. However, in the next section we provide interview data that indicate that learning about children's

Table 6
Content-Test Results for CMTE Students and for Control and MORE Students

Treatment	Pretest average	Posttest average	Change score	
			Average	Standard deviation
CMTEs ($n = 77$)	37.6	51.9	14.3	7.87
Control/MOREs ($n = 82$)	36.3	48.5	12.2	8.21

Note. Points possible = 82.

mathematical thinking may have supported PSTs' mathematics learning in ways that were not captured by the PSTs' change scores on the content test.

Qualitative Content Data

Three examples of students' comments provide the readers with a sense of how working with children supported the PSTs' mathematical content knowledge. Heidi found that the CMTE-V caused her to think more deeply about the mathematics she was learning: "First of all, I did learn about math. Because, sadly, I knew how to do it [the procedures], but the concepts, the . . . seriously, I was learning as much as the kids were learning. And it was *so* beneficial to me." This notion of thinking deeply about the concepts and not just learning the procedures also came through from Phil, a CMTE-L student interviewed midsemester:

Phil: One thing I got out of 296 [CMTE-L]—if I hadn't taken 296, I probably would have gone through [the subsequent mathematics courses] focusing on the thing that I already knew, the algorithm that I already knew, and thinking, "All right, that's the best." But now I realize that I have to take it all in, everything that the class is teaching, not just what I think is the most important. Because all of this is important. I probably wouldn't have realized that if I hadn't taken 296.

RP: Why is it important?

Phil: Because people think in different ways, and not everyone thinks like me.

RP: And you don't think you would have gotten that from [the mathematics course alone]?

Phil: No. No way.

We end this section with a statement made during the last day of class by Nora, a student enrolled in the CMTE-L, about what she might tell a friend she learned from taking the class:

For people who are going to take [the first mathematics class]—just because *a lot* of the times in class . . . people get *so* mad and *so* frustrated as to why they are learning what they are learning. And then you come [to the CMTE-L], and you see a kid do exactly what you are learning in [the mathematics] class. And it just makes sense, and it eliminates that whole frustration of feeling like "Why am I learning this? Where am I going to ever use this?" So by taking this class, you *see* how . . . the children actually apply what you are learning, the different styles or the different methods for solving problems.

These three PSTs' comments reflect how learning about children's mathematical thinking helped them to look in a new way at the mathematics they thought they already knew.

DISCUSSION

We conducted an experimental study using modified random assignment of prospective elementary school teachers, all of whom were enrolled in a mathematics course, to one of four treatments or to a control group. Results indicate that the beliefs of PSTs who studied children's mathematical thinking underwent more change than the beliefs of PSTs visiting classrooms. Results from the control group show that the curriculum in the university mathematics course supported some beliefs change. The beliefs of PSTs visiting conveniently located classes underwent less change than the beliefs of PSTs in either the other treatments or the control group, indicating that visiting these classes interfered with establishing the beliefs that the mathematics course might otherwise have fostered. Although no differences were found on change of either beliefs or knowledge between PSTs who focused on children's mathematical thinking by watching video and PSTs who focused on children's mathematical thinking through a combination of watching video and working with children, interview data indicated that work with children provided PSTs benefits we did not measure. Although other researchers have reported little change on PSTs' beliefs, we found significant changes on PSTs' beliefs; however, the beliefs-change variability within group was large. Although the mathematical content knowledge of PSTs who focused on children's mathematical thinking improved more than the mathematical content knowledge of those PSTs who did not, the overall change in content knowledge among PSTs was discouragingly low. We will situate some of these findings by returning to the distinction we drew between the apprenticeship approach and the laboratory approach to teacher education.

Studying Children's Mathematical Thinking Versus Visiting Classrooms: The Benefits of a Laboratory Approach in Mathematics Teacher Education

As stated previously, we viewed the CMTEs in our study as models for Dewey's (1904/1964b) laboratory approach, whereas we viewed the MOREs as models for the apprenticeship approach. An important difference in the experiences of the PSTs enrolled in these treatments related to the amount of variability within each treatment. PSTs in the CMTE were able to focus on the mathematical thinking of children because we had controlled many variables typically associated with teaching. By controlling the mathematical tasks used with children in the CMTEs, we were able to increase the likelihood that the PSTs encountered particular situations that had the potential to affect their beliefs. For example, the PSTs enrolled in the CMTE-L asked third graders to solve a multidigit subtraction problem and then posed questions designed to reveal the children's understanding of the standard algorithm.

