Math 150 Exam 3

Spring 2006 **Name:** Yellow Key

For credit on a problem, all work must be shown with a brief explanation of each step.

EXAM SCORES

1a	5pts	
1b	5pts	
1c	5pts	
1d	5pts	
1e	5pts	
1f	5pts	
2a	10pts	
2b	10pts	
3	25pts	
4	25pts	
	Total	

1a. If $y = \sqrt{x}\cos(2x)$ compute f '.

$$y' = \frac{1}{2\sqrt{x}}\cos(2x) - 2\sqrt{x}\sin(2x)$$

1b. If $y = \arctan(5x)$ compute f '.

$$y' = \frac{5}{1+(5x)^2}$$

1c. If $y = \exp\left(\frac{1}{x}\right)$ compute f '.

$$y' = -\frac{1}{x^2}e^{(1/x)}$$

1d. If $y = \ln\left(\frac{x-1}{2x+3}\right)$ compute f '.

$$y = \ln(x-1) - \ln(2x+3)$$

$$y' = \frac{1}{x-1} - \frac{2}{2x+3}$$

1e. Let $y = x^{\cos(x)}$ compute y '.

$\ln y = \cos(x) \ln(x)$

$\frac{y'}{y} = -\sin(x)\ln(x) + \frac{\cos(x)}{x}$ OR $y' = x^{\cos(x)}\left[-\sin x \ln(x) + \frac{\cos(x)}{x}\right]$

1f. Let $y = \left((x^5+6)^6+3\right)^8$ compute y '.

$y' = 8\left((x^5+6)^6+3\right)^7 \cdot 6(x^5+6)^5 \cdot 5x^4$

2a. Evaluate $\lim_{x\to\pi} \frac{-\cos(x)}{\sin(x)-1} = \frac{-(-1)}{0-1} = -1$

2b. Evaluate $\lim_{x\to 0} \frac{6\sin(x)-6x}{x^3} \to \frac{0}{0}$ Indet Form Apply L'Hospital's Rule

$\lim_{x\to 0} \frac{6\sin(x)-6x}{x^3} = \lim_{x\to 0} \frac{6\cos(x)-6}{3x^2} = \lim_{x\to 0} \frac{-6\sin(x)}{6x}$

$= \lim_{x\to 0} \frac{-6\cos(x)}{6} = -1$

3. Using the graphing guidelines, sketch the graph of the following function. Label all important points (zeros, critical pts, inflection pts) and all important intervals (dec., inc., concavity up, down). Show any asymptotes. **SHOW YOUR WORK**

