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DECIDING HOW TO RESPOND ON THE BASIS OF CHILDREN'S UNDERSTANDINGS¹

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The unraveling of the math lesson is a continuously reinvented process, with dozens of decision points at which the teacher moves on to the next activity format, which has only just emerged as a likely follow-on exercise, or switches to another exercise as a result of the drift of pupils' oral response, the level of pupils' task engagement, the time remaining until recess or the end of the period, or more likely, all these factors. This continuous readjustment results from what Lévi Strauss (1962) has called, felicitously, "engaging in a dialogue with the situation" as that situation unfolds. To tinker well here seems to depend on how quickly and accurately the teacher can read the situation.

Huberman (1993, pp. 15–16)

We appreciate Huberman's depiction of teaching as a fluid process requiring extensive and critical decision making on the basis of reading a situation in a specific moment (see also Franke, Kazemi, & Battey, 2007; Lampert, 2001; McDonald, 1992; Schoenfeld, 1998; Wells, 1999). Although the craft of teaching involves much more, we have chosen to focus on understanding this in-the-moment decision making both because of the centrality of this skill in effective teaching and because this expertise is so challenging to develop. In mathematics education, a particular type of in-the-moment instructional decision making has been emphasized—decision making in which children's thinking is central.

"Sizing up students' ideas and responding" has been identified as one of the core activities of teaching (Ball, Lubienski, & Mewborn, 2001, p. 453), and instruction that builds on children's mathematical thinking has been endorsed in many reform documents (National Council of Teachers of Mathematics [NCTM], 2000; National Research Council [NRC], 2001). This focus has been informed by the extensive and growing research base on children's mathematical

thinking and development (Lester, 2007; NRC, 2001), and instruction that builds on children's ways of thinking has been linked to rich instructional environments and documented gains in student achievement (Bobis et al., 2005; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Sowder, 2007; Wilson & Berne, 1999). In addition, focusing on the thinking of children can provide a constant source of professional development for teachers throughout their careers because they can continue to learn from their students' thinking on a daily basis, even after formal professional development support ends (Franke, Carpenter, Levi, & Fennema, 2001).

Despite these documented benefits for both students and teachers, creating instruction that builds on children's thinking has proven challenging. In this chapter, we use the construct of *noticing* to begin to unpack this practice and, in particular, the in-the-moment decision making that occurs, many times a day, when a child shares a verbal or written strategy explanation and the teacher needs to respond.

Noticing

For many years, psychologists have studied how individuals notice or attend to stimuli in their environments, and, more recently, researchers have been describing the distinct patterns of noticing particular to professions (see, e.g., Goodwin, 1994; Mason, 2002; Stevens & Hall, 1998). Those studying expert/novice differences have also acknowledged these professional patterns of noticing by confirming that experts in a field are more likely than novices to focus on and remember noteworthy aspects of complex situations that are relevant to future decision making (for a summary, see NRC, 2000). Mathematics educators have shown interest in the noticing construct as a way to understand how teachers make sense of complex classrooms in which attending and responding to everything is impossible, and they have defined *noticing* in a multitude of ways (as reflected in the chapters in this volume). Some have addressed solely where prospective and practicing teachers focus their attention (Star, Lynch, & Perova, this volume, Chapter 8; Star & Strickland, 2007), whereas others have also considered how teachers reason about what they see (Sherin 2007; Sherin & Han, 2004; van Es & Sherin, 2008), including their abilities to reflect on teaching strategies and consider alternatives (Santagata, this volume, chapter 10; Santagata, Zannoni, & Stigler, 2007).

This growing body of work on mathematics teacher noticing has underscored the idea that teachers see classrooms through different lenses and that understanding these lenses can be helpful in scaffolding teachers' abilities to notice in particular ways. We applaud these researchers' attention to the important role that noticing plays in teaching, and we build on their work by selecting a particular focus for noticing—children's mathematical thinking—and a particular slice of teaching—the hidden practice of in-the-moment decision making when teachers must respond to children's verbal or written strategy explanations. This type of

in-the-moment decision making is in contrast to the long-term decision making (or planning) that teachers do before or after school when children are not present. Specifically, we want to understand not only how teachers detect children's ideas that are embedded in comments, questions, notations, and actions but also how teachers make sense of what they observe in meaningful ways and use it in deciding how to respond. Thus, we are less interested in identifying the variety of what teachers notice and more interested in how and the extent to which teachers notice children's mathematical thinking. As such, we found merit in investigating a specialized type of mathematics teacher noticing that we call *professional noticing of children's mathematical thinking*. We conceptualize this expertise as a set of three interrelated skills: (a) *attending* to children's strategies, (b) *interpreting* children's understandings, and (c) *deciding how to respond* on the basis of children's understandings (Jacobs, Lamb, & Philipp, 2010).