Because most of the children interviewed correctly applied the standard algorithm but demonstrated a lack of understanding of place-value concepts, most PSTs in the CMTE-L experienced direct evidence supporting the belief that one can perform procedures without understanding underlying concepts, and those in the CMTE-V, who did not work with children, watched and discussed, under guidance of the instructor, a video of one such experience from the CMTE-L class. PSTs in the MOREs, whether they visited convenient classes or select classes, did not report having this kind of experience, because they were visiting a variety of classrooms in which various mathematical tasks were being used. When a child in a MORE classroom used a procedure without understanding it, this phenomenon would likely not have been noted by a PST observer unless the teacher probed the understanding of the child. Furthermore, even if a teacher probed a child's thinking, a PST visiting the complex classroom environment would be less likely to attend to this issue than would a PST enrolled in the more controlled laboratory environment of the CMTEs.

The controlled laboratory environment of the CMTE provided a platform for reflection that would be difficult to duplicate in a MORE. The PSTs in the CMTEs watched the same videos, enabling them to focus, in class discussion, on the ways that children engaged with tasks instead of having to describe the nature of the mathematical task and the context in which it had taken place. Even though PSTs in the CMTE-L worked with individual children, they shared common experiences because they were guided to use carefully selected questions and tasks designed to raise particular issues. Thus, even the few PSTs who worked with children whose knowledge of the procedure was connected to their underlying understanding of the concepts were able to hear from their peers about the more common experience. Because the PSTs with exceptional experiences had immediate access to the direct experiences of their peers, all PSTs could recognize what was the norm for children's mathematical thinking and what was the exception; consequently, all could grapple with the belief we intended for them to consider.

The instructors of the CMTEs could maintain a focus on the mathematics and the children's interpretations of the mathematics. Any group discussion designed to accompany the MOREs would likely have been less effective than the discussions in the CMTEs. Before analysis could take place, a shared context would have to be developed; PSTs would need a great deal of time to describe the lessons that had been observed. However, even under the best of classroom-observation circumstances, such as those the MORE-S PSTs experienced in selected reform-oriented classes, we expect that because the PSTs' experiences varied greatly and because the PSTs could share only their interpretations of the experiences, guiding the PSTs toward a particular theme (i.e., children can use procedures that they do not understand) would be difficult without the instructor's knowing what had transpired in the observed classrooms. We would expect PSTs to have difficulty using the group discussion as an opportunity to come to understand children's thinking more deeply.

Consider the amount of control the instructors had over each treatment. The CMTE-V was the most controlled because the PSTs saw only examples of children's

thinking selected by the instructors. Each video example was chosen to make a particular point, and we directed the PSTs' attention by generally filming only the child, rather than the child and the interviewer (Philipp & Sowder, 2002). PSTs might assume that such examples are contrived or atypical and might not consider them authentic evidence for challenging existing beliefs or supporting the creation of new beliefs. For example, the PSTs might view a video clip of a child struggling to understand nonstandard algorithms as an aberrant case instead of as evidence for the belief that children can use procedures without understanding them. The CMTE-V PSTs in our study did not react in this way, perhaps because they watched not only selected videos but also, on six occasions, videos filmed the previous day of PSTs enrolled in the CMTE-L conducting full interviews or tutorials with children. In the MOREs, the only control we exercised was in the initial placement with the teachers and through the requirement that the majority of what they observed be mathematics teaching. Even in the preferential MORE-S group, we could not ensure that the lessons PSTs observed would reveal children's mathematical thinking, that children would have opportunities to make sense of concepts and discuss their thinking, or that the PSTs would focus on children's strategies. We believe that the CMTEs were effective because we were able (a) to control for variables that might otherwise distract PSTs, (b) to still maintain sufficiently authentic experiences that PSTs found relevant to their future work as teachers, and (c) to provide PSTs opportunities for guided reflection.

