

Professional Noticing of Children's Mathematical Thinking

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The construct *professional noticing of children's mathematical thinking* is introduced as a way to begin to unpack the in-the-moment decision making that is foundational to the complex view of teaching endorsed in national reform documents. We define this expertise as a set of interrelated skills including (a) attending to children's strategies, (b) interpreting children's understandings, and (c) deciding how to respond on the basis of children's understandings. This construct was assessed in a cross-sectional study of 131 prospective and practicing teachers, differing in the amount of experience they had with children's mathematical thinking. The findings help to characterize what this expertise entails; provide snapshots of those with varied levels of expertise; and document that, given time, this expertise can be learned.

Key words: Children's strategies; Early childhood, K–4; In-service teacher education; Pedagogical knowledge; Planning, decision-making; Preservice teacher education; Professional development; Teaching practice

The range of what we think and do
is limited by what we fail to notice.
And because we fail to notice
that we fail to notice,
there is little we can do
to change
until we notice
how failing to notice
shapes our thoughts and deeds. (Goleman, 1985, p. 24)

Noticing is a common activity of teaching, but, as Goleman suggested, noticing effectively is both complex and challenging. For many years, psychologists have studied how we attend to stimuli in our environments, and researchers have learned not only that we have focusing and capacity limitations (Schneider & Shiffrin, 1977; Shiffrin & Schneider, 1977) but also that instead of perceiving the world objectively, we construct what we see (Gibson, 1979). Therefore, individuals looking at the same thing may see it in different ways. In addition, studies on inattention blindness and change blindness have highlighted what we do not see even when it is present (Most et al., 2001; Simons & Chabris, 1999). Consider Simons's (2000)

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popular study showing that a large percentage of individuals watching a video of two teams playing basketball completely miss the unexpected arrival of a man in a gorilla suit! Studies dating back to the classic study by Bartlett (1932) also have shown that individuals' knowledge, beliefs, and experiences influence what is seen and in what ways a stimulus is processed. More recently, Erickson (2007) compared viewing (in this case, video) to completing projective tests in which individuals are asked what they see in ambiguous stimuli (e.g., inkblots); their reactions extend beyond the information available to reflect the viewer as much as the stimulus itself.

Given the subjective nature of noticing, researchers have found that distinct patterns of noticing have evolved for groups of individuals who hold similar goals and experiences, such as groups of professionals. Goodwin (1994) used the term *professional vision* to capture how members of a profession develop perceptual frameworks that enable them to view complex situations in particular ways. For example, archeologists develop sensitivities to variations in color, texture, and consistencies of sand, and attending to these details is a critical component of their abilities to reason about a landscape. Similarly, Stevens and Hall (1998) used *disciplined perception* to describe the visual practices characteristic of particular professions (or disciplines), and Mason (2002) focused on the idea of *intentional noticing*, contrasting this type of noticing that is characteristic of a profession with everyday noticing (what everyone does). In short, learning to notice in particular ways is part of the development of expertise in a profession. In the next section, we explore research on the noticing expertise of a particular group of professionals, mathematics teachers.

NOTICING OF MATHEMATICS TEACHERS

Researchers have explored the noticing of mathematics teachers to understand how they make sense of complex classroom environments in which they cannot be aware of or respond to everything that is occurring. Sherin and van Es have offered the most extensive work on noticing in mathematics education (Sherin, 2001, 2007; Sherin & Han, 2004; Sherin & van Es, 2005, 2009; van Es & Sherin, 2002, 2006, 2008; van Es, in press). They have used Goodwin's (1994) concept of professional vision to examine teacher learning, most often in video clubs in which teachers watch and discuss video from their own classrooms. Their three-part learning-to-notice framework includes (a) identifying noteworthy aspects of a classroom situation, (b) using knowledge about the context to reason about the classroom interactions, and (c) making connections between the specific classroom events and broader principles of teaching and learning (van Es & Sherin, 2008). In a series of studies, they have found that teachers can improve their noticing by changing what they notice (e.g., moving from a focus on teachers' actions to students' conceptions) and how they reason (e.g., moving from mere reporting of events to synthesizing and generalizing, and moving from evaluative comments to interpretive comments based on evidence) (Sherin & Han, 2004; van Es & Sherin, 2008). They have also

demonstrated that the teacher learning that occurs in video clubs can extend to classroom instruction in similar ways (Sherin & van Es, 2009).

Of particular interest is the critical role that interpretation plays throughout the Sherin and van Es work—they have argued that how individuals analyze what they notice is as important as what they notice. In contrast, in the noticing work of Star and Strickland (2007), the focus was exclusively on identifying noteworthy aspects (Sherin and van Es's first component). Star and Strickland argued that their interest in what teachers do and do not attend to in classroom lessons is foundational for future instruction. Their results showed that, after a methods course, prospective secondary teachers were better able to notice classroom events in four observation categories (classroom environment, tasks, mathematical content, and communication). Thus, similar to Sherin and van Es, Star and Strickland found that noticing expertise could improve with support.

Other researchers have engaged in a variety of work closely related to noticing. For example, Santagata, Zannoni, and Stigler (2007) have offered a *lesson-analysis* framework as a means for helping prospective teachers gain expertise in observing and reasoning about classroom events. Their three-part framework highlights the identification of (a) learning goals, (b) student learning in relation to those goals, and (c) alternative teaching strategies to accomplish those goals. In their work, they asked teachers to analyze classroom videos that were collected as part of the video studies in the Third International Mathematics and Science Study (TIMSS) (Hiebert et al., 2003; Stigler & Hiebert, 1999). Because of the cultural nature of teaching, using videos from other countries can make instructional routines more visible and thus available for analysis. Miller and Zhou (2007) also used international work to explore teachers' noticing. In a comparison of what U.S. and Chinese elementary school teachers noticed in classroom videos, they found striking differences between the two cultures. For example, U.S. teachers were more likely to comment on pedagogical issues and the videotaped teachers' personalities, whereas Chinese teachers were more likely to comment on the mathematical content of the classes. These differences were consistent with the documented training and beliefs of U.S. and Chinese teachers.

Throughout all these studies, researchers define *noticing* in a multitude of ways, but the connecting thread is making sense of how individuals process complex situations. Furthermore, their findings all serve to underscore the idea that teachers see classrooms through different lenses depending on their experiences, educational philosophies, cultural backgrounds, and so on and that particular kinds of experiences can scaffold teachers' abilities to notice in particular ways. We build on the noticing research both inside and outside of mathematics education, and we have chosen to enter this dialogue by selecting a particular focus for noticing—children's mathematical thinking. In our study, we document groups of teachers' expertise with a specialized type of noticing, what we call *professional noticing of children's mathematical thinking*. By identifying a focus for noticing, we attend less to the variety of what teachers notice and more to how, and the extent to which, teachers notice children's mathematical thinking.

Professional Noticing of Children's Mathematical Thinking

During the past few decades, mathematics educators have gained substantial knowledge about both children's mathematical thinking within specific content domains and the power of teachers' regularly eliciting and building on children's thinking (Grouws, 1992; Kilpatrick, Swafford, & Findell, 2001; Lester, 2007; NCTM, 2000). Instruction that builds on children's ways of thinking has been linked to rich instructional environments for students (Clarke, 2008; Cobb et al., 1991; Gearhart & Saxe, 2004; Shifter, 1998; Sowder, 2007; Wilson & Berne, 1999) and documented gains in student achievement (Bobis et al., 2005; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema et al., 1996; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Villaseñor & Kepner, 1993). In addition, research has shown that teachers who, through professional development, learn how to learn from the thinking of the individual children in their classrooms can continue learning even after formal professional development support ends (Franke, Carpenter, Levi, & Fennema, 2001). Given these documented benefits for both students and teachers and the instructional vision of building on children's thinking suggested in national reform documents (Kilpatrick et al., 2001; NCTM, 2000), we have chosen to target teachers' expertise in *professional noticing of children's mathematical thinking*. We conceptualize this expertise as a set of three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings.