In this chapter, we have chosen to focus on the third component skill, deciding how to respond. Note that this skill reflects intended responding, not the actual execution of the response. We recognize that intended responding is not necessarily executed as planned, but we argue that teachers are not likely to respond on the basis of children's understandings without purposeful intention to do so. We are not looking for teachers to propose any particular responses (that is, there is no checklist of desired moves) but are instead interested in whether their decision making draws on and is consistent with the specifics of children's thinking in a given situation and the research on children's mathematical development (see also Jacobs & Philipp, 2010).

Other researchers have also included issues related to responding in their conceptualizations of noticing (see, e.g., Erickson, this volume, chapter 2; Santagata, this volume, chapter 10; Santagata et al., 2007), but we recognize that many may view decision making about how to respond as something that occurs *after* noticing. Both perspectives have advantages, but we argue for its inclusion as part of noticing given that deciding how to respond is both temporally and conceptually linked to the other two component skills of professional noticing of children's mathematical thinking (attending to children's strategies and interpreting children's understandings) during teachers' in-the-moment decision making. First, when a child offers a verbal or written strategy explanation, implementation of the three component skills must occur almost simultaneously—as if constituting a single, integrated teaching move—before the teacher responds. Second, expertise in deciding how to respond is nested within expertise in attending to children's strategies and interpreting children's understandings. In other words, teachers can decide how to respond on the basis of children's understandings only if they also have attended to children's strategies and interpreted the understandings reflected in those strategies. Thus, these three component skills are inextricably intertwined. Finally, the work of teaching orients teachers to constantly consider their next moves (Schoenfeld, 1998; Sherin, 2001); thus, the skills of attending to children's strategies and interpreting children's understandings are not ends

in themselves but are instead starting points for making effective instructional responses. By integrating teachers' reasoning about how to respond into the construct of professional noticing of children's mathematical thinking, we ensure that this ultimate goal of purposeful responding remains visible.

In this chapter, we characterize the component skill of deciding how to respond by investigating the expertise of four groups of participants with different amounts of experience with children's mathematical thinking. We also explore the specific connection between participants' expertise in deciding how to respond and their expertise in attending to children's strategies. Others have underscored the symbiotic relationship between the focus of attention and subsequent decision making. For example, Erickson (this volume, chapter 2) has argued that the selective attention of teachers is opportunistic in that they judiciously direct their attention to what is necessary to take action. Similarly, Sassi (2001), drawing on Aristotle's notion of *practical judgment*, argued that "learning to deliberate about the actions one should take is inseparable from cultivating perception of the salient features of one's situation" (p. 15). Thus we provide evidence for not only the developmental patterns of these two skills but also their connection.

Methods

The data were drawn from a cross-sectional study entitled "Studying Teachers' Evolving Perspectives" (STEP), in which we collected data on the professional noticing of teachers engaged in sustained professional development focused on children's mathematical thinking.

Participants

The 131 participants included three groups of practicing K–3 teachers and one group of prospective teachers who were just beginning their studies to become elementary school teachers (see Table 7.1).

Participant groups differed in their experience with children's mathematical thinking. Specifically, Prospective Teachers, by virtue of their lack of teaching experience and professional development, had the least experience with children's thinking, followed by Initial Participants, who had teaching experience but no sustained professional development, and then by Advancing Participants, who had teaching experience and 2 years of professional development. Emerging Teacher Leaders had the most experience with children's thinking because they had not only teaching experience coupled with 4 or more years of professional development but also engagement in at least a few leadership activities to support other teachers. These formal or informal activities included mentoring other teachers by visiting their classrooms, sharing mathematics problems with their grade level teams, or presenting at faculty meetings or at conferences.

Practicing teachers were drawn from three Southern California districts that

TABLE 7.1 Participant groups

<i>Participant group</i>	<i>Description</i>
Prospective Teachers (<i>n</i> = 36)	Undergraduates enrolled in a first mathematics content course for teachers
Experienced practicing teachers	
Initial Participants (<i>n</i> = 31)	Experienced K–3 teachers who were about to begin sustained professional development focused on children’s mathematical thinking
Advancing Participants (<i>n</i> = 31)	Experienced K–3 teachers who had engaged with sustained professional development focused on children’s mathematical thinking for 2 years
Emerging Teacher Leaders (<i>n</i> = 33)	Experienced K–3 teachers who had engaged with sustained professional development focused on children’s mathematical thinking for at least 4 years and were beginning to engage in formal or informal leadership activities to support other teachers

Note: All practicing teachers had at least 4 years of teaching experience (with a range of 4–33 years), and the number of years of teaching experience in each group averaged 14–16 years.

were similar in demographics, with one-third to one-half of the students classified as Hispanic, about one-fourth classified as English language learners, and one-fourth to one-half receiving free or reduced-cost lunch. Prospective teachers were undergraduates, generally in their first 2 years of study, in a nearby comprehensive urban university, and they had just begun their first mathematics content course for teachers.