Changing PSTs' Beliefs

Changing PSTs' beliefs has been difficult, and too often researchers assume a one-way relationship between beliefs and practice, whereby teachers' beliefs change and changes in practice follow (Philipp, 2007; A. G. Thompson, 1992). This study indicates that PSTs' beliefs *can* change. For some, the content of the mathematics course was sufficient to stimulate a change in beliefs. For many, their beliefs tended to change because they were engaged in mathematical activities designed to position them either to act or to consider how to act with children.

We were convinced that PSTs' interest in and care for children helped to engage them in the study of mathematics. Students in the CMTE-L and in both MORE groups spoke and wrote about how interactions with children shaped their thinking and motivated them to take the mathematics class seriously. Although caring was an important starting point, it was insufficient to support the specific beliefs change we envisioned. PSTs also needed a structured environment in which they interacted with children making sense of mathematics and collectively reflected on the significance of these interactions. In the absence of this structure (as in the MORE groups), PSTs wrote about the importance of teaching mathematics in a variety of ways to meet the needs of all students but had little appreciation for what doing so would entail. We posit that MORE PSTs, lacking an occasion to discuss their observations, failed to appreciate any phenomena that entailed children's mathematical thinking. We concluded that a structured laboratory environment was more likely

to support the beliefs change we hoped to cultivate than a more loosely organized apprenticeship structure.

We also concluded that mathematics educators should expect considerable variability in beliefs change when working with large groups of PSTs. For example, across the seven beliefs, scores for roughly one third to one half of the PSTs in the CMTEs underwent large change on each of the beliefs. However, scores for roughly one fifth to one third of the PSTs in these same treatments showed no change on each of the beliefs. This lack of change of beliefs could not be attributed to the students' general feelings about the CMTE experience; anonymous end-of-course evaluations indicated that the CMTE courses were deemed effective and valued by virtually all the students. Also, some students in the least effective treatment, the MORE-C, showed large score increases on most of the beliefs. What is one to make of such findings? We conclude that no holy grail exists for educating prospective elementary school mathematics teachers. Some PSTs are poised to benefit from a variety of experiences, and for these PSTs, whether their experience is laboratory based or apprenticeship based may be of no consequence, so long as they have opportunities to think about issues of teaching and learning. Furthermore, some PSTs may not be ready to undergo changes in their beliefs, regardless of the experience provided to them. This within-group variation makes generalizations from small samples to large groups suspect; for this reason, we consider the large-scale experimental design a critical aspect of this study.

Changing PSTs' Knowledge

PSTs, to be poised to engage their students in rich mathematical instruction, need to develop deep understanding of the mathematics they will teach, and mathematics courses specially designed for this purpose are offered throughout the country. The results of our quantitative study indicated that PSTs who undertook their first such course while focusing on children's mathematical thinking learned more mathematics than students who did not focus on children's mathematical thinking, with an effect size of about one-fourth standard deviation (Cohen's $d = 0.2645$). Quantitative pattern data supported this finding, with the most compelling result being that content-test scores increased at least 15 points between pretest and posttest for 50.7% of the CMTE students but for only 30.5% of the students who did not study children's mathematical thinking.

However, a troubling result lies beneath these data. On a content test designed to address the conceptual and procedural mathematics of the mathematics content course, the mean change scores were disappointingly low. On an 82-point test, the average increase for those who studied children's mathematical thinking was only 14.3 points (from 37.6 to 51.9 points), and the average increase of those who did not study children's mathematical thinking was 12.2 points (from 36.3 to 48.5). Although the CMTEs' change score was 17.2% higher than the change score for those who did not study children's mathematical thinking, we were disappointed. Why did the scores of those who completed the mathematics course concurrently

with the Children's Mathematical Thinking course between the pretest and posttest increase so little?

Perhaps more important than the CMTEs' effect on PSTs' content knowledge is the effect of broadening their abilities to navigate the mathematical landscape, to consider "What is this child's understanding of this topic? What should I do next to support this child? How might I react to an unanticipated response? How do I follow up a correct solution? Is a different representation called for?" That is, the CMTE students grew in their knowledge of content in the interface with the child. Measuring such hypothesized growth would require items different from most of those in our content instrument, perhaps a different kind of assessment focused instead on mathematical content knowledge for teaching (Hill, Rowan, & Ball, 2005).