Attending to Children's Strategies

Terms such as *highlighting* (Goodwin, 1994) or *making call-outs* (Frederiksen, Sipusic, Sherin, & Wolfe, 1998) have been used to describe how professionals attend to noteworthy aspects of complex situations. We are interested in the extent to which teachers attend to a particular aspect of instructional situations: the mathematical details in children's strategies. Research has shown that these strategies can be complex and that the strategy details are important because they provide a window into children's understandings (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, & Levi, 2003; Lester, 2007). Similar to those studying expertise in other areas, we hypothesize that teachers with more expertise in children's mathematical thinking will be better able to recall the details of children's strategies because they have developed meaningful ways to discern patterns and chunk information in complex situations (Bransford, Brown, & Cocking, 2000).

Interpreting Children's Mathematical Understandings

We are also interested in how teachers interpret children's understandings as reflected in their strategies. On the basis of a single problem, we do not expect a teacher to construct a complete picture of a child's understandings, but we are interested in the extent to which the teacher's reasoning is consistent with both the details of the specific child's strategies and the research on children's mathematical

development. Mason (2002) contrasted this type of productive (evidenced-based) interpretation with unproductive (snap evaluations based on minimal evidence) interpretation. We have chosen to exclude snap evaluations from our construct of interpretation, and our decision to separate interpretation and evaluation is consistent with the work of others (Blythe, Allen, & Powell, 1999; Seidel, 1998; van Es & Sherin, 2008).

Deciding How to Respond on the Basis of Children's Understandings

Our third component skill of interest is the reasoning that teachers use when deciding how to respond. We are not arguing that there is a single best response, but we are interested in the extent to which teachers use what they have learned about the children's understandings from the specific situation and whether their reasoning is consistent with the research on children's mathematical development. Note that we do not include the execution of the response in our conceptualization of professional noticing, so, in a sense, we are focusing on intended responding. With this focus, we join others interested in how potential instructional responses link to the other component skills of attending and interpreting. For example, Erickson (in press) has argued that teachers' selective attention is determined by consideration of next instructional steps in that teachers judiciously direct their attention to what is necessary to take action, and the lesson-analysis framework of Santagata and her colleagues (2007) included making sense of the teacher's actions and then proposing alternative instructional strategies to accomplish the same goal.

Investigation of Professional Noticing of Children's Mathematical Thinking

In investigating professional noticing of children's mathematical thinking, we explored these three component skills to begin to unpack the in-the-moment decision making that is often hidden, but foundational to the complex and challenging view of teaching endorsed in national reform documents (Kilpatrick et al., 2001; NCTM, 2000). We focus on a particular type of decision making—decision making that occurs on a daily basis in the classroom when a child offers a verbal- or written-strategy explanation. This type of in-the-moment decision making is in contrast to the long-term decision making (or planning) that teachers do after school when they are not interacting with children.

Because teachers cannot preplan in-the-moment responses, this improvisational part of teaching requires teachers to constantly analyze and connect specific situations to what they know about children's mathematical development (Franke, Kazemi, & Battey, 2007; Heaton, 2000; Lampert, 2001). We suggest that, before the teacher responds, the three component skills of professional noticing of children's mathematical thinking—attending, interpreting, and deciding how to respond—happen in the background, almost simultaneously, as if constituting a single, integrated teaching move. Thus, our conceptualization of the construct of professional noticing of children's mathematical thinking makes explicit the three

component skills but also identifies them as an integrated set that provides the foundation for teachers' responses.

Our assessment of professional noticing of children's mathematical thinking built on the scenario methodology used in a series of expert/novice studies conducted in the 1980s (for a summary, see Berliner, 1994). In these studies, researchers used scenario methodology to assess teachers' recall and analysis of classroom situations. For example, participants were shown slides or video and were asked, in structured interviews, to recall as much as possible of what they had seen and then to comment on the situation. Of most interest to researchers was the participants' recall of general pedagogical techniques such as management and organization. In our study, we also used scenario methodology but moved beyond a focus on recall of general pedagogical techniques to instead focus on the specific expertise needed to teach mathematics effectively by building on children's mathematical thinking. We also extended the earlier methodology by moving beyond comparison of only two groups (experts versus novices) in which only limited conclusions can be drawn about how teachers come to acquire skills (Peterson & Comeaux, 1987). Instead, we investigated the professional-noticing expertise of four groups of participants with differing amounts of experience with children's mathematical thinking. In the next section, we describe these participant groups and give further details about our measures and analysis approach.

METHOD

The data were drawn from a larger study entitled "Studying Teachers' Evolving Perspectives" (STEP), in which our overall goal was to map a trajectory for the changing needs and perspectives of teachers engaged in sustained professional development focused on children's mathematical thinking. In this cross-sectional study, we explored the professional noticing of children's mathematical thinking by 131 prospective and practicing teachers, and we used two written measures designed to assess the three component skills of attending, interpreting, and deciding how to respond.

Participants

Participants included three groups of practicing K–3 teachers and one group of prospective teachers who were beginning their studies to become elementary school teachers (see Table 1). Consistent with the population of K–3 teachers, our group of participants was overwhelmingly female (119 females and 12 males).

Participant groups differed in their experience with children's mathematical thinking. Specifically, Prospective Teachers, by virtue of their lack of teaching experience and professional development, had the least experience with children's thinking, followed by Initial Participants who had teaching experience but no professional development, and then by Advancing Participants who had teaching experience and 2 years of professional development. Emerging Teacher Leaders

Table 1
Participant Groups

Participant group	Description
Prospective Teachers (<i>n</i> = 36)	Undergraduates enrolled in a first mathematics-for-teachers content course
Experienced practicing teachers	
Initial Participants (<i>n</i> = 31)	Experienced K–3 teachers who were about to begin sustained professional development focused on children’s mathematical thinking
Advancing Participants (<i>n</i> = 31)	Experienced K–3 teachers engaged with sustained professional development focused on children’s mathematical thinking for 2 years
Emerging Teacher Leaders (<i>n</i> = 33)	Experienced K–3 teachers engaged with sustained professional development focused on children’s mathematical thinking for at least 4 years and beginning to engage in formal or informal leadership activities to support other teachers

Note. All practicing teachers had at least 4 years of teaching experience (with a range of 4–33 years), and the number of years of teaching experience in each group of practicing teachers averaged 14–16 years.

had the most experience with children’s thinking because they not only had teaching experience coupled with 4 or more years of professional development but also engagement in at least a few leadership activities to support other teachers. Examples of these formal and informal leadership activities include mentoring other teachers by visiting their classrooms, sharing mathematics problems with their grade-level teams, and presenting at faculty meetings or conferences.

Note that the average number of years of teaching in the three groups of practicing teachers was similar (14–16 years). We did not choose to group teachers by the number of years they had been teaching because, although we recognized that many important aspects of teaching improve with experience, we hypothesized that teachers needed more than teaching experience alone to learn to teach mathematics by building on children’s mathematical thinking in ways suggested in reform documents. Our larger study is specifically designed to explore this hypothesis by comparing the knowledge, beliefs, and practices of teachers at three points during sustained professional development. The prospective teachers were included as an anchor point for this trajectory in which additional experience with children’s mathematical thinking is hypothesized to be connected with enhanced expertise. In this article, we focus on the participants’ professional noticing of children’s mathematical thinking, both to investigate the hypothetical developmental trajectory related to this expertise and to characterize the range of expertise in each of the three component skills of professional noticing.

Practicing teachers were drawn from three districts in Southern California that were similar in demographics, with one third to one half of the students in these

districts classified as Hispanic, about one fourth classified as English Language Learners, and one fourth to one half receiving free or reduced-cost lunch. Prospective teachers were undergraduates, generally in their first 2 years of study, in a nearby comprehensive urban university, and they had just begun their first mathematics content course for teachers.