Professional Development

The professional development occurred prior to the study and was almost always facilitated by the same experienced mathematics-program specialist. It drew heavily from the research and professional development project Cognitively Guided Instruction [CGI] (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, & Levi, 2003), and the overarching goals were to help teachers learn (a) how children think about and develop understandings in particular mathematical domains and (b) how teachers can use this knowledge to inform their instruction. Participation was voluntary and consisted of about 5 full days of workshops per year (in either half- or full-day increments spread throughout the year). In workshops, teachers analyzed classroom artifacts (video and written student work), explored underlying mathematical concepts and children’s understandings of those concepts, and considered how those understandings could be used to inform instruction. Between meetings, teachers were asked to pose problems to their students and bring their student work to the next meeting for

discussion and reflection. (See Lamb, Philipp, Jacobs, & Schappelle, 2009, for more details about the professional development.)

Measures

We developed a written assessment to capture participants' professional-noticing expertise in terms of the component skills of deciding how to respond and attending. Specifically, participants were asked to watch a video of a one-on-one problem-solving interview between a teacher and a kindergartner (Rex), shown in two parts. After viewing each part, participants were asked to react, in writing, to a prompt. We allowed participants to view the video only once, because we wanted it to serve as a proxy for actual instructional situations in which children often share their thinking verbally and a rewind button does not exist.

Part I: Deciding How to Respond on the Basis of Children's Understandings

Participants watched Part I of the video (3 minutes), in which Rex solved two problems and was asked to solve a third problem. Unifix cubes and paper and pencil were available for Rex's use. Participants were provided background information that the video was filmed in June of Rex's kindergarten year. The three problems follow:

- Rex had 13 cookies. He ate 6 of them. How many cookies does Rex have left?
- Today is June 5 and your birthday is June 19. How many days away is your birthday?
- Rex had 15 tadpoles. He put 3 tadpoles in each jar. How many jars did Rex put tadpoles in?

On the first (cookie) problem, Rex used his fingers to count back 6 from 13 ("12, 11, 10, 9, 8, 7") to answer, "Seven." On the second (birthday) problem, Rex initially declared, "I can't figure that one out," so the teacher restated the problem and asked, "What do you think we could do to figure that out?" Rex offered, "Use our fingers or something," and then, after that minimal encouragement, was able to begin counting up from June 5th to June 19th on his fingers. When he reached June 15th and had all 10 fingers extended, he announced, "That's 10" before successfully counting up 4 more to June 19th. Next, counting on from 10, he recounted the four fingers for the dates after June 15th to answer, "Fourteen." Part I of the video concluded after the teacher had presented the third (tadpole) problem and Rex had commented, "I don't even know that one. That's hard."

To assess participants' expertise in deciding how to respond on the basis of children's understandings, we requested, "Describe some ways you might respond

to Rex, and explain why you chose those responses.” We coded responses on a 3-point scale that reflected the extent of the evidence we had of participants’ deciding how to respond on the basis of Rex’s understandings: robust evidence (2), limited evidence (1), or lack of evidence (0).

We purposefully selected Part I of the video because it included Rex’s solving a series of problems so that participants could draw on Rex’s previous performance when deciding how to respond to his struggles with the tadpole problem. For example, participants could not only learn that Rex successfully solved a subtraction and a missing-addend problem but also see his range of counting strategies, emerging understanding of tens, and comfort level with using fingers as a tool during problem solving. Furthermore, because of Rex’s successful use of counting strategies on the first two problems, participants might reasonably assume that he should be able to solve the measurement-division (tadpole) problem. Research has shown that measurement-division problems are not substantially more difficult for young children than the other two problems when solved by direct modeling (i.e., a basic, yet powerful, strategy in which children represent all the quantities and the action or relationship described in the problem) (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993). Thus, given that Rex solved the first two problems with counting strategies, which are more sophisticated than direct-modeling strategies,² one might reasonably assume that the tadpole problem was accessible because Rex could always return to a direct-modeling strategy.

Part II: Attending to Children’s Strategies

After sharing how they would support Rex on the tadpole problem, participants watched Rex solve the problem in Part II of the video (1.5 minutes), which began exactly where Part I ended. Following the teacher’s repetition of the problem, Rex began linking cubes into groups of 3 until he had five groups. For the first three groups, he counted by 3s (“3, 6, 9”), and then he used his fingers to count up by 1s for the last two groups (“10, 11, 12” and “13, 14, 15”). Next Rex answered “Four,” but immediately self-corrected to “Five,” and then recounted his 15 cubes by again counting by 3s for the first three groups and by 1s for the last two groups. When the teacher asked how many jars were needed, Rex answered “Fifteen,” but again immediately indicated that he knew his answer was wrong. In response, the teacher acknowledged that there were 15 tadpoles and asked again how many jars were needed. Rex hastily answered “Four,” but, when the teacher then asked how many jars he had on the table, Rex looked at his groups of cubes and confidently answered “Five.”

To assess participants’ expertise in attending to children’s strategies, we requested “Please describe in detail what Rex said and did in response to this tadpole problem.” We coded responses on a 2-point scale that reflected whether we had evidence for participants’ attending to Rex’s strategy: evidence (1) or lack of evidence (0).