Additionally, we hypothesize that PSTs across all treatments learned more mathematics than the results of our content instrument indicated but that the content instrument was not sensitive enough to capture these changes. Perhaps teaching for Profound Understanding of Fundamental Mathematics (Ma, 1999) is even more difficult than we realized, and students have more to learn about the mathematical terrain of elementary school topics than we expected. We were surprised, for example, to find that 16% of the PSTs in the pretest did not identify 1/2 and 3/6 as equal, and 10% in the pretest failed to identify 3/7 as larger than 2/7. We were chagrined that even at the posttest these two items were answered incorrectly by 11% and 8% of the students, respectively; we had expected nearly perfect responses to these items after the mathematics content course.

One explanation for these findings is that some students, for a variety of reasons, do not seem to benefit from a particular class at a particular time. We are led to make the following recommendation: Study the contributions of a mathematics course more carefully. Perhaps the modest improvement in this study reflects only a limitation of the particular items and the mode of testing, or even the particular course offered, but possibly such improvement is typical of such courses. Because college courses usually do not have pretest-posttest designs, we have no bases for comparison. The relatively scant content-knowledge growth as measured for PSTs in this study, however, indicates a need to examine more closely what such courses do accomplish and to compare results within the community.

IMPLICATIONS: REFLECTION AND THE INFUSION OF CHILDREN'S MATHEMATICAL THINKING INTO MATHEMATICS TEACHER EDUCATION

The notion that we first teach PSTs mathematics content and later address issues of teaching and learning was inverted in this study. Learning about children's mathematical thinking facilitated the learning of mathematics while supporting the development of the PSTs' beliefs. After considering two models for helping PSTs learn about children's mathematical thinking, we conclude that for those who have access only to a video experience, the CMTE-V has the potential to lead to significant

changes, in many ways comparable to those resulting from experiences in which video use and work with children are combined. However, we think that the time PSTs spend working directly with children, under the guidance of an instructor who helps the PSTs learn how to interview and use carefully selected questions, is well spent because we found that although the students in the CMTE-Ls showed the same measured increase as the students in the CMTE-V, they also reported benefits not reported by students in the CMTE-V. Some PSTs reported that they appreciated the experience of simply working with the child. Some PSTs had had few experiences working with children, and others who had worked with children appreciated engaging with the children on the topic of mathematics. Many PSTs reported that aspects of working with children, such as devising a question to reveal a child's understanding or a task to promote a child's learning, were challenging. However, even challenging experiences were positive for the PSTs, who not only found that they learned from working with the children but also reported, almost unanimously, feeling that their interaction was positive for the child. Several PSTs spoke of valuing the feeling of learning to successfully work with children and said that these experiences confirmed for them their decisions to become teachers; a few students in the CMTE-L group decided, however, *not* to become teachers.

Another result of our study with implications for teacher education relates to the role of early field experiences often used in teacher education. PSTs' feedback (like Gail's, reported previously in this article) often conveys the sense that early field experiences are magical when PSTs and children are brought together. We recognize value in enabling prospective teachers to work closely with children; however, the results of our study indicate that the magic the PSTs experience may not be of the kind of mathematics that educators value, at least for promoting generative beliefs about mathematics, mathematics learning, and understanding.

We suspect that an important component distinguishing our treatments was the *type* of experiences the PSTs had and their opportunities to reflect on these experiences. In the same way that a physics lab is offered in conjunction with a physics class as a means to ground the theory in practice so as to enhance the learning of both, we see the CMTE as a lab for a mathematics class for PSTs. The laboratory model of the CMTEs provided opportunities for students to engage in guided reflection about issues related to mathematics, teaching, and learning. This finding both supports and extends previous work on prospective teachers' reflection.

Earlier we noted Dewey's notion that PSTs often want recipes for teaching. The laboratory approach was designed to move beyond simply teaching recipes to providing PSTs with opportunities to become more reflective about issues of teaching and learning. For Dewey, amassing and retaining information that was not understood was nothing more than an "undigested burden" (1933/1964a, p. 249). For information to become knowledge, it must be comprehended, and "understanding, comprehension, means that the various parts of the information acquired are grasped in their relations to one another—a result that is attained only when acquisition is accompanied by constant reflection upon the meaning of what is studied" (p. 249).