Professional Development

The Initial Participants, Advancing Participants, and Emerging Teacher Leaders were all volunteer participants in sustained professional development focused on children's mathematical thinking (Lamb, Philipp, Jacobs, & Schappelle, 2009), although the Initial Participants had yet to begin.¹ The professional development occurred prior to the study and was almost always facilitated by the same experienced mathematics program specialist. It included about 5 full days of workshops per year (in either half- or full-day increments) and drew heavily from the research and professional development project Cognitively Guided Instruction (CGI) (Carpenter et al., 1999; Carpenter et al., 2003). CGI is based on the idea that instruction can be improved by providing teachers access to research-based knowledge about children's thinking and by helping them to explore instruction that builds on children's thinking. CGI has documented gains in student achievement and teacher learning (Carpenter et al., 1989; Fennema et al., 1996), including in urban classrooms (Jacobs et al., 2007; Villaseñor & Kepner, 1993).

The overarching goals of the professional development were to help teachers learn how children think about and develop understandings in particular mathematical domains and how teachers can elicit and respond to children's ideas in ways that support those understandings. Children's mathematical thinking served as the focus for interactions, and conversations were informed by research on children's thinking, but the goal was not to provide teachers with a completed framework that summarized children's reasoning. Instead, teachers worked together to create their own frameworks that made explicit the similarities and differences in children's strategies. Teachers were given opportunities to recognize the power of attending to the subtle details in individual children's strategies—details that reflected mathematically relevant differences in the understandings children bring to their problem solving. Teachers also engaged in conversations about how mathematical tasks, classroom interactions, and classroom norms could be used to support and extend children's understandings within particular mathematical domains.

¹We initially worried that, because of participant dropout, those teachers who persisted 2 years as Advancing Participants or 4-or-more years as Emerging Teacher Leaders were different from the teachers who chose to leave the professional development early. If so, the Initial Participants (who had yet to begin professional development) might differ in important ways from the Advancing Participants and Emerging Teacher Leaders for reasons other than the number of years of professional development. We examined the enrollment records and concluded that dropout was not an issue in our study because fewer than 10% of the Advancing Participants or Emerging Teacher Leaders dropped out by choice, for programmatic reasons. Furthermore, even though some individuals were forced to discontinue their participation for reasons outside their control (e.g., funding), they overwhelmingly chose to re-enroll when the opportunity became available.

During professional development sessions, teachers engaged by solving mathematics problems, reading research, and analyzing video and written student work derived from their own classrooms as well as from artifacts provided by the facilitator. Between professional development sessions, teachers were asked to pose problems to their students and to bring the written student work to the next session. With the help of their colleagues, they then worked to make sense of their students' thinking in ways that highlighted core mathematical ideas and their developing frameworks of children's understandings related to those ideas.

Measures

In this article, we share a compilation of results from two assessments that we developed to capture participants' professional noticing of children's mathematical thinking. Each assessment was structured around an artifact of K–3 classroom practice focused on problem solving involving whole-number operations: a classroom video clip (Lunch Count) or a set of written student work (M&M's[®]). Participants were asked to watch the video clip or examine the set of written student work and then to respond, in writing, to prompts about attending, interpreting, and deciding how to respond.

Artifacts of Practice

The Lunch Count and M&M's artifacts provided the core of the two assessments.

Lunch Count video clip. The participants watched an edited 9-minute video clip of a 40-minute lesson taught in February in a combination class with Grades 1 and 2. This lesson was selected because of its complexity, including on-task and off-task behaviors, extensive teacher questioning, and sharing of various strategies reflecting a range of understandings. Additionally, a child (rather than the teacher) posed the problem for all children to solve. This lesson included many characteristics of instruction recommended in reform documents.

In the video clip, a child posed the problem “We have 19 children, and 7 are hot lunch. How many are cold lunch?”² The teacher then allowed the children to work on the problem individually or in pairs, solving the problem in any way that made sense to them. Various tools were available. While the children were solving the problem, the classroom was noisy, and when the children came together to share their solutions, the teacher reminded them to put their tools aside and listen to their classmates. Three correct strategies were shared. The first pair of children (Katie and Sam) wrote $19 - 7 = \square$ and shared that they had counted back 7 from 19 on their fingers. The second pair of children (Annette and Maureen) drew 19 individual tally marks (not grouped in 5s) and erased 7 to find the number of cold lunches.

²*Hot lunch* was the classroom terminology used to refer to students who were buying their lunches, and *cold lunch* referred to students who had brought their lunches that day.

The third strategy was shared by Sunny, a boy who used a counting frame (10 rods with 10 beads on each rod). He first built 19 lunches by isolating 2 beads on 9 rods and 1 bead on 1 rod. Next, he removed 7 (hot lunches) by removing 1 bead from each of 7 rods so that what remained was 2 rods with 2 beads and 8 rods with 1 bead. Finally, he counted the remaining beads to find the number of cold lunches; he subitized 4 (the 2 rods with 2 beads) before counting-on by 1s to reach the correct answer of 12. During the sharing, the teacher questioned the children about their reasoning and corrected some off-task behavior.

M&M's written student work. The participants were provided three samples of written student work in response to the problem "Todd has 6 bags of M&M's. Each bag has 43 M&M's. How many M&M's does Todd have?" (See the Appendix for the three samples.) The set of written student work came from a second-grade class and was selected because the strategies reflected a range of base-ten understandings. Alexis's strategy provided the least evidence of base-ten understanding in that she drew six groups of 43 tally marks (grouped by 5s), but exactly how she counted the tallies (by 1s, 5s, 10s, or 40s) to arrive at the correct answer of 258 is unclear. Cassandra's strategy provided the most evidence of base-ten understanding in that she decomposed numbers in several ways to combine quantities. After writing six 43s, she worked with the 43s in pairs, and by combining the tens and then the ones, she arrived at three 86s. She then combined two (of the three) 86s by combining the tens to get 160 and the ones to get 12 for a total of 172. She then took 20 from the 70 (in 172) to add to the 80 (in the third 86) to make 100. After adding this 100 to the 100 (in 172) to get 200, she added the 52 (remaining from $72 - 20$) to arrive at her answer of 252, but she made a minor error, forgetting to add the 6 from the third 86. Josie's strategy was between the other two strategies in terms of providing evidence of base-ten understanding. She first represented six groups of 43 by decomposing each 43 into the numeral 40 and 3 tally marks, and although her counting strategy is not completely clear, she appears to have skip-counted by 40s to arrive at 240. She then counted-on from there (but whether she counted-on by 1s or 3s is difficult to tell) to correctly arrive at 258.

Writing Prompts and Coding Schemes

For each artifact, participants were asked to write in response to three prompts related to the three component skills of professional noticing of children's mathematical thinking. We coded the responses on scales indicating the extent to which we had evidence for participants' engagement with children's mathematical thinking.

Attending prompt. To assess participants' expertise in attending to children's strategies, we requested, "Please describe in detail what you think each child did in response to this problem." The specific names of the children whose strategies were shown in the video clip or in the written work were listed after the prompt to

ensure that participants commented on each strategy. Coding responses was a three-step process. First, for each of the six strategies, we identified the mathematically significant details. For example, how children counted was considered mathematically significant, whereas whether children shared their strategy standing at the board or sitting on the floor was not. Second, for each of the six strategies, we determined whether the response demonstrated attention to most of these mathematical details or only a few. Third, for each artifact, we aggregated the three strategy codes to identify whether we had evidence for each participant's attention to children's strategies: evidence (1) or lack of evidence (0). Participants who provided most details for at least two of three strategies were considered to have provided evidence of attention to children's strategies for that artifact. Note that we did not require most details for all three strategies because demonstrating expertise does not require that individuals always recall and understand everything; even teachers who have acquired expertise in attending to children's strategies can lose focus and miss a strategy or not fully understand a particular aspect of a strategy.

Interpreting prompt. To assess participants' expertise in interpreting children's understandings, we requested, "Please explain what you learned about these children's understandings." We coded responses on a 3-point scale that reflected the extent of the evidence we had of participants' interpretation of children's understandings: robust evidence (2), limited evidence (1), or lack of evidence (0). Prior to the study, we determined our focus on interpreting children's understandings, but the number of categories and their characterizations emerged from the data.

