Math 150 Exam 2

Spring 2006 **Name:** Gold Key

For credit on a problem, all work must be shown with a brief explanation of each step.

EXAM SCORES

1a	5pts	
1b	5pts	
1c	5pts	
1d	5pts	
1e	5pts	
1f	5pts	
2a	10pts	
2b	10pts	
3	25pts	
4	25pts	
	Total	

1. For the given function y compute the derivative y' (Do not simplify)

1a. $y=\frac{x^2-11x+10}{x-1}$

$$y'=\frac{(x-1)(2x-11)-(x^2-11x+10)}{(x-1)^2}$$

1b. $y=\frac{1+\cos(2x)}{x+\sin(2x)}$

$$y'=\frac{(x+\sin(2x))(-2\sin(2x))-(1+\cos(2x))(1+2\cos(2x))}{(x+\sin(2x))^2}$$

1c. $y=e^{-3x}\cos(4x)$

$$y'=(-3)e^{-3x}\cos 4x-4e^{-3x}\sin(4x)$$

1d. $y = \ln\left(\frac{x-1}{2x+3}\right) = \ln(x-1) - \ln(2x+3)$

$$y' = \frac{1}{x-1} - \frac{2}{2x+3}$$

1e. $y = \left(\left(x^2+1\right)^2+1\right)^3$

$$y' = 3\left(\left(x^2+1\right)^2+1\right)^2 \cdot 2 \cdot \left(x^2+1\right)^1 \cdot 2x$$

1f. $y = \arctan(\sqrt{x})$

$$y' = \frac{1}{1+x}\left(\frac{1}{2\sqrt{x}}\right)$$

2a. Let $y = \frac{x}{2x-1}$ compute $\frac{d^3y}{dx^3}$.

$$y' = \frac{(2x-1) - 2 \cdot x}{(2x-1)^2} = \frac{-1}{(2x-1)^2} = -(2x-1)^{-2}$$

$$y'' = (-1)(-2)(2x-1)^{-3}(2)$$

$$y''' = (-1)(-2)(-3)(2x-1)^{-4} 2 \cdot 2$$

2b. Let $y = x^{\ln(x)}$ compute y'.

$$\ln y = \ln\left(x^{\ln x}\right) = \ln(x) \cdot \ln(x) = (\ln(x))^2$$

$$\frac{y'}{y} = 2\,\frac{\ln(x)}{x}$$

$$y' = x^{\ln(x)}\left[2\,\frac{\ln(x)}{x}\right]$$

3. Find an equation for the tangent line to the curve $x^3 + y^3 = 3xy$ at the point $P = \left(2^{1/3}, 2^{2/3}\right)$.

Compute the slope

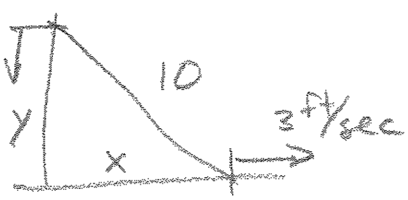
$$3x^2 + 3y^2 y' = 3y + 3xy' \Rightarrow x^2 + y^2 y' = y + xy'$$

$$\text{OR } y' = \frac{y - x^2}{y^2 - x} \Rightarrow y'\big|_P = \frac{2^{2/3} - 2^{2/3}}{2^{4/3} - 2^{1/3}} = 0$$

$$\boxed{y = 2^{2/3}}$$

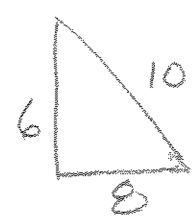
4. A ladder 10ft long rests against a vertical wall. If the bottom of the ladder slides horizontally away from the wall at a speed of 3ft/sec:
a. Sketch and label a diagram for this problem.
b. How fast is the top of the ladder sliding toward the ground when the distance between the ground and the top of the ladder is 6ft?
c. How fast is the area of the triangle changing when the distance between the ground and the top of the ladder is 6ft?

(a)



(b) $x^2 + y^2 = 100$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$



$$\text{OR } \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt}\bigg|_{y=6} = -\frac{8}{6}3 = -4 \text{ ft/sec}$$

(c) $A = \frac{1}{2}xy$

$$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + \frac{1}{2}y\frac{dx}{dt}$$

$$\frac{dA}{dt}\bigg|_{y=6} = \frac{1}{2}8(-4) + \frac{1}{2}6 \cdot 3 = -16 + 9 = -7 \text{ ft}^2/\text{sec}$$