Math 150 Exam 4

Spring 2006 **Name:** Blue Key

For credit on a problem, all work must be shown with a brief explanation of each step.

EXAM SCORES

1a	**5pts**	
1b	**5pts**	
1c	**5pts**	
1d	**5pts**	
1e	**5pts**	
1f	**5pts**	
2a	**10pts**	
2b	**10pts**	
3	**25pts**	
4	**25pts**	
	Total	

1a. Evaluate $\lim\limits_{x \to \infty} \dfrac{\ln(x)}{x}$. $\rightarrow \frac{\infty}{\infty}$

By L'Hospital's Rule

$$= \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = 0$$

1b. Compute $\dfrac{d}{dx} \ln\left(\dfrac{\sqrt[3]{x-1}}{x^2+1}\right)$

$$= \frac{d}{dx}\left[\frac{1}{3}\ln(x-1) - \ln(x^2+1)\right]$$

$$= \frac{1}{3}\,\frac{1}{x-1} - \frac{2x}{x^2+1}$$

1c. Compute $\dfrac{d}{dx}\left[\displaystyle\int_x^{\pi} \cos(t^2)\,dt\right]$ $= -\cos(x^2)$ (by F. Thm)

1d. Evaluate $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$ $= \dfrac{1}{2}\displaystyle\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\arcsin(x^2)$

Let $u = x^2$

$du = 2x\,dx$

1e. If $f'(x) = 3x^2 + 4x - 6$ and $f(0) = 9$ find f.

$f(x) - f(0) = \int_0^x f'(x)\,dx = \int_0^x (3x^2 + 4x - 6)\,dx = x^3 + 2x^2 - 6x$

$f(x) = x^3 + 2x^2 - 6x + 9$

1f. If $f''(x) = \cos(x)$ find **all** antiderivatives of f.

$f'(x) = \sin(x) + C$

$f''(x) = -\cos(x) + Cx + D$

2a. Evaluate $\int_{-1}^{0} \frac{1}{(1-x)^3}dx$ $= -\int_2^1 u^{-3}du = \int_1^2 u^{-3}du = \frac{u^{-2}}{-2}\Big|_1^2$

Let $u = 1 - x$

$du = -dx$

$= -\frac{1}{2}\left[\frac{1}{4} - 1\right] = \frac{3}{8}$

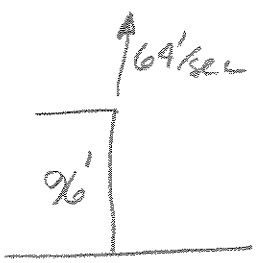
2b. Evaluate $\int_{1}^{e} \frac{(\ln(t))^2}{t}dt$ $= \int_0^1 u^2 du = \frac{u^3}{3}\Big|_0^1 = \frac{1}{3}$

Let $u = \ln(t)$

$du = \frac{dt}{t}$

3. A stone is thrown straight up from the edge of a roof, 96 feet above the ground, at a speed of 64 ft per second. If the acceleration of gravity is -32 feet per second squared,

i) At what time does the stone hit the ground?
ii) What is the velocity of the stone when it hits the ground?
iii) At what time does the stone reach its maximum height?
iv) What is the maximum height?



$s(0) = 96$

$v(0) = 64$

$a(t) = -32$

$v(t) - v(0) = \int_0^t \frac{dv}{dt}\,dt = \int_0^t -32\,dt = -32t$

$v(t) = -32t + 64$

$s(t) - s(0) = \int_0^t \frac{ds}{dt}\,dt = \int_0^t (-32t+64)\,dt = -16t^2 + 64t$

$s(t) = -16t^2 + 64t + 96 = -16(t^2 - 4t - 6)$

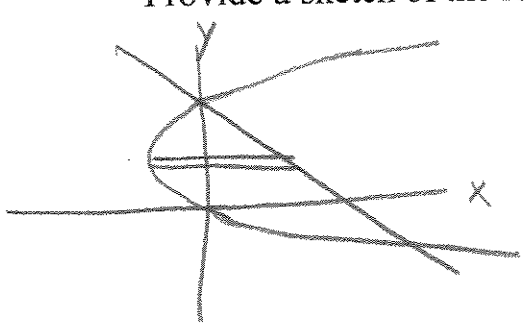
(i) $t_g = 2 + \sqrt{10}$ sec since $s(t_g) = 0$

(ii) $v(t_g) = -32(2+\sqrt{10}) + 64 = -32 \cdot \sqrt{10}$ ft/sec

(iii) $t_{max} = 2$ sec since $v(t_{max}) = 0$

(iv) $s(t_{max}) = -16(4 - 8 - 6) = 160$ ft

4. Find the area between the curves $x + y = 3$ and $x = y^2 - 3y$. Provide a sketch of the region.



$x = 3 - y$ and $x = y^2 - 3y$

$\Rightarrow 3 - y = y^2 - 3y$

OR $0 = y^2 - 2y - 3 = (y-3)(y+1)$

$y = -1$ and $y = 3$

$$\int_{-1}^{3} \left((3-y) - (y^2 - 3y)\right) dy = \int_{-1}^{3} (3 + 2y - y^2)\,dy$$

$$= \left(3y + y^2 - \frac{y^3}{3}\right)\Big|_{-1}^{3} = (9+9-9) - (-3+1+1/3) = \frac{32}{3}$$