Deciding-how-to-respond prompt. To assess participants' expertise in deciding how to respond on the basis of children's understandings, we asked, "Pretend that you are the teacher of these children. What problem or problems might you pose next?" A part of the response space was labeled "Problem(s)" and another part was labeled "Rationale" to ensure that participants provided both a next problem and their reasoning.³ We coded responses on a 3-point scale that reflected the extent of the evidence we had of participants' deciding how to respond on the basis of children's understandings: robust evidence (2), limited evidence (1), or lack of evidence (0). Similar to our coding of interpreting data, our coding of these data reflected our prestudy focus on children's understandings, but the number of categories and their characterizations emerged from the data.

³ We recognize that selecting a next problem is only one of the many ways that a teacher can respond to a child. Other types of responses include probing existing strategies, facilitating comparison of strategies, purposefully pairing children to share ideas, and so on. In this study, we chose to focus on participants' reasoning when selecting a next problem as a way to extend children's understandings after a correct answer was given (Jacobs & Ambrose, 2008). This focus seemed appropriate because the six strategies presented were all valid (even though Cassandra's answer was incorrect, given her minor error of forgetting the last 6). See Jacobs, Lamb, Philipp, and Schappelle (in press) for a discussion of similar results when participants' noticing was focused on a situation in which a child needed support to solve a problem correctly.

Participants watched the Lunch Count video clip before being provided with the prompts, but they were given the M&M's written student work and the prompts together and were able to refer to this written work while writing their responses. In the Lunch Count assessment, we chose to delay the presentation of the prompts and allow the participants to view the video clip only once because we wanted the video to serve as a proxy for actual instructional situations in which children often share their thinking verbally and a rewind button does not exist. Similarly, in the M&M's assessment, we allowed participants to view the written student work while writing their responses because in many instructional situations, teachers have access to written work while they are trying to make sense of and respond to children's thinking.

Analyses

We began our analyses by coding the three professional-noticing skills of attending, interpreting, and deciding how to respond for each of the two assessments. Because the three skills each had unique prompts in the assessments, we first examined the written response linked to the relevant prompt and then reviewed the responses on the entire assessment to see whether related information was included elsewhere. Data from all four participant groups were mixed and blinded so that group membership was hidden during coding. All data were double-coded by the first two authors, and interrater reliability for each set of coding was 80% or more. Discrepancies were resolved through discussion.

In this study, we combined the results from the two assessments to achieve a more stable measure of professional noticing of children's mathematical thinking—one that reflects participants' noticing of children's thinking when interacting with both their verbal- and written-strategy explanations. By focusing on these two common and important teaching activities, we addressed participants' abilities to articulate some of the types of noticing that teachers typically do in classrooms on a daily basis. Specifically, we constructed an overall score for each of the three component skills for each participant, and in creating each score, we wanted to capture what a participant *could* do—the highest level of expertise demonstrated by that participant. Thus, we used the participant's score that showed the greater engagement with children's thinking on the two assessments as his or her overall score for that skill. This decision stemmed, in part, from our desire to minimize underrepresenting participants' expertise, given the limitations of written assessments without follow-up questioning.⁴

Means were then calculated for these overall scores for each participant group, and we used our cross-sectional design to capture the development of these professional-noticing skills. Group differences were tested with four planned comparisons: a monotonic trend reflecting increased experience with children's

⁴We felt comfortable combining the two assessments because, for each assessment, the patterns of results across participant groups generally mirrored those for the overall scores. (We describe the overall-scores patterns in the following section.)

mathematical thinking and three pairwise comparisons of adjacent groups (Prospective Teachers vs. Initial Participants, Initial Participants vs. Advancing Participants, and Advancing Participants vs. Emerging Teacher Leaders). One-tailed tests were conducted because we hypothesized that more experience with children’s mathematical thinking should bring gains in professional-noticing expertise. The Type I error rate of .05 was split among the four comparisons using Holm’s procedure.

FINDINGS AND DISCUSSION

We examined the professional noticing of children’s mathematical thinking across our four participant groups. We begin with an overview of our findings about the differences between groups, and then we examine each component skill individually to characterize in more detail the various levels of expertise.

Overview of Participant-Group Differences

One of our major goals in this study was to identify group differences among the four participant groups to capture the development of professional-noticing expertise. Thus, means were calculated for each participant group for the overall scores of each component skill, with higher numbers indicating more evidence for engagement with children’s mathematical thinking (see Table 2). Monotonic trends for all three component skills were significant, indicating that increased experience with children’s thinking was related to increased engagement with children’s thinking on the professional-noticing tasks.

To better understand the development of professional-noticing expertise, we also looked at the pairwise comparisons of adjacent groups for the three component skills. For attending to children’s strategies, the pairwise comparisons of Prospective

Table 2
Participant Group Means (Standard Deviations) for the Overall Scores of the Component Skills

Component skill	Scale	Prospective Teachers	Initial Participants	Advancing Participants	Emerging Teacher Leaders
Attending to children’s strategies	0–1	0.42 (0.50)	0.65 (0.49)	0.90 (0.30)	0.97 (0.17)
Interpreting children’s understandings	0–2	0.47 (0.51)	0.94 (0.63)	1.19 (0.54)	1.76 (0.44)
Deciding how to respond on the basis of children’s understandings	0–2	0.14 (0.35)	0.29 (0.53)	0.84 (0.73)	1.45 (0.79)

Teachers versus Initial Participants and Initial Participants versus Advancing Participants were significant with effect sizes of 0.58 and 0.66, respectively. These findings provided evidence that expertise in attending to children's strategies grew with teaching experience and continued to grow with 2 years of professional development. The pairwise comparison between Advancing Participants and Emerging Teacher Leaders was not significant and therefore did not provide evidence for additional gains with more years of professional development and opportunities to engage in leadership activities, perhaps because performance was already at a high (almost ceiling) level.

For interpreting children's understandings, all three pairwise comparisons were significant with effect sizes ranging from 0.49 to 1.06. Thus, expertise in interpreting children's understandings grew with teaching experience and 2 years of professional development. Furthermore, unlike expertise in attending to children's strategies, expertise in interpreting children's understandings continued to grow significantly when teachers had engaged in 4 or more years of professional development and leadership activities.

For deciding how to respond on the basis of children's understandings, the pairwise comparisons of Initial Participants versus Advancing Participants and Advancing Participants versus Emerging Teacher Leaders were significant, with effect sizes of 0.88 and 0.99, respectively. Because the comparison of Prospective Teachers versus Initial Participants was not significant, we found no evidence that expertise in deciding how to respond on the basis of children's understandings resulted from teaching experience alone. We did find evidence, however, that expertise grew with 2 years of professional development and again when teachers had engaged in 4 or more years of professional development and leadership activities.

In summary, we have begun to construct a picture of the development of professional-noticing expertise.⁵ Teaching experience seems to provide support for individuals to begin developing expertise in attending to children's strategies and interpreting children's understandings, but we did not find similar evidence for expertise in deciding how to respond on the basis of children's understandings. In contrast, professional development seems to provide support for developing expertise in all three component skills. Furthermore, when professional development is sustained beyond 2 years and coupled with leadership activities, teachers continue to gain in their abilities to interpret children's understandings and to use those understandings in deciding how to respond. In the next three sections, we further characterize professional-noticing expertise by sharing a range of sample responses for each of the component skills.

⁵ We recognize the limitations of written assessments in that individuals have different proclivities (and abilities) to articulate their ideas in writing, but we had no reason to believe that writing expertise differed systematically across our four participant groups. Thus, we argue that the patterns we identified were not simply a reflection of writing ability.

Attending to Children's Strategies

Professional noticing of children's mathematical thinking requires the ability to attend to the mathematically important details of children's strategies—details that could inform a teacher's instruction. In the sections that follow, we share sample responses that provided evidence of attention to children's strategies and responses that did not.

