Math 150 Exam 1

Spring 2006 **Name:** KEY

For credit on a problem, all work must be shown with a brief explanation of each step.

Exam Scores

1a	10pts	
1b	10pts	
2	20pts	
3	20pts	
4	20pts	
5	20pts	
Total		

1a. Evaluate $\lim_{x\to 2} \frac{x^4-16}{x-2}$

$= \lim_{x\to 2} \frac{(x^2-4)(x^2+4)}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)(x^2+4)}{\cancel{x-2}}$

$= \lim_{x\to 2} (x+2)(x^2+4) = \boxed{32}$

1b. Evaluate $\lim_{x\to 4} \frac{\left(\frac{1}{x^{1/2}} - \frac{1}{2}\right)}{x-4}$

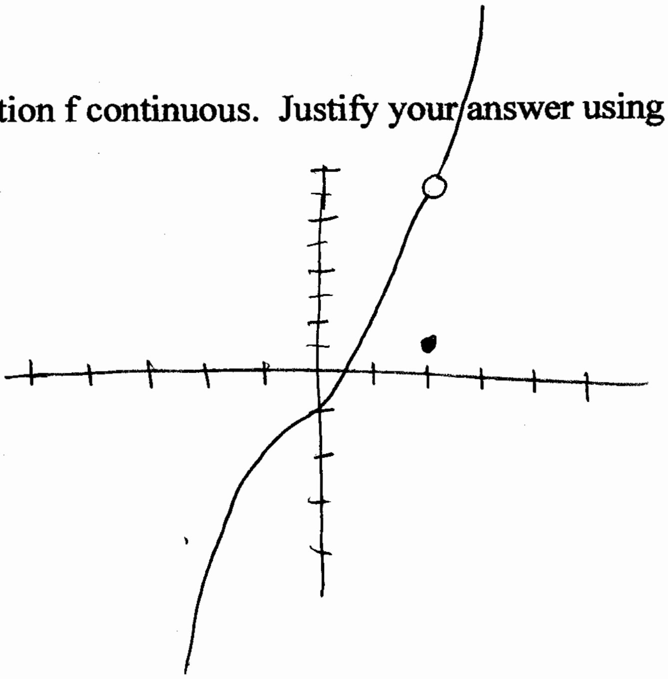
$= \lim_{x\to 4} \frac{\frac{2-x^{1/2}}{2x^{1/2}}}{\frac{x-4}{1}} = \lim_{x\to 4} \frac{2-x^{1/2}}{2x^{1/2}(x-4)} \cdot \frac{(2+x^{1/2})}{(2+x^{1/2})}$

$= \lim_{x\to 4} \frac{4-x}{2x^{1/2}(x-4)(2+x^{1/2})} = \lim_{x\to 4} \frac{-\cancel{(x-4)}}{2x^{1/2}\cancel{(x-4)}(2+x^{1/2})}$

$= \lim_{x\to 4} \frac{-1}{2x^{1/2}(2+x^{1/2})} = \boxed{\frac{-1}{16}}$

2. For what values of x is the function f continuous. Justify your answer using the various limit theorems.

$$f(x)=\begin{cases} x^2+3 & x>2 \\ x-1 & x=2 \\ x^3-1 & x<2 \end{cases}$$



f(x) is not continuous at $x=2$

since $\lim_{x\to2} f(x) = 7$, but $f(2) = 1$

f(x) is continuous for all other values since all polynomials are continuous.

3. Using the definition of the derivative, find $f'(x)$ for $f(x)=\sqrt{x}$.

Definition: $f'(x) = \lim_{h\to0} \dfrac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to0} \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to0}\frac{x+h-x}{(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to0}\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to0}\frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

4. Find an equation for the tangent line to the curve $y = \dfrac{3}{1-5x}$ at the point (-1, ½).

$$\text{slope} = f'(-1) = \lim_{x \to -1} \frac{\frac{3}{1-5x} - \frac{3}{1-5(-1)}}{x+1} = \lim_{x \to -1} \frac{\frac{3}{1-5x} - \frac{1}{2}}{x+1}$$

$$= \lim_{x \to -1} \frac{\frac{6-(1-5x)}{2(1-5x)}}{\frac{x+1}{1}} = \lim_{x \to -1} \frac{5+5x}{2(1-5x)(x+1)}$$

$$= \lim_{x \to -1} \frac{5\cancel{(1+x)}}{2(1-5x)\cancel{(x+1)}} = \frac{5}{2(1-5(-1))} = \frac{5}{12}$$

y-int: $y = \frac{5}{12}x + b$, $\frac{1}{2} = \frac{5}{12}(-1) + b \Rightarrow b = \frac{11}{12} \Rightarrow \boxed{y = \frac{5}{12}x + \frac{11}{12}}$

5. Find all the horizontal and vertical asymptotes for the curve $y = \dfrac{x^3+1}{x^3+x}$.

Sketch the graph.

$x = \pm i$ ← not vertical asymptotes

vertical: $x^3 + x = 0 \Rightarrow x(x^2+1) = 0 \Rightarrow \boxed{x = 0}$ ← vertical asymptote

Horizontal: $$\lim_{x\to\infty} \frac{x^3+1}{x^3+x} = \lim_{x \to \infty} \frac{\frac{x^3}{x^3} + \frac{1}{x^3}}{\frac{x^3}{x^3} + \frac{x}{x^3}} = \lim_{x\to\infty} \frac{1 + \frac{1}{x^3}}{1 + \frac{1}{x^2}} = 1$$

$$\lim_{x\to-\infty} \frac{\frac{x^3}{x^3} + \frac{1}{x^3}}{\frac{x^3}{x^3} + \frac{x}{x^3}} = \lim_{x \to -\infty} \frac{1 + \frac{1}{x^3}}{1 + \frac{1}{x^2}} = 1$$

same