We purposefully selected Part II of the video because it included a variety of mathematically important details that could inform a teacher's instruction. First, Rex used a direct-modeling strategy in which he represented all the tadpoles in groups of 3 cubes, with each group signifying a jar (Carpenter et al., 1999). Second, Rex's strategy included two types of counting (by 3s and by 1s), which is critical information for teachers of young children. How children count, the ability to switch between two types of counting, and the ability to recognize a group of cubes as a single entity are important indicators of emerging mathematical understandings. Finally, when asked for the number of jars, Rex showed some confusion with units when he answered 15 (the number of tadpoles) instead of 5 (the number of jars). Distinguishing these units is an important mathematical goal for children learning to make sense of this type of division problem.

Analyses

We began our analyses by coding the two professional-noticing skills of deciding how to respond and attending. We double-coded all data (in a blinded format), and interrater reliability was 80% or more. All discrepancies were resolved through discussion. We then used our cross-sectional design to capture the development of these professional-noticing skills. Means were calculated for each participant group, and group differences were tested with four planned comparisons: a monotonic trend reflecting increased experience with children's mathematical thinking and three pairwise comparisons of adjacent groups (Prospective Teachers vs. Initial Participants, Initial Participants vs. Advancing Participants, and Advancing Participants vs. Emerging Teacher Leaders). One-tailed tests were conducted, because we hypothesized that more experience with children's mathematical thinking should bring gains in professional-noticing expertise. The Type I error rate of 0.05 was split among the four comparisons using the Holm's procedure.

Findings

Using responses to the Rex video, we characterized the two skills of deciding how to respond on the basis of children's understandings and attending to children's strategies, and then, for each, we considered developmental patterns across our four participant groups.

Deciding How to Respond on the Basis of Children's Understandings

We reviewed participants' reasoning about how to respond to Rex after he had shared that the tadpole problem was hard and he did not know how to solve it. We noted (a) whether the participants' reasoning explicitly referenced Rex's thinking on the first two problems and (b) whether the proposed interaction left

space for Rex's future thinking (not just the teacher's thinking). In both cases, we looked to see that the ideas were consistent with the research on children's mathematical development. Below we share sample responses for each level of the scale: robust evidence, limited evidence, and lack of evidence of deciding how to respond on the basis of Rex's understandings.

Robust Evidence of Deciding How to Respond on the Basis of Rex's Understandings

We begin by sharing an example of a response that reflected robust evidence of deciding how to respond on the basis of Rex's understandings:

Rex really prefers to use his fingers as a tool to solve problems. In the first problem he used them to count down from 13, keeping track of when he'd counted down 6 times. In the second problem he counted on from June 5th to June 19th, but was thrown—ever so slightly—when his counting on continued beyond his 10 fingers.

Considering this, I think the third problem caused some difficulty because he couldn't represent 15 tadpoles with his fingers. Also, since his other strategies involved counting on and counting back he might think he could use that here.

Okay—the original question, what to do from here: I'd start by asking him why that problem was hard. Is it because of the language and context of tadpoles? Is it because he can't use a counting on or back strategy? Does he recognize that his *previous* counting strategies won't work?

Where I'd go from there would really depend on his response: I'm going to assume that he understands what the problem is asking.

I might adjust the numbers to (16, 2) to see if he'd skip-count by 2s up to 16 and keep track on his fingers.

If Rex explained that it was hard to use his fingers for this one, I might ask if there's another tool that would help him.

In coding this response as robust evidence, we were not evaluating whether the suggested moves were the best moves (given that we do not believe that best moves even exist). Instead, we tracked this participant's extensive consideration of Rex's understandings on the previous problems and her awareness of the importance of his future thinking in solving the tadpole problem. Specifically, in the first half of the response, we learn that this participant attended carefully to how Rex solved the first two problems, including his facility and preference in using fingers to count up and down. She then used her observation that Rex was thrown "ever so slightly" when the numbers went beyond 10 in the second problem to hypothesize why Rex might be struggling with the tadpole problem ("he couldn't represent 15 tadpoles with his fingers"). Note that her reasoning

is not generic reasoning about this mathematics problem but instead is particular to how she thinks Rex might engage with this problem on the basis of what she learned from his strategies on the previous two problems.

In the second half of the response, the participant returned to the original question about what she should do next and chose to explore the issue of problem difficulty with Rex (“asking him why that problem was hard”), leaving space for Rex’s thinking while considering connections to his past work (“Is it because he can’t use a counting on or back strategy? Does he recognize that his *previous* counting strategies won’t work?”). Next, the participant explicitly stated that her responses “would really depend on [Rex’s] response,” indicating that Rex’s thinking would play a strong role in the proposed interaction. She continued by proposing supporting moves she might consider, all of which were consistent with what the video showed about Rex’s understandings and what the research shows about children’s mathematical development. Specifically, she considered whether Rex understood the problem and whether she could adjust the problem so that he could use a strategy similar to one he had used earlier. When children use a counting strategy to solve measurement–division problems, they often skip-count (Carpenter et al., 1999), and this participant chose numbers (16 tadpoles with 2 in each jar) to make the skip-counting easier (2s instead of 3s) while still enabling the use of a familiar tool (i.e., Rex could use each finger to represent two tadpoles and thus count by 2s to 16 without having to count beyond his two hands). With her final suggestion, the participant acknowledged that fingers might be a challenging tool for solving the original problem and other tools might be useful.