Evidence of Attention to Children's Strategies

When responses provided evidence of attention to children's strategies, the descriptions included mathematically significant details such as how children counted, used tools or drawings to represent quantities, or decomposed numbers to make them easier to manipulate. For example, consider the following description of Cassandra's strategy in the M&M's written student work (see the Appendix for Cassandra's strategy):⁶

I think that Cassandra made 6 circles with the number 43 in each one. Then she combined every 2 circles by adding the 10s together and then adding the 1s together for each pair. Next, she added the 10s ($80 + 80$) and the 1s ($6 + 6$) for the first 4 circles. After adding $160 + 12$ to equal 172, she needed to add 86. Knowing that $80 + 20 = 100$ (a familiar #), she took 20 from the 70 to get to 100. Then she figured she needed to add the 52 left from the 172. What she forgot about was the 6 left from the 86. That's why her answer is off by 6.

This response captured the mathematical essence of the strategy. Specifically, the participant articulated the decomposition of numbers into place values to combine the pairs of 43s and the two 86s; the decomposition of 172 to create a familiar number of 100; and the omission of the final 6 (from the third 86), which led to the incorrect answer. Responses demonstrating evidence of attention to children's strategies were phrased in many ways, but they all tracked the entire strategy with substantial detail about the mathematically important aspects of that strategy.

Lack of Evidence of Attention to Children's Strategies

When responses did not provide evidence of attention to the details of children's strategies, comments tended toward general features of the strategies, such as identifying a tool or mentioning that the problem was solved successfully, but omitted details of how the problem was solved. For example, consider this vague description of how Cassandra solved the problem: "Wrote 43 down 6 times, then added them together in groups of 2. Then added those answers together to come up with her final answer." This description is general, missing all references to place value and decomposition, and leaving open the question of whether the participant

⁶To provide a sense of the data in both assessments and for ease of comparison among scores within a component skill, we have chosen to provide examples from the M&M's assessment when discussing attending and deciding how to respond and examples from the Lunch Count assessment when discussing interpreting.

fully understood Cassandra's approach. At times, these lack-of-evidence responses also included information that was inconsistent with the written work provided or showed the participant's confusion about Cassandra's work, often implying that it was problematic because of its complexity:

Cassandra's work is very practical and simple too, but it's not understandable. Why did she subtract 20 and where did she get the 70 from? Her work was not very clean, and she probably lost herself with too many numbers and lots of adding.

Developmental Patterns

Almost all the participants in the professional development groups (90% of the Advancing Participants and 97% of the Emerging Teacher Leaders) provided evidence of attention to children's strategies. In contrast, only 65% of the Initial Participants and 42% of the Prospective Teachers did so. Thus, expertise in attending to children's strategies is neither something adults routinely know how to do nor is it expertise that teachers generally develop solely from many years of teaching. Given the high level of performance of the Advancing Participants and Emerging Teacher Leaders, we suspect that they may have been better able to recall details of the strategies because they had learned to chunk strategy details in meaningful ways, and these findings corroborate those in the expert/novice studies in that experts tend to chunk information in ways that significantly enhance later recall (Bransford et al., 2000).

A final note. We initially wondered whether the Prospective Teachers' and Initial Participants' performance was lower than that of the other groups simply because they did not understand the task of describing children's strategies. However, we concluded that misunderstanding the task was not a sufficient explanation for the differences among participant groups. Although the participant-group patterns for each of the six strategies generally mirrored those of the overall pattern, there was one exception. In the Lunch Count assessment, all groups described well the Pair 2 strategy (the most basic of the six strategies), with more than 80% of each group providing most details of the strategy. Given that all participants (not just those in professional development) could successfully describe strategy details for certain strategies, we concluded that all groups understood what was expected. Thus, the participant-group differences on the overall attending score were reflective of differing expertise in attending to children's strategies rather than of some groups' misunderstanding of the task.

Interpreting Children's Understandings

Professional noticing of children's mathematical thinking requires not only attention to children's strategies but also interpretation of the mathematical understandings reflected in those strategies. When identifying the extent of the evidence participants demonstrated in interpreting children's understandings, we were not seeking a single best interpretation but were instead interested in the extent to which

participants' reasoning was consistent with the details of the specific children's strategies and the research on children's mathematical development. In the sections that follow, we share sample responses for each level of the scale: robust evidence, limited evidence, and lack of evidence of interpretation of children's understandings.

Robust Evidence of Interpretation of Children's Understandings

We begin with a sample response showing robust evidence of interpretation of the understandings of children shown in the Lunch Count video:

The first pair understands the problem is a [subtraction problem] by writing a number sentence that showed $19 - 7 = \square$. They did not need to count out 19 and take away 7 to get 12. They simply used their fingers to count backwards from 19. They seem to have good number sense.

The second pair has a simpler strategy than the first because they have to count out 19 tallies and then take away 7. They still need to make the amount. They can't hold it in their head yet like the first pair. Also they did not group their tallies into 5's which [would] allow them to keep better track of their numbers.

The last boy has good number sense and understands different amounts. He was able to count by groups of 2's and switch to a group of 1 to make 19. He then took away 7 and counted what remained. He was able to start with 4 and count on by 1's which shows he has some understanding of amounts. He still needs to make 19 and so I think the 1st pair has the best number sense because they were able to start right at 19 and count down.

This participant interpreted the children's understandings in several ways. First, she made sense of the details of each strategy and noted how these details reflected what the children *did* understand. For example, when discussing Sunny's ("the last boy's") understandings about quantities, this participant recognized Sunny's ability to count by 2s, his ability to switch between counting by 2s and 1s, and his ability to subitize an amount of 4 and count on from that quantity. These comments all point to mathematically relevant details that reflect Sunny's understandings. Second, the participant also recognized what strategies and understandings the children did not demonstrate. For example, when discussing Pair 2's understandings, this participant recognized that they did not group their tallies into 5s, which would have been a more efficient strategy, perhaps less prone to error. Finally, the participant compared the strategies by recognizing that the ability to mentally abstract a quantity was a required understanding only for Pair 1's counting-back strategy, which meant that this strategy reflected better "number sense" than the strategies of Pair 2 and Sunny. Responses demonstrating robust evidence of interpretation of children's understandings focused on making sense of strategy details in a variety of ways, but these interpretations were all consistent with the strategy presented and the research on children's mathematical development.

Limited Evidence of Interpretation of Children's Understandings

The middle level of the scale included responses in which participants maintained a focus on interpreting children's understandings but with less depth than responses that demonstrated robust evidence. The following is a sample response that provided limited evidence of interpretation of children's understandings:

The first set that shared had computational and representational understanding of the problem. They knew what the algorithm would be to solve the word problem.

The second set was very one sided. They were at the one-to-one correspondence picture stage and could have easily miscounted or made a computational mistake.

The third boy seemed to have very good number sense. He was able to group beads, skip count, and explain his thinking very clearly. I would say that he would be able to solve much more complex problems.

This participant described the children's understandings, but often in broad terms that were sometimes undefined (e.g., Pair 1's having "computational and representational understanding" and Pair 2's being at the "one-to-one correspondence picture stage"). Specific connections to the children's strategies existed, but they were more limited than in responses with robust evidence, and conclusions were sometimes overgeneralized, going beyond the evidence provided. For example, another participant with a limited-evidence response wrote, "These children understand subtraction and addition—and which to choose when presented with a problem. . . . They know how to write a number sentence." These broad conclusions about addition and subtraction are difficult to justify on the basis of the children's performance on a single problem in which they all used a separating action. Furthermore, only Pair 1 wrote a number sentence, but this participant seemed to imply that Pair 1's understandings were necessarily shared by the other children. Thus generality, sometimes coupled with overgeneralization, characterized the limited-evidence responses, but unlike responses described in the next section, these limited-evidence responses were still focused on interpreting the children's understandings.

Lack of Evidence of Interpretation of Children's Understandings

Some of the responses did not provide any evidence of interpretation of children's understandings, even though participants had been explicitly prompted to do so ("Please explain what you learned about these children's understandings"). These responses had alternative foci, such as something learned about mathematics teaching and learning in general, as in the following response:

I learned that it's important to allow students to use different tools to come up with mathematical problem solutions. Of course with this, it's vital to provide lessons on how to use several different tools. Only after that, can students decide what's easiest for them, and in turn choose tools which best work for the individual. I also learned that a math lesson can be so much more than just math. This teacher invited the students to a lesson in communication, listening, and respect in addition to subtraction (no pun intended).