$$y = \frac{x^2}{x+8}$$

Vert Asym $x = -8$

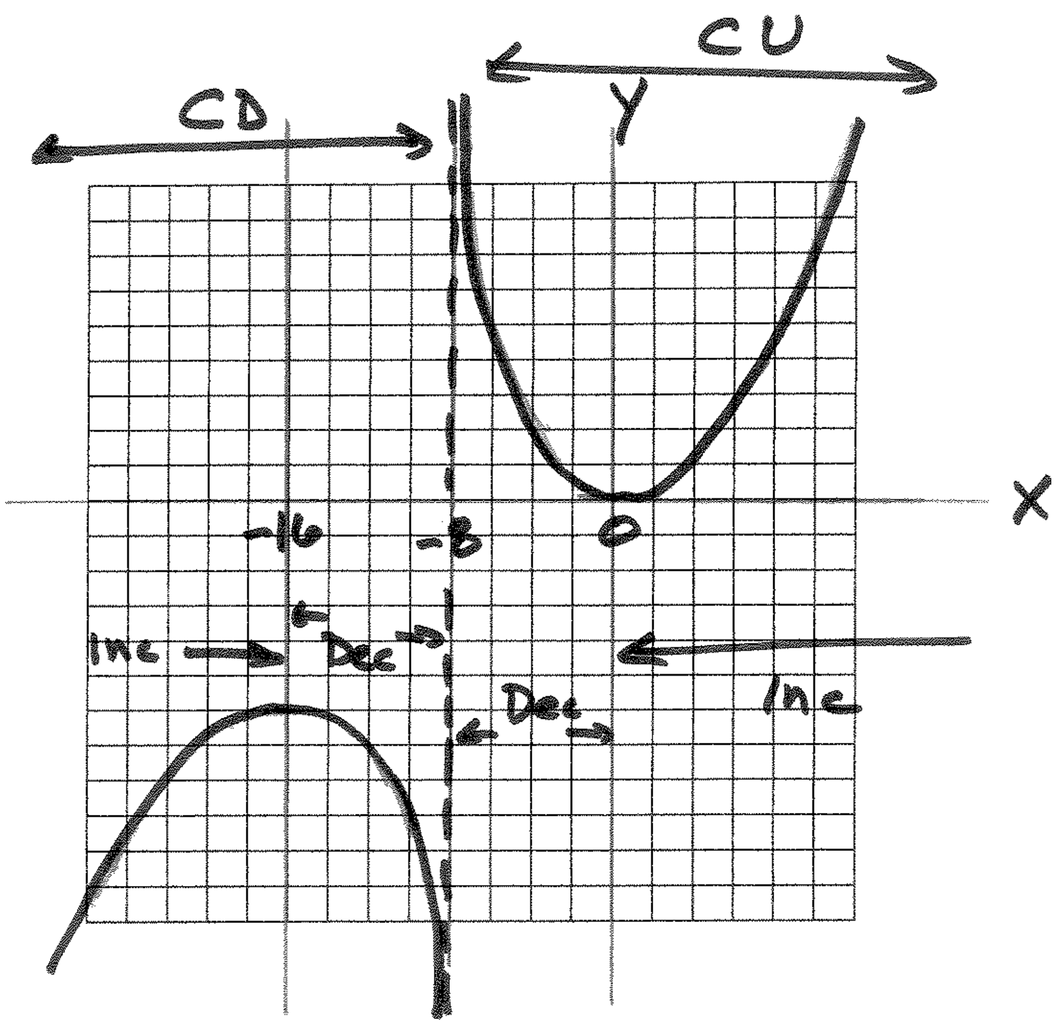
No Horiz Asym

Zero at $(0,0)$

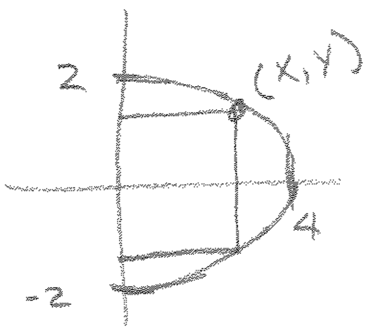
$$y' = \frac{(x+8)2x - x^2}{(x+8)^2} = \frac{x^2 + 16x}{(x+8)^2} \qquad \text{C.P.} = \{0, -16, -8\}$$

$$y'' = \frac{(x+8)^2(2x+16) - (x^2+16x)2(x+8)}{(x+8)^4} = \frac{2(x+8)\left((x+8)^2 - (x^2+16x)\right)}{(x+8)^4}$$

$$y'' = \frac{128}{(x+8)^3} \qquad \text{No Pts of Inflex}$$



4. Consider the area of the largest rectangle that can be inscribed (2 corners on the curve and one side on the y-axis) in the right half of the ellipse given by $x^2 + 4y^2 = 16$.
a. Draw a sketch labeling the variables and list the relevant formulas.
b. Find the co-ordinates of this rectangle's corner in the first quadrant.
c. Find the area of this largest rectangle.



$A = 2xy$

$x^2 + 4y^2 = 16$

$\frac{dA}{dx} = 2y + 2xy'$

$2x + 8yy' = 0$

$y' = \frac{-x}{4y}$

$\frac{dA}{dx} = 0 \Rightarrow 2y + 2xy' = 0$

OR $y + x\left(\frac{-x}{4y}\right) = 0$

OR $4y^2 = x^2$ at the maximum

$\Rightarrow x^2 + x^2 = 16$ OR $x^2 = 8$ OR $x = 2\sqrt{2}$

and $4y^2 + 4y^2 = 16$ or $y^2 = 2$ or $y = \sqrt{2}$

Corner $= (2\sqrt{2}, \sqrt{2})$

Max Area $= 2 \cdot 2\sqrt{2} \cdot \sqrt{2} = 8$