Limited Evidence of Deciding How to Respond on the Basis of Rex’s Understandings

Some responses were similar to robust-evidence responses in that they maintained a focus on drawing on Rex’s understandings on the past problems and providing a space for Rex’s future thinking, but they did so with less depth. Consider how the following response offers limited evidence of deciding how to respond on the basis of Rex’s understandings:

I would encourage him to try because of how successful he was with the other two questions. Then I would show him tools/manipulatives to use (connecting cubes, paper, pencil or chalkboard, yarn loops, etc.). I think he was just intimidated because it wasn’t a counting question that he seems so familiar with. With tools, I believe he could at least get through the problem with or without assistance. He has a good sense of number and [is] able to count backwards from at least 13 to 6, so he seems ready for this type of problem.

This participant used Rex’s understandings but in a more general way than they were used in the previous example. Specifically, she referred to Rex’s success and

counting strategies on the first two problems to conclude that Rex should be able to engage with the tadpole problem and that tools might be useful. However, this participant provided fewer details linking the proposed interaction to Rex's past and future thinking. Note that the length of the response was not the determining factor for robust or limited evidence; instead we focused on the depth of the use of Rex's understandings.

Lack of Evidence of Deciding How to Respond on the Basis of Rex's Understandings

Other responses provided no evidence of deciding how to respond on the basis of Rex's understandings and instead focused on either general comments or comments dominated by the teachers' (instead of Rex's) thinking.

Focus on General Comments

Some responses included few specifics in terms of the instructional next steps or the underlying reasoning. For example, they consisted of a single, general suggestion (e.g., offering tools) with little rationale, or mention of the importance of questioning without any articulation of specific questions or even types of questions to be posed ("... I would ask questions along the way as a guide to get him started. I think questioning is a way to guide students in the process of how to start and where to go next"). Other responses focused on broad curriculum issues ("... This question might actually be too hard for a kindergartner. I am not sure what the average kindergartner learns in a math class, but I think it's pretty basic...") or on nurturing Rex's affect without any reference to his past or future mathematical understandings ("It is hard but let's try—teachers love it when you try! I would always try to keep the child's self-esteem high. I wouldn't want him to feel like he wasn't smart"). In summary, lack of specificity with respect to Rex's mathematical understandings and the teacher's role in nurturing those understandings characterized these responses as being focused on general comments.

Focus on Teachers' Thinking

Some responses were focused on the teachers' thinking instead of Rex's past or future thinking. In these responses, reaching a correct answer was generally emphasized and details of the proposed strategies and teacher's instructional moves were provided. However, the suggested moves were typically focused on guiding Rex through the solving of the tadpole problem, with little concern for how (or even if) he was making sense of the mathematics or how these experiences would link with his work on the first two problems. In fact, it was almost as if participants could have generated these exact responses without having seen Rex's work on the first two problems. For example, one participant suggested:

I would help him draw a picture and guide him through the problem. I would ask him to draw 15 dots or lines to represent the 15 tadpoles. Then I would tell him that there will be 3 in each jar, so to represent each jar he could circle tadpoles in groups of 3. I would then ask him how many circles he has.

Another method I would guide him through would be to use the cubes that were on the table. I would ask him to count out 15 cubes, and then make them into sticks of 3 (stick them together). I would then ask him to count how many sticks he has.

Both suggestions describe specific and effective strategies for solving the tadpole problem, and these strategies are ones that children are likely to use. However, in this case, the strategies are the *teacher's* strategies, and whether any consideration has been (or would be) given to Rex's understandings of these strategies is unclear.

Attending to Children's Strategies

Because of the foundational role that attending to children's strategies plays in deciding how to respond, we also examined whether participants provided evidence of attending to Rex's strategy on the tadpole problem.

Responses that provided evidence of the participants' attending to Rex's strategy included most of the mathematically significant details of the strategy: (a) grouping of the cubes into five sets of 3, (b) counting by 3s to 9 and then by 1s to 15, and (c) demonstrating confusion about the answer (i.e., offering 4, 5, and 15 as the answer at different times). For example, a participant offered:

After the teacher reread the problem Rex started to grab unifix cubes in groups of 3. He confidently went 3, 6, 9. He then stopped to state 9 tadpoles, that's 3 (groups). He then had to use fingers to count up another group of 3—10, 11, 12. "That's 4." He did it one more time—13, 14, 15. He then stated that's 15. When the teacher prompted how many groups, he at first said 4. When she asked how many groups he had made, he recounted and then said 5.

Note that not every detail is included, but this participant showed that she attended to the mathematical essence of the strategy.

In contrast, the following response demonstrated lack of evidence of attending to Rex's tadpole strategy.

Rex said that the problem was too hard although he attempted it. He then used visual blocks to set aside 15 tadpoles. He used his counting to figure out 15 tadpoles among 5 jars. Rex then had to make sure that his process

was right. He finished the problem knowing that there were 5 jars for 15 tadpoles. Rex knew how to solve the problem; he just needed the necessary help and motivation.