Other responses with these alternative foci included a positive evaluation of the teaching in the video (e.g., “. . . I was glad that the teacher allowed her students to use multiple ways of arriving to the correct answer”) or suggestions for improving that teaching (e.g., “. . . I would have liked to hear the word difference and would have liked to have seen a way of checking or proving answers were correct”). Finally, some responses included commentary on the children but not on their understandings (e.g., “I noticed all eager to try . . .”).

A lack of focus on individual children was another characteristic of these responses. Fewer than half (43%) of the participants with an overall interpreting score of lack of evidence differentiated their comments about the various children who shared their work. Thus, the majority of these participants shared nothing—not even something unrelated to the children’s understandings, such as behaviors or affect—that provided evidence that they had noted anything about the individual children on either assessment. This result is in contrast to participants whose overall interpreting scores demonstrated limited or robust understanding: 66% of participants with an overall score of limited evidence and 100% of participants with an overall score of robust evidence differentiated their discussions to explicitly address individual children on at least one of the assessments. This differential approach to our request for what was learned about the children’s understandings—commenting on individual children versus discussing children only as a group—is also reflected in the sample responses shared earlier. This distinction is critical because when participants view groups of children only as a group, identifying the understandings reflected in specific strategies becomes challenging, if not impossible.

Developmental Patterns

About half of the Prospective Teachers and three fourths of the Initial Participants provided analyses with some evidence of interpretation of children’s understandings, although no Prospective Teachers and only about one sixth of the Initial Participants provided robust evidence. In contrast, every Emerging Teacher Leader and all but two Advancing Participants focused on interpreting children’s understandings in their responses (see Table 3 for a comparison of participant groups on

Table 3
Percentage Within Each Participant Group Demonstrating Each Level of Evidence of Interpreting Children’s Understandings

	Prospective Teachers	Initial Participants	Advancing Participants	Emerging Teacher Leaders
Robust evidence	0%	16%	26%	76%
Limited evidence	47%	61%	68%	24%
Lack of evidence	53%	23%	7%	0%

their overall interpreting scores). Thus, like expertise in attending, expertise in interpreting children's understandings is neither expertise that adults routinely possess nor something that teachers generally develop solely from years of teaching. Furthermore, providing robust evidence is particularly challenging, and this expertise takes years to develop; almost three times the percentage of Emerging Teacher Leaders compared to Advancing Participants generated responses that demonstrated robust evidence. Thus, these results underscore the importance of professional development that extends beyond 2 years.

Deciding How to Respond on the Basis of Children's Understandings

Professional noticing of children's mathematical thinking requires not only attending to children's strategies and interpreting the understandings reflected in those strategies but also expertise in using those understandings in deciding how to respond. Although teachers' responses could be of many types, we chose to focus on the reasoning involved in selecting the next problem. When identifying the extent of evidence that participants demonstrated in deciding how to respond on the basis of children's understandings, we were not seeking a particular next problem or rationale but were instead interested in the extent to which participants based their decisions on what they had learned about the children's understandings from the specific situation and how consistent their reasoning was with the research on children's mathematical development. In the sections that follow, we share sample responses for each level of the scale: robust evidence, limited evidence, and lack of evidence of deciding how to respond on the basis of children's understandings.

Robust Evidence of Deciding How to Respond on the Basis of Children's Understandings

Consider the following sample response showing robust evidence of deciding how to respond on the basis of children's understandings in relation to the M&M's written student work:

Problems and Rationale

For Alexis, I would use more round numbers to see if she could use a more efficient strategy, like 6 bags of 50.

For Cassandra, I would try similar numbers again to see if she could perform her calculations without error. Maybe 8 bags of 48—8 would leave even numbers of bags to add up.

For Josie, I'd go with a larger number of M&M's and more bags. Maybe she would try a different strategy. 13 bags of 77. I'd be interested to see if she'd use the same strategy and if so how would she break the numbers up?

This participant customized her suggestions for each child, explicitly considering the child's existing strategy and, in some cases, anticipating a possible next strategy.

Each suggestion reflects not only expertise in interpreting a child's understandings as reflected in the strategy used on the M&M's problem but also, based on these understandings, knowledge about next steps that research on children's mathematical development has shown are likely to further this child's understandings. For example, on the M&M's problem, Alexis showed evidence of grouping her representation of 43s into 5s, but she also made all 258 tallies. Providing an opportunity to add groups of 50 (instead of 43) might encourage Alexis to move away from representing all 300 M&M's. Fifty is a familiar number that children often know in combination (e.g., $50 + 50 = 100$), and the inclusion of a decade number can facilitate use of mental strategies or strategies that do not involve counting by 1s (or 5s). For each child, this participant suggested not that she would force or even show a particular strategy but instead that she would strategically provide a next problem to invite use of a more sophisticated strategy as a reasonable next step for that child. Note that we were not evaluating whether the suggested moves were the best moves (if that assessment is even possible). Instead, when responses demonstrated robust evidence, we tracked the participant's consideration of the children's understandings reflected in the strategies already used and how those understandings could be building blocks when proposing a new problem.

Limited Evidence of Deciding How to Respond on the Basis of Children's Understandings

As in the scale for interpreting, the middle level of this scale included responses in which participants used children's understandings in their reasoning but in a more general way. The following is a sample response that provided limited evidence of deciding how to respond on the basis of children's understandings:

Problem: I think I would give them the same problem using a 100 number, such as 6 bags of 154 M&M's.

Rationale: All of these kids know how to break numbers into 10's plus 1's. I think they are ready to go to the next step and look at 100's, 10's, plus 1's.

This participant shared a problem with a rationale that accounted for the children's past performance ("these kids know how to break numbers into 10's plus 1's") and anticipated next strategies ("they are ready to go to the next step and look at 100's, 10's, plus 1's") but did so in a general, somewhat vague, fashion. Furthermore, all three children were assumed to have similar understandings and to need a similar next step, even though their strategies on the M&M's problem showed mathematically important distinctions, such as their different ways of breaking numbers into 10s plus 1s. Nonetheless, despite the minimal specificity and lack of customization in the reasoning, this participant clearly considered these children's strategies and understandings in deciding how to respond, a characteristic missing from the responses described in the next section.

Lack of Evidence of Deciding How to Respond on the Basis of Children's Understandings

Some responses provided no evidence of deciding how to respond on the basis of children's understandings; these responses included little or no reference to building on the children's understandings or anticipating future strategies for the proposed problem. Sometimes these responses did identify the operation used or the children's success in solving the given problem but not their thinking on that problem. In fact, the proposed next steps often seemed as if they could have been generated without the participants' having seen the children's strategies, and reasoning other than the children's existing understandings were offered to justify them.

For example, some lack-of-evidence responses identified other multiplication problems ("The zoo field trip requires 4 buses. If each bus can hold 33 students, how many students can go?") so that the students could practice ("I would definitely try a similar problem but with different wording. This is until I knew that the students have become pretty familiar with those problems."), but without reference to existing understandings or anticipated strategies. Others focused on problem difficulty, again without any specific link to these children's existing understandings. Instead, they included problems that would generally be considered more difficult regardless of which children were solving them ("I would continue with the same type but give more difficult numbers such as 110 in each group . . ."). Finally, responses such as the following identified problems that introduced a new mathematical topic (often a related operation) but did not specify how future work related to this new focus could link to the children's existing understandings:

Problem: Johnny has 56 blocks. He can put 7 blocks into each toy chest. How many toy chests does Johnny have?

Rationale: This is obviously a lesson on an intro to division and multiplication. The students are learning how to divide quantities and how to add up these quantities. Therefore, since the students just learned how to add up certain amounts they should learn to divide the amounts next.

