Math 150 Exam 3

Spring 2006 **Name:** Blue Key

For credit on a problem, all work must be shown with a brief explanation of each step.

EXAM SCORES

1a	5pts	
1b	5pts	
1c	5pts	
1d	5pts	
1e	5pts	
1f	5pts	
2a	10pts	
2b	10pts	
3	25pts	
4	25pts	
	Total	

1a. If $y = \sqrt{x}\tan(2x)$ compute f '.

$$y' = \frac{1}{2\sqrt{x}}\tan(2x) + 2\sqrt{x}\sec^2(2x)$$

1b. If $y = \arcsin\left(\frac{x}{2}\right)$ compute f '.

$$y' = \frac{1}{2}\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$$

1c. If $y = \exp(-(x-3)^2)$ compute f '.

$$y' = -2(x-3)e^{-(x-3)^2}$$

1d. If $y = \ln\left(\frac{2x-2}{3x+3}\right)$ compute f '.

$$y = \ln(2x-2) - \ln(3x+3)$$

$$y' = \frac{2}{2x-2} - \frac{3}{3x+3}$$

1e. Let $y = x^{\sin(x)}$ compute y '.

$\ln y = \sin(x) \ln x$

$\frac{y'}{y} = \frac{\sin(x)}{x} + \cos(x)\ln(x)$ OR $y' = x^{\sin(x)}\left[\frac{\sin(x)}{x} + \ln(x)\cos(x)\right]$

1f. Let $y = \left(\left(x^8+6\right)^5+3\right)^6$ compute y '.

$y' = 6\left(\left(x^8+6\right)^5+3\right)^5 \cdot 5\left(x^8+6\right)^4 \cdot 8x^7$

2a. Evaluate $\lim_{x\to\pi} \frac{-\cos(x)}{\sin(x)-1} = \frac{-1(-1)}{0-1} = -1$

2b. Evaluate $\lim_{x\to 0} \frac{\sin(x)-x}{x^3} \to \frac{0}{0}$ indet form Using L'Hopital's Rule

$\lim_{x\to 0} \frac{\sin(x)-x}{x^3} = \lim_{x\to 0} \frac{\cos(x)-1}{3x^2} = \lim_{x\to 0} \frac{-\sin(x)}{6x} = \lim_{x\to 0} \frac{-\cos(x)}{6}$

$\lim_{x\to 0} \frac{\sin(x)-x}{x^3} = -\frac{1}{6}$

3. Using the graphing guidelines, sketch the graph of the following function. Label all important points (zeros, critical pts, inflection pts) and all important intervals (dec., inc., concavity up, down). Show any asymptotes. **SHOW YOUR WORK**

$$y = \frac{1}{x} + \frac{1}{x+1} = \frac{2x+1}{x(x+1)}$$

$x=0$ & $x=-1$ Vertical Asym.

$y=0$ Horizontal Asym.