This participant mentioned that Rex solved the problem correctly and used blocks, but information about how Rex used the blocks, how Rex counted, and how Rex determined the answer was missing. Thus, unlike the case in the previous example, this response provided insufficient information for one to reconstruct Rex's solution. Strategy descriptions demonstrating lack of evidence of attending to Rex's strategy often included mention of the success in solving the problem but omitted details about *how* Rex solved the problem. The absence of these details is problematic, because strategy details provide a window into a child's understandings and should form the basis for teachers' decisions about how to respond.

Developmental Patterns

Using our cross-sectional design, we captured the developmental patterns of expertise in deciding how to respond and attending. Means were calculated for each participant group for the scores on each component skill, with higher numbers indicating more evidence for engagement with children's mathematical thinking (see Table 7.2). In both cases, we found a statistically significant monotonic trend, indicating that increased experience with children's thinking was related to increased engagement with children's thinking on the professional-noticing tasks.

In examining the three pairwise comparisons of adjacent groups for deciding how to respond, we found no significant differences between Prospective Teachers and Initial Participants, but we did find significant differences between Initial Participants and Advancing Participants and between Advancing Participants and Emerging Teacher Leaders, with effect sizes of 0.68 and 0.77, respectively. Thus we found no evidence that expertise in deciding how to respond on the basis of children's understandings resulted from teaching experience alone. Instead,

TABLE 7.2 Participant-group means (standard deviations) for the two component skills

<i>Component skill</i>	<i>Scale</i>	<i>Prospective Teachers</i>	<i>Initial Participants</i>	<i>Advancing Participants</i>	<i>Emerging Teacher Leaders</i>
Deciding how to respond on the basis of children's understandings	0–2	0 (0)	0.19 (0.40)	0.61 (0.80)	1.09 (0.88)
Attending to children's strategies	0–1	0.19 (0.40)	0.35 (0.49)	0.77 (0.43)	0.88 (0.33)

expertise in this skill grew with 2 years of professional development and again when teachers had engaged in 4 or more years of professional development and leadership activities.

We also examined the three pairwise comparisons of adjacent groups in attending, and we found only one significant difference—between Initial Participants and Advancing Participants, with a large effect size of 1.02. This finding is evidence that expertise in attending to children’s strategies grew after 2 years of professional development. Unlike with deciding how to respond, however, there was no significant difference between the Advancing Participants and Emerging Teacher Leaders, perhaps because both were already at a high level of performance, with more than three-fourths of each group providing evidence of attending to Rex’s tadpole strategy.

Discussion

Building on research that connected teachers’ use of children’s mathematical thinking with rich instructional environments, gains in student achievement, and teacher learning, we conceptualized the construct of professional noticing of children’s mathematical thinking to begin to unpack the in-the-moment decision making that occurs when a child shares a verbal or written strategy explanation and the teacher needs to respond. This conceptualization contributes to efforts to make explicit the work of teaching, and our main focus in this chapter has been to explore one of the components skills, deciding how to respond on the basis of children’s understandings.

From our cross-sectional data, we learned that developing expertise in deciding how to respond is challenging but can be achieved with engagement in professional development that is sustained over many years. We recognize that decision making in relation to Rex captures only one type of responding that teachers do, but the results shared in this chapter are consistent with patterns found with the same participants (Jacobs et al., 2010) when their deciding-how-to-respond expertise was assessed in relation to classroom video and written student work. These artifacts were different from the Rex video not only in the form of the instructional setting but also in that they depicted situations in which children generally solved problems correctly. Thus the participants’ decision making was focused on extending the children’s understandings rather than supporting their efforts to solve a problem correctly. Given the similarities between the developmental patterns in these supporting and extending situations, we suggest that the extent of teachers’ focus on children’s understandings may permeate teachers’ decision making across the range of responding in which teachers engage. We reiterate that, in contrast to the typical, short-term model of professional development (Hawley & Valli, 1999; Hill, 2004), long-term support is needed for the development of this expertise in deciding how to respond on the basis of children’s understandings.

Our secondary focus in this chapter was to explore the connection between participants' expertise in deciding how to respond and their expertise in attending to children's strategies. We found that 20 participants (across the 4 participant groups) provided responses demonstrating robust evidence of deciding how to respond on the basis of Rex's understandings, and 19 of those 20 also provided evidence of attending to Rex's strategy on the tadpole problem. The reverse, however, was not true. The 71 participants (across the 4 participant groups) who provided evidence of attending to Rex's tadpole strategy generated responses at all three levels of the deciding-how-to-respond scale (i.e., robust, limited, and lack of evidence of deciding how to respond on the basis of Rex's understandings). Thus, if teachers decide how to respond on the basis of children's understandings, they are likely to also attend to children's strategies. However, if teachers attend to children's strategies, they may or may not decide how to respond on the basis of the understandings reflected in those strategies. In short, expertise in attending to children's strategies is foundational to deciding how to respond on the basis of children's understandings, and our cross-sectional findings showed that neither form of expertise is something that adults routinely possess but is something they can gain with support. Participants' struggles to attend to children's strategies were particularly salient in this study, in which much of the complexity of classrooms was removed by use of a video that depicted only a single child engaged in problem solving. Therefore we argue that teachers need support in learning to attend to children's strategies, and they need additional support to learn how to use those details in deciding how to respond so that their instruction maintains children's thinking as central. We suggest that building on teachers' existing perspectives can be helpful in this endeavor.