More recently, researchers studying teacher education have added to our understanding of the role that reflection plays in teacher education. Cooney et al. (1998) found that reflection played an important role in the growth of prospective secondary school teachers over their last year in an undergraduate teacher preparation program. Those student teachers who were poised to emerge as reflective practitioners were those most reflective about their own beliefs as compared to the beliefs of others. Additionally, in a study of reflection and its role in the education of four prospective elementary school teachers in a field-based mathematics methodology course, Mewborn (1999) found that the PSTs were able to engage reflectively, but they needed support in learning to observe mathematics teaching and learning environments and, in particular, in developing an internal locus of authority for pedagogical ideas. Mewborn identified five elements of the design of the field experience that she considered critical to the PSTs' becoming reflective about teaching and learning mathematics: (a) The field experience was approached from an inquiry perspective; (b) the PST, teacher, and teacher educator participated as a community of learners; (c) the community was nonevaluative; (d) the PSTs were given time to reflect; and (e) the field experience was subject specific. Four of these five components were elements of our study, and the only one not present, the nonevaluative aspect of the experience, was not a major issue, because generally the students were not worried about their grades in the CMTEs.

Reflection in a community of learners helped the PSTs to comprehend the children's mathematical thinking they encountered in the laboratory setting. The next issues to consider are how to offer more such experiences and what factors play out in faculty's ability to offer such a course. Currently, many mathematics methodology instructors offer such experiences, but how might these experiences be infused earlier? At our institution we have developed, as a course, a children's mathematical thinking experience that is required of all Liberal Studies majors (the major taken by PSTs), and local community colleges are also beginning to offer this course. We do not expect most institutions to develop such a course. However, we think that a promising way to infuse children's mathematical thinking early in PSTs' undergraduate experiences, to at least modestly affect their concurrent and subsequent learning of mathematics, is to infuse children's thinking into mathematics content courses. Reactions to experiments with this infusion, with faculty both at our institution and around the country, have been positive. For example, a faculty member at another institution expressed enthusiasm for the way that including video clips of children's mathematical thinking positively affected his college mathematics content course:

> I have used the tape to show my prospective elementary teachers the kind of creative and "different" thinking students use to reason and make calculations. The video clips became motivational clips and saved me having to make the argument for PUFM [Profound Understanding of Fundamental Mathematics, (Ma, 1999)]. (George Poole, personal communication, November 12, 2001)

In conclusion, we set out to determine whether an integration of mathematical-content learning with a focus on children's mathematical thinking would enhance

PSTs' learning of mathematics and foster their developing reform-oriented beliefs. We found that a laboratory model facilitated this integration. Although we expected greater change in the PSTs' mathematics knowledge, we were impressed with changes in their beliefs. We emphasize that these changes took place early in the PSTs' teaching training. We are hopeful that these changes will help the PSTs to approach their future mathematical experiences from a meaning-making perspective so that they might take full advantage of future mathematics content and methods courses. Ideally they would have further experiences learning about and experimenting with children's mathematical thinking throughout their teacher education programs so that they would be poised to build on children's thinking when they become teachers.

REFERENCES

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education, 7*, 91–119.

Ambrose, R., Clement, L., Philipp, R., & Chauvot, J. (2004). Assessing prospective elementary school teachers' beliefs about mathematics and mathematics learning: Rationale and development of a constructed-response-format beliefs survey. *School Science and Mathematics, 104*, 56–69.

Ball, D. L. (1990). Breaking with experience in learning to teach mathematics: The role of a preservice methods course. *For the Learning of Mathematics, 10*(2), 10–16.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*, 499–531.

Clarke, D. M. (1997). The changing role of the mathematics teacher. *Journal for Research in Mathematics Education, 28*, 278–308.

Cooney, T. J., Shealy, B. E., & Arvold, B. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education, 29*, 306–333.

D'Ambrosio, B., & Campos, T. M. M. (1992). Pre-service teachers' representations of children's understanding of mathematical concepts: Conflicts and conflict resolution. *Educational Studies in Mathematics, 23*, 213–230.

