Multiresolution Analysis: from Meyer and Mallat's Wavelets to Diffusion Wavelets

Cristina Pereyra
Department of Mathematics and Statistics
University of New Mexico
Albuquerque, New Mexico, USA

In this talk we survey the crucial notion of multiresolution analysis (MRA) in wavelet theory. Meyer and Mallat's theorem says that given an orthogonal MRA we can find a function whose dilates and translates will create an orthonormal basis. Furthermore fast algorithms are available, and have become part of the toolbox of applied mathematicians, engineers, geophysicists, statisticians, etc.

Once the general framework is understood, variations on the same theme can be performed. In the setting of classical wavelets we can now understand orthogonal and biorthogonal (MRA), MRAs associated to multiscaling functions and generating multiwavelets, 2-dimensional MRAs (or N-dimensional). These first generation of wavelets have found uncountable applications, among the most famous in image compression, retrieval and denoising. We will describe a particular class of biorthogonal divergence-free vector-valued multiwavelets (developed in joint work with J. Lakey) as an example on how to obtain bases with special features (in this case: compact support, smoothness, incompressible, adapted to life on the unit cube). These bases where built with applications to fluid mechanics in mind, however so far have been used for statistical applications (joint work with S. Efromovich et al).

Time permitting we will move to a setting that includes digital data clouds, manifolds, graphs, etc. Here a multiresolution analysis is defined via a diffusion operator which replaces scaling in the classical setting (notice that scaling and translation are "rigid" operations associated to Euclidean spaces, which are not any more available when dealing with more geometric situations). Here we will report mostly on very recent work by R. Coifman and M. Maggioni. Examples include MRA on the sphere, "diffusion wavelets" on graphs (the graph could be anything: Erdos number, publications connected by increasingly large number of common themes). One can only imagine the implications these ideas can have for the google of the future!