Building Professional Development on Teachers' Existing Perspectives

Just as teachers need to first determine what children understand so that they can use that understanding as a starting point for instruction, we argue that professional developers can use an understanding of teachers' reasoning in deciding how to respond to inform their professional development. A note of caution is necessary. Although reasoning patterns existed for each participant group and can be useful as starting points, we found a range of perspectives in each group, and thus professional developers also need to look beyond group membership to consider individuals' perspectives.

When helping teachers to develop expertise in deciding how to respond on the basis of children's understandings, we encourage professional developers to recognize the positive attributes of all perspectives, including those currently demonstrating lack of evidence. In this way, responses focused on general comments and teachers' thinking can be viewed as resources rather than as ways of reasoning that need to be replaced.

Resources in Responses Focused on General Comments

Participants whose responses were focused on general comments lacked specificity about mathematics thinking and teachers' moves to support that thinking, but they also often indicated the need to promote confidence and positive feelings toward mathematics. These affective goals have been shown to be important by research connecting students' lack of confidence or dislike of mathematics with low achievement (Ma, 1999). Thus professional developers could view this concern with children's affect as a productive starting point for discussions about teaching and learning mathematics. Instead of trying to replace this concern, professional developers could work to augment it so that, in addition, children's understandings are considered when teachers decide how to respond. Our data support this additive goal in that 70% of participants who demonstrated robust evidence of deciding how to respond on the basis of Rex's understandings also made comments reflecting concern with Rex's affect.

Resources in Responses Focused on Teachers' Thinking

Participants whose responses were focused on teachers' thinking typically provided extensive details about strategies and teachers' instructional moves. Even though the specificity in these responses was related to teachers' strategies and instructional moves (rather than children's thinking), professional developers could use it as a starting point for helping teachers learn to attend to and use the specific details of children's strategies. Given that 30% of all responses (across participant groups) were focused on general comments, we know that specificity is not something that all participants demonstrated, and thus professional developers could build on this expertise.

Our cross-sectional results also revealed an interesting phenomenon related to this perspective in that almost half of the Advancing Participants—experienced practicing teachers who had completed 2 years of professional development—offered responses focused on teachers' thinking. Given that the professional development emphasized *children's* mathematical thinking, one might have expected otherwise, but we hypothesize that the Advancing Participants were in a transition period. Sustained professional development focused on children's mathematical thinking tends to fundamentally change the ways that teachers engage with children and mathematics; a shift to understanding, valuing, eliciting, and building on children's mathematical ideas is challenging and takes many years to develop (Fennema et al., 1996; Franke et al., 2001). During the first 2 years of professional development, the Advancing Participants were exposed to many new mathematical strategies, patterns of children's development in relation to these strategies, and the role of the teacher in carefully selecting tasks and posing follow-up questions to support children's construction of these strategies. We suspect that Advancing Participants may not yet have coordinated the knowledge, beliefs, and skills needed not only to believe that Rex could generate a strategy to

solve the tadpole problem on his own but also to determine their role in supporting Rex's thinking (vs. imposing their own thinking) during this problem solving. In contrast, after 4 or more years of sustained professional development and opportunities to engage in leadership activities, the transition seems to have been more consolidated in that fewer than one-fifth of the Emerging Teacher Leaders generated responses focused on teachers' thinking. This shift again points to the power of long-term professional development and the need to identify and build on the positive attributes in teachers' existing perspectives, in part because they may reflect skill development that is in transition.

Final Thoughts

We close by suggesting that this work on professional noticing of children's mathematical thinking may serve as a resource for professional developers beyond providing them with information about teachers' existing perspectives and expertise. Although the construct of noticing was not explicitly discussed in the professional development in this study, we wonder about the possible benefits of talking directly with teachers about professional noticing of children's mathematical thinking. Teachers who have engaged with our work have found our conceptualization of professional noticing of children's mathematical thinking, and in particular our characterization of teachers' reasoning in deciding how to respond, to be a useful self-reflection tool. By seeing themselves in each level of the scale, perhaps in different situations or at different times in their own development, they were able not only to see their own growth but also to consider paths for future growth. Thus an open question remains about the multitude of ways that the construct of professional noticing of children's mathematical thinking can be useful in supporting teachers' development.

Notes

- 1 An earlier version of this chapter was presented at the 2009 annual conference of the American Educational Research Association. This research was supported in part by a grant from the National Science Foundation (ESI0455785). The opinions expressed in this chapter do not necessarily reflect the position, policy, or endorsement of the supporting agency.
- 2 When using counting strategies, children do not need to represent all quantities (e.g., Rex did not need to represent all 13 cookies and instead started counting backward at 12, using his fingers to represent counts rather than cookies).