Darling-Hammond, L., & Sclan, E. M. (1996). Who teaches and why: Dilemmas of building a profession for twenty-first century schools. In J. Sikula (Ed.), *Handbook of research on teacher education* (2nd ed., pp. 67–101). New York: Simon & Schuster Macmillan.

Dewey, J. (1929). *The sources of a science of education*. New York: Liveright.

Dewey, J. (1964a). *The process and product of reflective activity: Psychological process and logical form*. New York: Random House. (Original work published 1933)

Dewey, J. (1964b). *The relation of theory to practice in education*. New York: Random House. (Original work published 1904)

Dewey, J. (1990). *The school and society: The child and the curriculum*. Chicago: The University of Chicago Press. (Original work published 1902)

Fennema, E., Carpenter, T., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). Mathematics instruction and teachers' beliefs: A longitudinal study of using children's thinking. *Journal for Research in Mathematics Education, 27*, 403–434.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147–164). New York: Macmillan.

Franke, M. L., Carpenter, T. P., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal, 38*, 653–690.

Grant, T. J., Hiebert, J., & Wearne, D. (1998). Observing and teaching reform-minded lessons: What do teachers see? *Journal of Mathematics Teacher Education, 1*, 217–236.

Green, T. F. (1971). *The activities of teaching*. New York: McGraw-Hill.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*, 371–406.

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Reston, VA: National Council of Teachers of Mathematics.

Hochberg, Y. (1988). A sharper Bonferroni procedure for multiple tests of significance. *Biometrika, 75,* 800–802.

Long, J. S., & Freese, J. (2001). *Regression models for categorical dependent variables using STATA*. College Station, TX: STATA Corporation.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

McDonough, A., Clarke, B. A., & Clarke, D. M. (2002). Understanding assessing and developing young children's mathematical thinking: The power of the one-to-one interview for preservice teachers in providing insights into appropriate pedagogical practices. *International Journal of Education Research, 37*, 211–226.

McIntyre, D. J., Byrd, D. M., & Foxx, S. M. (1996). Field and laboratory experiences. In J. Sikula (Ed.), *Handbook of research on teacher education* (2nd ed., pp. 171–193). New York: Simon & Schuster Macmillan.

Mewborn, D. S. (1999). Reflective thinking among preservice elementary mathematics teachers. *Journal for Research in Mathematics Education, 30*, 316–341.

National Center for Education Statistics. (1999). *Highlights from the TIMSS 1999 video study of eighth-grade mathematics teaching*. Washington, DC: National Center for Education Statistics. Retrieved May 12, 2005, from http://nces.ed.gov/pubs2003/timssvideo/

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Noddings, N. (1984). *Caring*. Berkeley: University of California Press.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*, 307–332.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Reston, VA: National Council of Teachers of Mathematics.

Philipp, R. A., Cabral, C., & Schappelle, B. P. (2005). *Integrating mathematics and pedagogy to illustrate children's reasoning*. Upper Saddle River, NJ: Pearson Education.

Philipp, R. A., & Sowder, J. T. (2002). Using eye-tracking technology to determine the best use of video with prospective and practicing teachers. In A. D. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th annual meeting of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 233–240). Norwich, United Kingdom: University of East Anglia.

Philipp, R. A., Thanheiser, E., & Clement, L. (2002). The role of a children's mathematical thinking experience in the preparation of prospective elementary school teachers. *International Journal of Educational Reform, 37*, 195–210.

Phillips, D. C. (1998). John Dewey's *The Child and the Curriculum*: A century later. *The Elementary School Journal, 98*, 403–414.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28*, 550–576.

Rokeach, M. (1968). *Beliefs, attitudes, and values: A theory of organization and change.* San Francisco: Jossey-Bass.

Shulman, L. S. (1998). Theory, practice, and the education of professionals. *The Elementary School Journal, 98*, 511–526.

Skemp, R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher, 26*(3), 9–15.

Sowder, J. T., Philipp, R. A., Armstrong, B. E., & Schappelle, B. P. (1998). *Middle-grade teachers' mathematical knowledge and its relationship to instruction*. New York: State University of New York Press.

