Math 150 Exam 3

Spring 2006 **Name:** White Key

For credit on a problem, all work must be shown with a brief explanation of each step.

EXAM SCORES

1a	5pts	
1b	5pts	
1c	5pts	
1d	5pts	
1e	5pts	
1f	5pts	
2a	10pts	
2b	10pts	
3	25pts	
4	25pts	
	Total	

1a. If $y = \sqrt{x}\,\sec(2x)$ compute f '.

$$y' = \frac{1}{2\sqrt{x}}\sec(2x) + 2\sqrt{x}\,\sec(2x)\tan(2x)$$

1b. If $y = \arcsin\left(x^2\right)$ compute f '.

$$y' = \frac{2x}{\sqrt{1-x^4}}$$

1c. If $y = \exp(\sin(3x))$ compute f '.

$$y' = 3\cos(3x)\,e^{\sin(3x)}$$

1d. If $y = \ln\left(\frac{3x-1}{4x-3}\right)$ compute f '.

$$y = \ln(3x-1) - \ln(4x-3)$$

$$y' = \frac{3}{3x-1} - \frac{4}{4x-3}$$

1e. Let $y = x^{\tan(x)}$ compute y'. $\ln y = \tan(x)\ln(x)$

$$\frac{y'}{y} = \sec^2(x)\ln(x) + \frac{\tan(x)}{x} \quad \text{OR} \quad y' = x^{\tan(x)}\left[\sec^2(x)\ln(x) + \frac{\tan(x)}{x}\right]$$

1f. Let $y = \left(\left(x^7+6\right)^5+3\right)^9$ compute y'.

$$y' = 9\left(\left(x^7+6\right)^5+3\right)^8 \cdot 5\left(x^7+6\right)^4 \cdot 7x^6$$

2a. Evaluate $\lim\limits_{x\to\pi} \frac{\cos(x)}{\sin(x)-1}$ $= \frac{-1}{-1} = 1$

2b. Evaluate $\lim\limits_{x\to 0} \frac{\cos(x)-1}{x^2}$ $\rightarrow \frac{0}{0}$ Indet Form Apply L'Hospital Rule

$$\lim_{x\to 0} \frac{\cos(x)-1}{x^2} = \lim_{x\to 0} \frac{-\sin(x)}{2x} = \lim_{x\to 0} \frac{-\cos(x)}{2} = \frac{-1}{2}$$

3. Using the graphing guidelines, sketch the graph of the following function. Label all important points (zeros, critical pts, inflection pts) and all important intervals (dec., inc., concavity up, down). Show any asymptotes. **SHOW YOUR WORK**

$$y = \frac{x^2}{x^2+3}$$

$f(x) = f(-x)$ even function

No Vert Asym.