$(-\frac{1}{2}, 0)$ Zero

$$y' = \frac{-1}{x^2} - \frac{1}{(x+1)^2} < 0$$

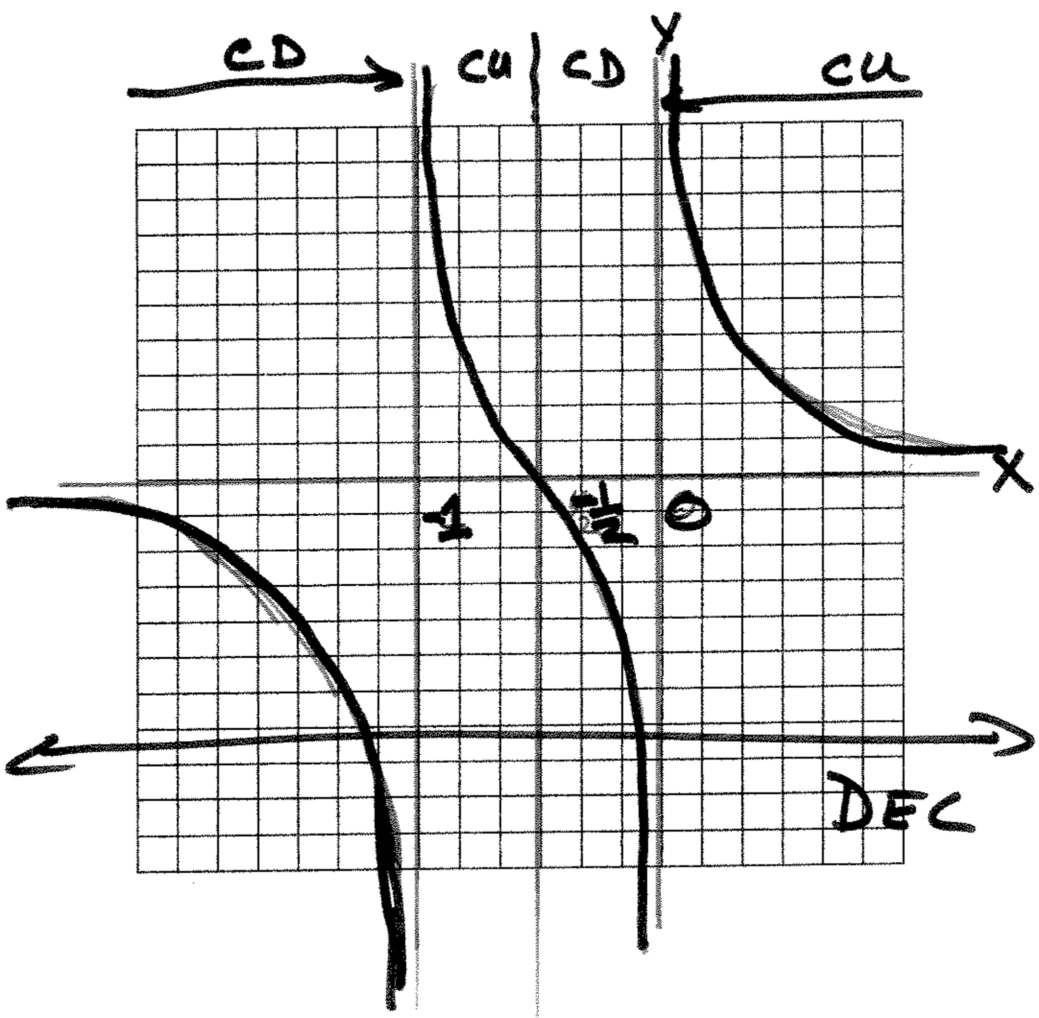
Decreasing for all x

No flat spots

$$y'' = \frac{2}{x^3} + \frac{2}{(x+1)^3} = 2\left[\frac{(x+1)^3 + x^3}{x^3(x+1)^3}\right] \Rightarrow y''=0 \text{ is given by } (x+1)^3 = -x^3$$

OR $x+1 = -x$

OR $x = -\frac{1}{2}$ Pt of Inf.

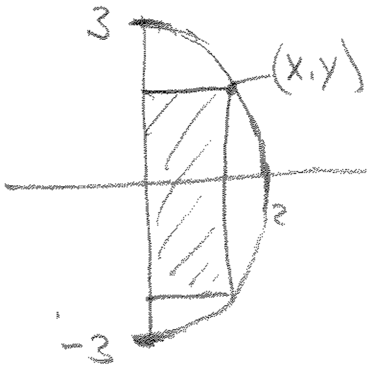


4. Consider the area of the largest rectangle that can be inscribed (2 corners on the curve and one side on the y-axis) in the right half of the ellipse given by $9x^2 + 4y^2 = 36$.

a. Draw a sketch labeling the variables and list the relevant formulas.

b. Find the co-ordinates of this rectangle's corner in the first quadrant.

c. Find the area of this largest rectangle.



$$A = 2xy \qquad 9x^2 + 4y^2 = 36$$

$$\frac{dA}{dx} = 2y + 2xy' \qquad 18x + 8yy' = 0$$

OR

$$y' = \frac{-18x}{8y} = -\frac{9}{4}\frac{x}{y}$$

$$\frac{dA}{dx} = 0 \Rightarrow 2y + 2xy' = 0$$

OR $\quad y + x\left(-\frac{9}{4}\frac{x}{y}\right) = 0$

OR $\quad 4y^2 = 9x^2$ at the max Area

$\Rightarrow \quad 9x^2 + 9x^2 = 36$ OR $x^2 = 2$ OR $x = \sqrt{2}$

$\Rightarrow \quad 18 + 4y^2 = 36$ OR $4y^2 = 18$ OR $y^2 = \frac{18}{4}$ OR $y = \frac{3\sqrt{2}}{2}$

The Corner is at $\left(\sqrt{2}, \frac{3\sqrt{2}}{2}\right)$

The max Area is $= 2\sqrt{2}\,\frac{3\sqrt{2}}{2} = 6$