Sowder, J., Sowder, L., Thompson, A., & Thompson, P. (1999). *Reconceptualizing mathematics: Courseware for elementary and middle grade teachers: Number and number sense*. San Diego, CA: San Diego State University, Center for Research in Mathematics and Science Education.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). New York: Macmillan.

Thompson, P. W., & Thompson, A. G. (1994). Talking about rates conceptually, part I: A teacher's struggle. *Journal for Research in Mathematics Education, 25*, 279–303.

Vacc, N. N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education, 30*, 89–110.

Villaseñor, A. J., & Kepner, H. S. J. (1993). Arithmetic from a problem-solving perspective: An urban implementation. *Journal for Research in Mathematics Education, 24*, 62–69.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. *Review of Research in Education, 24,* 173–209.

Authors

Randolph A. Philipp, San Diego State University, School of Teacher Education & Center for Research in Mathematics and Science Education (CRMSE), 6475 Alvarado Road, Suite 206, San Diego, CA 92120; rphilipp@mail.sdsu.edu

Rebecca Ambrose, University of California, Davis, One Shields Avenue, Davis, CA 95616; rcambrose @ucdavis.edu

Lisa L. C. Lamb, San Diego State University, School of Teacher Education & CRMSE, 6475 Alvarado Road, Suite 206, San Diego, CA 92120; Lisa.Lamb@sdsu.edu

Judith T. Sowder, San Diego State University, Mathematics Department (emeritus) & CRMSE, 6475 Alvarado Road, Suite 206, San Diego, CA 92120; jsowder@sciences.sdsu.edu

Bonnie P. Schappelle, San Diego State University, CRMSE, Alvarado Road, Suite 206, San Diego, CA 92120; bschappe@sunstroke.sdsu.edu

Larry Sowder, San Diego State University, Mathematics Department (emeritus) & CRMSE, 6475 Alvarado Road, Suite 206, San Diego, CA 92120; lsowder@sciences.sdsu.edu

Eva Thanheiser, Rutgers University, Graduate School of Education, 10 Seminary Place, New Brunswick, NJ 08901; evat@rci.rutgers.edu

Jennifer Chauvot, University of Houston, Department of Curriculum & Instruction, 256 Farish Hall, Houston, TX 77204-5027; jchauvot@uh.edu

APPENDIX A

IMAP's Seven Beliefs Assessed

(Note. These statements were used by the research team to describe what we intended to measure with our survey, but they were not shown to the study participants.)

Beliefs about mathematics

1. Mathematics is a web of interrelated concepts and procedures (school mathematics should be too.)

Beliefs about knowing or learning mathematics, or both

2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts. That is, students or adults may know a procedure they do not understand.
3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.
4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

Beliefs about children's (students') doing and learning mathematics

5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than their teachers, or even their parents, expect.
6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.
7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

APPENDIX B

Odds Ratios

Because some readers may not have experience with odds ratios, we provide a simple example using only two categories per group: Suppose that the same number of males and females apply for admission to a university, and 9 of 10 men applying are accepted, whereas 8 of 10 women are accepted. How might one compare the number of men accepted to the number of women accepted? One could state that there are 9/8 times as many male applicants admitted as female applicants. One could also state that there are twice as many female applicants denied admission as male applicants. These two statements, although both true, leave one with a different sense. How is one to deal with this difference? One solution is to standardize the comparison by use of an odds ratio that multiplicatively compares the number accepted to the number rejected for each group. To do so, one composes a ratio of those *in* to those *not in* for each group and then compares these ratios. In this example, the ratio of males' acceptances to males' rejections is 9:1, whereas the ratio of females' acceptances to females' rejections is 8:2, or 4:1. Or, another way to put this is that for every male rejected, 9 are admitted, whereas for every female rejected, 4 are admitted. The odds ratio is a means for capturing all this information; it is created by taking the ratio of the odds, which in this case would be 9:1 / 8:2, or 9/4, or 2 1/4. One interprets this odds ratio by noting that the odds of acceptance among males are 2 1/4 times the odds of acceptance among females. This does *not* mean that 2 1/4 times as many males are admitted as females, nor does it mean that males are 2 1/4 times as likely to be admitted as females. It means that for every male rejected, the number of males accepted is 2 1/4 times the number of females accepted for every female rejected.