Lack of discussion about number selection was another characteristic of these responses in which participants used reasoning other than children's understandings to determine their next steps. Only 38% of participants with an overall deciding-how-to-respond score of lack of evidence included any discussion of number selection in relation to their proposed problem(s) on either assessment. Thus, the majority of these participants included no reasoning about number selection (e.g., specific numbers or classes of numbers such as larger numbers or decade numbers). This result is in contrast to participants whose overall deciding-how-to-respond scores demonstrated limited or robust evidence: 85% of participants with an overall score of limited evidence and 96% of participants with an overall score of robust evidence discussed the reasoning underlying their number selection for at least one of the assessments. We argue that attention to the details of number selection,

similar to attention to the details in children's strategies, reflects an orientation to teaching mathematics that is critical for providing tailored instruction that builds on children's existing understandings.

Developmental Patterns

Note that about two thirds of the Advancing Participants and more than four fifths of the Emerging Teacher Leaders provided some evidence of using children's understandings, whereas only about one fourth or fewer of the Initial Participants and Prospective Teachers did so (see Table 4 for a comparison of participant groups on their overall deciding-how-to-respond scores). Furthermore, although almost two thirds of the Emerging Teacher Leaders exhibited robust evidence, fewer than one fifth of participants in each of the other groups did so. Thus, these results again provide evidence that professional development, especially professional development focused on children's mathematical thinking that extends beyond 2 years, can help teachers increase their engagement with children's mathematical thinking.

Table 4

Percentage Within Each Participant Group Demonstrating Each Level of Evidence of Deciding How to Respond on the Basis of Children's Understandings

	Prospective Teachers	Initial Participants	Advancing Participants	Emerging Teacher Leaders
Robust evidence	0%	3%	19%	64%
Limited evidence	14%	23%	45%	18%
Lack of evidence	86%	74%	36%	18%

CONCLUSIONS

Evidence from our cross-sectional study indicates that the construct of professional noticing of children's mathematical thinking merits attention from teachers, professional developers, and researchers working toward the vision of successful classrooms put forth in national reform documents (Kilpatrick et al., 2001; NCTM, 2000). On the one hand, the Prospective Teachers' minimal engagement with children's thinking in all three component skills showed that professional noticing of children's mathematical thinking is challenging and not something that adults routinely know how to do. On the other hand, the accomplished performance of the Emerging Teacher Leaders and the consistently significant monotonic trend (capturing increasing experience with children's mathematical thinking) showed that this expertise can be learned and that both teaching experience and professional development support this endeavor.

We suggest that our study provides three types of resources for educators working toward this vision: (a) In our conceptualization of professional noticing of children's mathematical thinking, we identify three specific skills—attending, interpreting, and deciding how to respond—worthy of consideration; (b) our cross-sectional data

provide a nuanced story of the development of this expertise; and (c) our assessments and results can serve as useful tools for professional developers.

Conceptualization of Professional Noticing of Children's Mathematical Thinking

Noticing is a promising construct that contributes to efforts to make explicit the work of teaching, and researchers in mathematics education are just beginning to mine this construct to explore how teachers process complex instructional situations.⁷ Thus, in characterizing a range of noticing expertise, we contribute to the growing research base on how prospective and practicing teachers see and make sense of classrooms in different ways and how particular types of experiences can support the development of their abilities to notice in particular ways (see, e.g., Santagata et al., 2007; Sherin & van Es, 2009; van Es & Sherin, 2008).

We chose to focus on a specialized type of noticing—professional noticing of children's mathematical thinking—and a particular slice of teaching—the hidden practice of in-the-moment decision making when teachers must respond to children's verbal- or written-strategy explanations. In these situations, if instruction is to build on children's thinking, teachers must be able to attend to children's strategies, interpret their understandings, and use these understandings in deciding how to respond. Furthermore, they must execute these three skills in an integrated way, almost simultaneously, while they are making these in-the-moment decisions.

We believe that the ability to effectively integrate these three component skills is a necessary, but not sufficient, condition for responding on the basis of children's understandings, a core tenet of the vision of instruction promoted in reform documents (Kilpatrick et al., 2001; NCTM, 2000). In other words, attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings collectively provide a foundation for responding on the basis of children's understandings. We recognize that effective integration of these three skills—professional noticing of children's mathematical thinking—precedes a response and thus does not necessarily translate into effective execution of the response, because execution requires yet another set of complex skills. However, we argue that teachers are unlikely to base their responses on children's understandings without purposeful intention to do so, and it is this purposeful intention that we have tried to capture with our construct of professional noticing. In the next section, we summarize what we have learned about the development of this expertise.

Developmental Trajectories of Professional-Noticing Expertise

Our cross-sectional design enabled us to capture each participant group's patterns of engagement with children's mathematical thinking on all three component skills

⁷A forthcoming book, *Mathematics Teacher Noticing: Seeing Through Teachers' Eyes* (Sherin, Jacobs, & Philipp, in press), will present a compilation of the noticing work in mathematics education.

of professional noticing. Although variability existed within each participant group, we found the consistent patterns across groups to be convincing and worthy of attention. Through these patterns, we can begin to paint a developmental trajectory of professional-noticing expertise, highlighting which component skills developed with teaching experience and which seemed to require the support of professional development.

Prospective Teachers struggled with all the component skills, and for each skill, fewer than half demonstrated evidence of engaging with children's thinking. In contrast, more than half of the Initial Participants showed evidence of attending to children's strategies and interpreting their understandings, but fewer than one fifth provided robust evidence of interpreting children's understandings. Thus, teaching experience alone seemed to provide support for at least the initial development of expertise in attending to children's strategies and interpreting children's understandings. It did not, however, provide much support for development of the expertise needed in deciding how to respond on the basis of children's understandings; only about one fourth of the Initial Participants demonstrated any evidence of deciding how to respond on the basis of children's understandings, and only one participant demonstrated robust evidence.

For teachers who had engaged in professional development, the group patterns changed substantially, in that engagement with children's thinking in attending, interpreting, and deciding how to respond was the norm. Nonetheless, the extent of professional development was important, and potential for growth remained for both groups. Specifically, almost all the Advancing Participants and Emerging Teacher Leaders demonstrated evidence of attending to children's strategies. Furthermore, although almost all these teachers also provided some evidence of interpreting children's understandings, about three fourths of the Emerging Teacher Leaders provided robust evidence, whereas only about one fourth of the Advancing Participants did so. Thus, all gains from professional development in attending to children's strategies seemed to come within the first 2 years, but additional gains were found in interpreting children's understandings when professional development continued through 4 or more years and included opportunities to engage in leadership activities.

Emerging Teacher Leaders also showed superior expertise in deciding how to respond on the basis of children's understandings. Although a large majority of both professional development groups demonstrated some evidence of deciding how to respond on the basis of children's understandings, about two thirds of the Emerging Teacher Leaders offered robust evidence, whereas fewer than one fifth of the Advancing Participants did so. Because this deciding-how-to-respond expertise is essential for achieving instruction consistent with the reform vision, this study provides strong evidence of the need for professional development that is sustained over not only months but many years.

Teaching is a learning profession (Darling-Hammond & Sykes, 1999), and, as such, teachers need opportunities to learn, with support, throughout their teaching careers. Unfortunately, professional development has typically been short term and

fragmented (Hawley & Valli, 1999; Hill, 2004; Sowder, 2007; Sparks & Hirsh, 1997); however, our study context provided a unique opportunity for investigation of the effects of professional development that was sustained over 4 or more years. We found that sustained engagement was valuable, and in the next section, we provide specific suggestions to support the development of professional-noticing expertise in professional development.

Supporting Professional Noticing of Children's Mathematical Thinking in Professional Development

Our conceptualization of professional noticing of children's mathematical thinking and the findings from our cross-sectional study provide several resources for professional developers who want to support the development of professional-noticing expertise for prospective and practicing teachers. We highlight two of these resources: discussion prompts and growth indicators.

Discussion Prompts

Our specific prompts to assess participants' expertise in attending, interpreting, and deciding how to respond could be useful discussion prompts during professional development. Not only would these discussions provide the facilitator with valuable information about participants' perspectives but participants would also have targeted opportunities to explore these important instructional skills. In the sections that follow, we offer some additional considerations for discussions focused on each component skill.