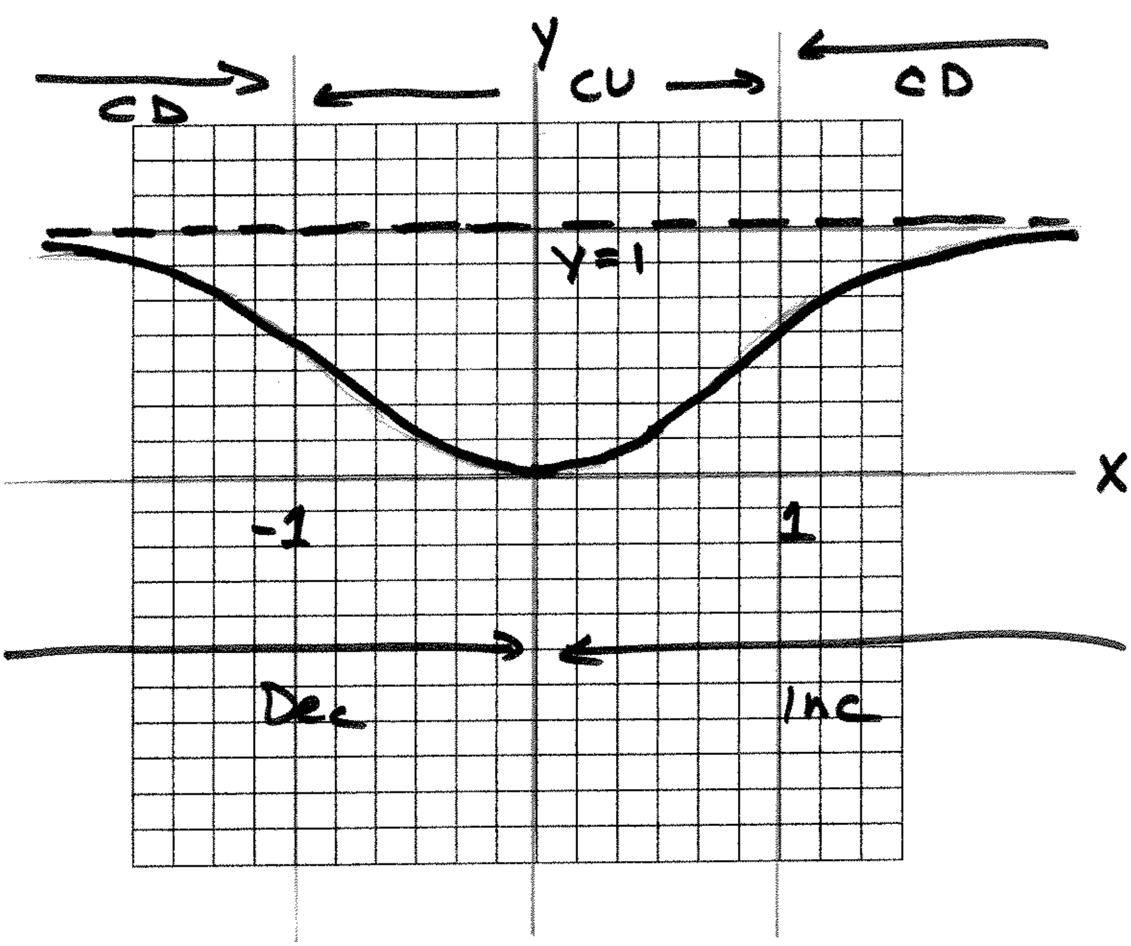
$y=1$ Horizontal Asym

zero at $(0,0)$

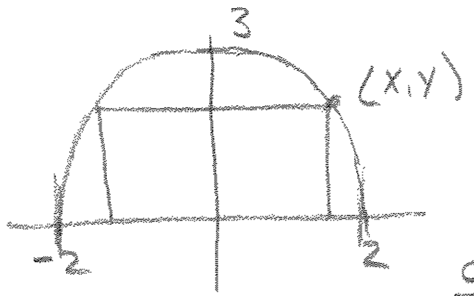
$$y' = \frac{(x^2+3)2x - x^2 \cdot 2x}{(x^2+3)^2} = \frac{6x}{(x^2+3)^2}$$ Crit. Pt $= \{x=0\}$

$$y'' = \frac{(x^2+3)^2 6 - 6x \cdot 2(x^2+3)2x}{(x^2+3)^4} = \frac{(x^2+3)(6x^2+18-24x^2)}{(x^2+3)^4}$$

$$y'' = \frac{18-18x^2}{(x^2+3)^3} \Rightarrow x = \pm 1 \text{ Pts of Inflexion.}$$



4. Consider the area of the largest rectangle that can be inscribed (2 corners on the curve and one side on the x-axis) in the top half of the ellipse given by $9x^2 + 4y^2 = 36$.
a. Draw a sketch labeling the variables and list the relevant formulas.
b. Find the co-ordinates of this rectangle's corner in the first quadrant.
c. Find the area of this largest rectangle.



$A = 2xy \qquad 9x^2 + 4y^2 = 36$

$\frac{dA}{dx} = 2y + 2xy' \qquad 18x + 8yy' = 0$

$y' = \frac{-18x}{8y} = -\frac{9}{4}\frac{x}{y}$

$\frac{dA}{dx} = 0 \Rightarrow 2y + 2xy' = 0$

OR $y + x\left(\frac{-9}{4}\right)\frac{x}{y} = 0$

OR $4y^2 = 9x^2$ at the maximum

$\Rightarrow 9x^2 + 9x^2 = 36 \Rightarrow x^2 = 2$ OR $x = \sqrt{2}$

and $4y^2 + 4y^2 = 36 \Rightarrow y^2 = \frac{36}{8} \Rightarrow y = \frac{6}{2\sqrt{2}} = \frac{3\sqrt{2}}{2}$

Corner $= \left(\sqrt{2}, \frac{3\sqrt{2}}{2}\right)$

Max Area $= 2 \cdot \sqrt{2} \cdot \frac{3\sqrt{2}}{2} = 6$