Math 150 Exam 4

Spring 2006 **Name:** White Key

For credit on a problem, all work must be shown with a brief explanation of each step.

EXAM SCORES

1a	5pts	
1b	5pts	
1c	5pts	
1d	5pts	
1e	5pts	
1f	5pts	
2a	10pts	
2b	10pts	
3	25pts	
4	25pts	
	Total	

1a. Evaluate $\lim_{x \to 1} \frac{\ln(x)}{\cos\left(\frac{\pi x}{2}\right)}$. $\to \frac{0}{0}$

By L'Hospital's Rule

$$= \lim_{x \to 1} \frac{\frac{1}{x}}{-\frac{\pi}{2}\sin\left(\frac{\pi x}{2}\right)} = \frac{1}{-\frac{\pi}{2}} = \frac{-2}{\pi}$$

1b. Compute $\frac{d}{dx}\ln\left(\frac{\sqrt[3]{2x-1}}{\sqrt{5x^2+1}}\right)$

$$= \frac{d}{dx}\left[\frac{1}{3}\ln(2x-1) - \frac{1}{2}\ln(5x^2+1)\right]$$

$$= \frac{1}{3}\frac{2}{2x-1} - \frac{1}{2}\frac{10x}{5x^2+1}$$

1c. Compute $\frac{d}{dx}\left[\int_x^{\pi} \sec(t^2)dt\right]$ $= -\sec(x^2)$ (F. Thm)

1d. Evaluate $\int \frac{x}{\sqrt{1-x^4}}dx$ $= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\arcsin(x^2)$

Let $u = x^2$

$du = 2x\,dx$

1e. If $f'(x) = 3x^2 + 2x - 6$ and $f(0) = 8$ find f.

$$f(x) - f(0) = \int_0^x f'(x)\,dx = \int_0^x (3x^2 + 2x - 6)\,dx = x^3 + x^2 - 6x$$

$$f(x) = x^3 + x^2 - 6x + 8$$

1f. If $f''(x) = \cos(x)$ find **all** antiderivatives of f.

$$f'(x) = \sin(x) + C$$

$$f(x) = -\cos(x) + Cx + D$$

2a. Evaluate $\int_{-1}^{0} \frac{1}{(3+x)^4} dx = \int_2^3 u^{-4}\,du = \left.\frac{u^{-3}}{-3}\right|_2^3 = \frac{-1}{3}\left[\frac{1}{27} - \frac{1}{8}\right]$

Let $u = 3 + x$

$du = dx$

$$= \frac{1}{3}\left[\frac{1}{8} - \frac{1}{27}\right]$$

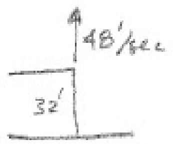
2b. Evaluate $\int_1^e \frac{(\ln(t))^2}{t} dt = \int_0^1 u^2\,du = \left.\frac{u^3}{3}\right|_0^1 = \frac{1}{3}$

Let $u = \ln(t)$

$du = \frac{dt}{t}$

3. A stone is thrown straight up from the edge of a roof, 32 feet above the ground, at a speed of 48 ft per second. If the acceleration of gravity is -32 feet per second squared,

i) At what time does the stone hit the ground?
ii) What is the velocity of the stone when it hits the ground?
iii) At what time does the stone reach its maximum height?
iv) What is the maximum height?



$s(0) = 32 \qquad v(t) - v(0) = \int_0^t \frac{dv}{dt}\,dt = \int_0^t -32\,dt = -32t$

$v(0) = 48$

$a(t) = -32 \qquad v(t) = -32t + 48$

$s(t) - s(0) = \int_0^t \frac{ds}{dt}\,dt = \int_0^t (-32t + 48)\,dt = -16t^2 + 48t$

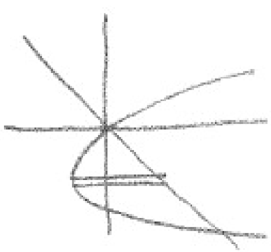
$s(t) = -16t^2 + 48t + 32 = -16(t^2 - 3t + 2)$

(i) $t_g = \frac{3+\sqrt{17}}{2}$ sec since $s(t_g) = 0$

(ii) $v(t_g) = -32\left(\frac{3+\sqrt{17}}{2}\right) + 48 = -16\cdot\sqrt{17}$ ft/sec

(iii) $t_{max} = \frac{48}{32} = 3/2$ sec (iv) $s(t_{max}) = -16(9/4 - 9/2 - 2) = 68$ ft

4. Find the area between the curves $x + y = 0$ and $x = y^2 + 3y$.
Provide a sketch of the region.



$x = -y \quad x = y^2 + 3y$

$\Rightarrow -y = y^2 + 3y$ or $0 = y^2 + 4y$

$\Rightarrow y = 0$ or $y = -4$

$$\int_{-4}^{0} (-y - (y^2 + 3y))\,dy = \int_{-4}^{0} -y^2 - 4y\,dy$$

$$= \frac{-y^3}{3} - \frac{4y^2}{2}\Big|_{-4}^{0} = \frac{-4^3}{3} + \frac{4^3}{2} = \frac{32}{3}$$