References

- Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433–456). Washington, DC: American Educational Research Association.

- Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, R., Young-Loveridge, J., et al. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large-scale cases. *Mathematics Education Research Journal*, 16(3), 27–57.
- Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education*, 24, 428–441.
- Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.
- Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Lof, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26, 499–531.
- Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.
- Erickson, F. (this volume, chapter 2). *On noticing teacher noticing*.
- Fennema, E., Carpenter, T., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). Mathematics instruction and teachers' beliefs: A longitudinal study of using children's thinking. *Journal for Research in Mathematics Education*, 27, 403–434.
- Franke, M. L., Carpenter, T. P., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal*, 38, 653–689.
- Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Charlotte, NC: Information Age.
- Goodwin, C. (1994). Professional vision. *American Anthropologist*, 96, 606–633.
- Hawley, W. D., & Valli, L. (1999). The essentials of effective professional development: A new consensus. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession*. San Francisco: Jossey-Bass.
- Hill, H. C. (2004). Professional development standards and practices in elementary school mathematics. *The Elementary School Journal*, 104, 215–231.
- Huberman, M. (1993). The model of the independent artisan in teachers' professional relations. In J. W. Little & M. W. McLaughlin (Eds.), *Teachers' work: Individuals, colleagues, and contexts* (pp. 11–50). New York: Teachers College Press.
- Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary school. *Journal for Research in Mathematics Education*, 38, 258–288.
- Jacobs, V. R., Lamb, L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41, 169–202.
- Jacobs, V. R., & Philipp, R. A. (2010). Supporting children's problem solving. *Teaching Children Mathematics*, 17(2), 98–105.
- Lamb, L. C., Philipp, R. A., Jacobs, V. R., & Schappelle, B. P. (2009). Developing teachers' stances of inquiry: Studying teachers' evolving perspectives. In D. Slavit, T. Holmlund Nelson, & A. Kennedy (Eds.), *Perspectives on supported collaborative teacher inquiry* (pp. 16–45). New York: Taylor & Francis.
- Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

- Lester, F. K. (2007). *Second handbook of research on mathematics teaching and learning*. Charlotte, NC: Information Age.
- Lévi-Strauss, C. (1962). *La pensée sauvage*. Paris: Plon.
- Ma, X. (1999). A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. *Journal for Research in Mathematics Education*, 20, 520–540.
- Mason, J. (2002). *Researching your own practice: The discipline of noticing*. London: RoutledgeFalmer.
- McDonald, J. P. (1992). *Teaching: Making sense of an uncertain craft*. New York: Teachers College Press.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Research Council (Eds.). (2000). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academy Press.
- National Research Council (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.
- Santagata, R. (this volume, chapter 10). *From teacher noticing to a framework for analyzing and improving classroom lessons*.
- Santagata, R., Zannoni, C., & Stigler, J. W. (2007). The role of lesson analysis in pre-service teacher education: An empirical investigation of teacher learning from a virtual video-based field experience. *Journal of Mathematics Teacher Education*, 10, 123–140.
- Sassi, A. (2001). Cultivating perception: Helping teachers attend to the salient features of their mathematics classrooms. Newton, MA: Center for the Development of Teaching, Education Development Center. Retrieved October 29, 2005, from http://www2.edc.org/cdt/cdt/cdt_researchpapers.html#paper15
- Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education*, 4(1), 1–94.
- Sherin, M. G. (2001). Developing a professional vision of classroom events. In T. Wood, B. S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 75–93). Hillsdale, NJ: Erlbaum.
- Sherin, M. G. (2007). The development of teachers' professional vision in video clubs. In R. Goldman, R. Pea, B. Barron, & S. J. Derry (Eds.), *Video research in the learning sciences* (pp. 383–395). Mahwah, NJ: Erlbaum.
- Sherin, M. G., & Han, S. Y. (2004). Teacher learning in the context of a video club. *Teaching and Teacher Education*, 20, 163–183.
- Sowder, J. (2007). The mathematics education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157–223). Charlotte, NC: Information Age.
- Star, J. R., Lynch, K., & Perova, N. (this volume, chapter 8). *Using video to improve preservice mathematics teachers' abilities to attend to classroom features: A replication study*.
- Star, J. R., & Strickland, S. K. (2007). Learning to observe: Using video to improve preservice teachers' ability to notice. *Journal of Mathematics Teacher Education*, 11, 107–125.
- Stevens, R., & Hall, R. (1998). Disciplined perception: Learning to see in technoscience. In M. Lampert & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 107–150). New York: Cambridge University Press.
- van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education*, 24, 244–276.

Wells, G. (1999). *Dialogic inquiry: Toward a sociocultural practice and theory of education*. New York: Cambridge University Press.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. In A. Iran-Nejad & P. D. Pearson (Eds.), *Review of research in education* (Vol. 24, pp. 173–209). Washington, DC: American Educational Research Association.