Attending to children's strategies. We suspect that the skill of attending to children's strategies is the skill that is most likely to be overlooked. Professional developers may assume that all adults possess this skill and that everyone sees the same details in children's strategies. We found otherwise, in that a substantial number of Prospective Teachers and Initial Participants struggled to provide evidence of attending to children's strategies. In a related study, we found that even when asked to watch a video that depicted only one child engaged in problem solving in an interview setting, some adults still struggled to recall the mathematically important details of the child's strategy (Jacobs, Lamb, Philipp, & Schappelle, in press). Thus, the challenge associated with attending to children's strategies is not simply that teachers need to pay attention to many things in a busy classroom with numerous distractions. Given that some individuals still struggled to attend to a child's strategy in an interview setting, these challenges extend beyond a processing-capacity issue; attending to children's strategies requires not only the ability to focus on important features in a complex environment but also knowledge of what is mathematically significant and skill in finding those mathematically significant indicators in children's messy, and often incomplete, strategy explanations.

These results are consistent with other studies that have underscored the importance and challenge of description (Blythe et al., 1999; Rodgers, 2002), especially

for teachers whose work orients them to immediately consider their next moves (Sherin, 2001). Given the role of attending to children's strategies as a foundational skill for interpreting and deciding how to respond, professional developers should consider the development of expertise in attending to children's strategies to be a worthwhile goal that will require time and targeted support to learn.

Interpreting children's understandings. Although the majority of practicing teachers in our study provided some evidence of interpreting children's understandings, fewer than half of the Prospective Teachers did so. Furthermore, the Emerging Teacher Leaders were the only group to have a substantial number of participants who provided robust evidence of this expertise. We were intrigued with the range of issues addressed when responses did not focus on interpreting children's understandings, especially given that one prompt explicitly asked participants to do so: "Please explain what you learned about these children's understandings." We suspect that the task was too challenging for some; to interpret children's understandings, one must not only attend to children's strategies but also have sufficient understanding of the mathematical landscape to connect how those strategies reflect understanding of mathematical concepts. When participants—especially Prospective Teachers—did not focus on children's understandings, they instead discussed other issues such as the child's affect, what they learned more generally about mathematics teaching and learning (e.g., the importance of tool use or the idea that multiple ways exist to solve a problem), or their evaluation of the teacher's actions, sometimes suggesting improvements. We encourage facilitators to be watchful for the appearance of these alternative topics—topics that are important to discuss but that may also derail an explicit conversation about children's understandings. Avoiding discussion of children's understandings may be reflective of a lack of attention to children's strategies or a lack of mathematical knowledge for how to make sense of those strategies, and professional developers will need to address these challenges.

Deciding how to respond on the basis of children's understandings. In this study, we chose to focus on the reasoning behind one type of instructional response: selecting the next problem. This expertise requires not only expertise in attending to children's strategies and interpreting children's understandings but also knowledge about children's mathematical development to identify a reasonable next step and the ability to facilitate that next step by selecting a problem that will be accessible yet also challenge children's thinking. Thus, selecting a problem that builds on children's existing understandings is complex and worthy of discussion.

However, selecting the next problem is only one of many ways that teachers can build on children's understandings after they have solved a problem correctly. Other effective ways to respond include probing for the reasoning underlying children's strategies, asking children to compare strategies, encouraging the symbolic representation of mental or tool-based strategies, and so on. In other situations, unlike those reflected in the artifacts used in this study, children may struggle to even solve

a problem, and teachers can again choose from an array of responses including clarifying the problem, probing children's initial solution attempts, pairing children so that they can help each other, and so on. All these types of responding are part of the complex work of teaching (see Jacobs & Ambrose, 2008, for a more extended discussion of the range of potential responses that build on children's understandings).

We argue that the reasoning behind all these types of responses is similar in that the reasoning is more likely to be productive when it includes consideration of the children's existing understandings. Therefore, we suspect that many of the issues and developmental trajectories identified in this study, in which the focus was exclusively on selecting next problems, would also apply to other types of teachers' responses. We have, in fact, explored teachers' reasoning when deciding how to respond to a child who is struggling to solve a problem, and the patterns of results across participant groups were similar (Jacobs et al., in press). Nonetheless, professional developers (and researchers) need to address the professional noticing of children's mathematical thinking across the range of responding in which teachers engage.

Growth Indicators

Our coding schemes and cross-sectional findings provide growth indicators that can help professional developers identify and celebrate shifts in teachers' professional noticing of children's mathematical thinking. Specifically, we encourage attention to the following shifts:

- A shift from general strategy descriptions to descriptions that include the mathematically important details;
- A shift from general comments about teaching and learning to comments specifically addressing the children's understandings;
- A shift from overgeneralizing children's understandings to carefully linking interpretations to specific details of the situation;
- A shift from considering children only as a group to considering individual children, both in terms of their understandings and what follow-up problems will extend those understandings;
- A shift from reasoning about next steps in the abstract (e.g., considering what might come next in the curriculum) to reasoning that includes consideration of children's existing understandings and anticipation of their future strategies; and
- A shift from providing suggestions for next problems that are general (e.g., practice problems or harder problems) to specific problems with careful attention to number selection.

Note that some of these shifts may be minimal at first. Thus, professional developers need to be patient and initially expect limited, rather than robust, evidence of shifts. Expertise in professional noticing of children's mathematical thinking is

complex, and, as our cross-sectional results illustrate, may require years to develop. We conclude with a caution. In conceptualizing the construct of professional noticing of children's mathematical thinking, we envisioned the existence of a nested relationship among the three component skills such that deciding how to respond on the basis of children's understandings can occur only if teachers interpret children's understandings, and these interpretations can be made only if teachers attend to the details of children's strategies. Given this nested relationship, one could conclude that professional development should focus exclusively on attending before interpreting and interpreting before deciding how to respond. We worry that an approach that addresses these skills only independently and sequentially may seem too removed from teachers' everyday work. Instead, we argue that professional developers can focus on all three skills in integrated ways but be aware of the component skills and their growth indicators.

Next Steps

Through this study, we have become further convinced of the complexity of the expertise needed to teach in ways that are consistent with the reform vision. We focused specifically on the expertise underlying the hidden practice of in-the-moment decision making that is needed to respond to children's verbal- and written-strategy explanations. Our theoretical conceptualization of professional noticing of children's mathematical thinking proved useful for characterizing this expertise; providing snapshots of those with varied levels of expertise; and documenting that, given time, this expertise can be learned. We recognize that our results are tied to the particular type of professional development experienced, and future studies will need to confirm the generalizability of our findings to other professional development in which learning about children's mathematical thinking is central. We also recognize that the ultimate utility of this construct will depend on the ways in which future studies can connect teachers' professional noticing of children's mathematical thinking with the execution of their in-the-moment responses. Nonetheless, we hope that our study provides a starting point for researchers and professional developers seeking to identify teachers' existing perspectives and productive next steps for their learning in much the same way that frameworks for children's thinking have assisted teachers in identifying children's existing understandings and productive next steps.

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APPENDIX

M&M's Written Student Work

Alexis

Todd has 6 bags of M&Ms. Each bag has 43 M&Ms. How many M&Ms does Todd have?

The student's work consists of six hand-drawn bags of M&Ms. Each bag is filled with tally marks representing 43 M&Ms. The number 40 is written on each bag. At the bottom, a separate bag contains the number 258, which is the correct answer to the problem.

M&M's Written Student Work (continued)

Cassandra

Todd has 6 bags of M&Ms. Each bag has 43 M&Ms. How many M&Ms does Todd have?

It is going to be 252 because I took away 20 from the 70.

$20 + 80 = 100$

$200 + 52 = 252$

Josie

Todd has 6 bags of M&Ms. Each bag has 43 M&Ms. How many M&Ms does Todd have?

40	80	120	160	200	240
40 	40 	40 	40 	40 	40
243	246	249	252	255	258

258
answer