Math 150 Exam 2

Spring 2006 **Name:** White Key II

For credit on a problem, all work must be shown with a brief explanation of each step.

EXAM SCORES

1a	**5pts**	
1b	**5pts**	
1c	**5pts**	
1d	**5pts**	
1e	**5pts**	
1f	**5pts**	
2a	**10pts**	
2b	**10pts**	
3	**25pts**	
4	**25pts**	
	Total	

1. For the given function y compute the derivative y' (Do not simplify)

1a. $y = \dfrac{x^2 + 21x - 10}{(x-1)^2}$

$$y' = \frac{(x-1)^2(2x+21) - (x^2+21x-10)\,2(x-1)}{(x-1)^4}$$

1b. $y = \dfrac{x^2 + \sin(5x)}{x + \tan(3x)}$

$$y' = \frac{(x+\tan(3x))(2x+5\cos(5x)) - (x^2+\sin(5x))(1+3\sec^2(3x))}{(x+\tan(3x))^2}$$

1c. $y = e^{-5x}\cos(4x+3)$

$$y' = -5e^{-5x}\cos(4x+3) - 4e^{-5x}\sin(4x+3)$$

1d. $y = \ln\left(\dfrac{2x+1}{(x-2)^4}\right)$ $= \ln(2x+1) - 4\ln(x-2)$

$y' = \dfrac{2}{2x+1} - \dfrac{4}{x-2}$

1e. $y = \left(\left(x^5+5\right)^6+7\right)^4$

$y' = 4\left(\left(x^5+5\right)^6+7\right)^3 \cdot 6\left(x^5+5\right)^5 \cdot 5x^4$

1f. $y = 3\arcsin(\sqrt{x+1})$

$y' = \dfrac{3}{\sqrt{1-(\sqrt{x+1})^2}} \cdot \dfrac{1}{2}\dfrac{1}{\sqrt{x+1}}$

2a. Let $y = \dfrac{x}{2x-1}$ compute $\dfrac{d^3y}{dx^3}$.

$y' = \dfrac{2x-1-x\cdot 2}{(2x-1)^2} = \dfrac{-1}{(2x-1)^2} = (-1)(2x-1)^{-2}$

$y'' = 4(2x-1)^{-3}$

$y''' = -24(2x-1)^{-4}$

2b. Let $y = x^{\sin(2x)}$ compute y'.

$\ln y = \sin(2x)\ln(x)$

$\dfrac{y'}{y} = 2\cos(2x)\ln(x) + \dfrac{\sin(2x)}{x}$

$y' = x^{\sin(2x)}\left[2\cos(2x)\ln(x) + \dfrac{\sin(2x)}{x}\right]$

3. Find an equation for the tangent line to the curve $y^2 - 3y = x^3 + 3x^2$ at the point $P = (-2, 4)$.

Calc Derivative $2yy' - 3y' = 3x^2 + 6x$ OR

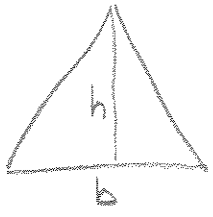
$$y' = \frac{3x^2+6x}{2y-3} \Rightarrow y'|_p = \frac{12-12}{8-3} = 0$$

Thus $\boxed{y = 4}$

4. The altitude of a triangle is increasing at a rate of 1cm/min while the area of the triangle is increasing at a rate of $10\text{cm}^2/\text{min}$:

a. Sketch and label a diagram for this problem.

b. At what rate is the base of the triangle changing when the altitude is 10cm and the area is 100cm^2?



$\frac{dh}{dt} = 1\text{ cm/min}$ $\frac{dA}{dt} = 10\text{ cm}^2/\text{min}$ $\frac{db}{dt} = ?$

$$A = \tfrac{1}{2}bh \Rightarrow \frac{dA}{dt} = \tfrac{1}{2}b\frac{dh}{dt} + \tfrac{1}{2}h\frac{db}{dt}$$

$100\text{cm}^2 = \tfrac{1}{2}b \cdot 10\text{cm} \Rightarrow b = 20\text{cm}$

OR $10\text{cm}^2/\text{min} = \tfrac{1}{2}\,20\text{cm}\,1\text{ cm/min} + \tfrac{1}{2}\,10\text{cm}\,\frac{db}{dt}$

Thus $\boxed{\frac{db}{dt} = 0\text{ cm/min}}$