Math 150 Exam 2

Spring 2006 **Name:** Blue Key II

For credit on a problem, all work must be shown with a brief explanation of each step.

EXAM SCORES

1a	**5pts**	
1b	**5pts**	
1c	**5pts**	
1d	**5pts**	
1e	**5pts**	
1f	**5pts**	
2a	**10pts**	
2b	**10pts**	
3	**25pts**	
4	**25pts**	
	Total	

1. For the given function y compute the derivative y' (Do not simplify)

1a. $y = \dfrac{x^2 + 41x - 10}{(x+1)^2}$

$$y' = \frac{(x+1)^2(2x+41) - (x^2+41x-10)\,2(x+1)}{(x+1)^4}$$

1b. $y = \dfrac{x^2 + \cos(5x)}{x + \tan(2x)}$

$$y' = \frac{(x+\tan(2x))(2x - 5\sin(5x)) - (x^2+\cos(5x))(1+2\sec^2(2x))}{(x+\tan(2x))^2}$$

1c. $y = e^{-2x}\sin(4x+3)$

$$y' = e^{-2x}\,4\cos(4x+3) - 2e^{-2x}\sin(4x+3)$$

1d. $y = \ln\left(\frac{2x+1}{(x-2)^3}\right) = \ln(2x+1) - 3\ln(x-2)$

$y' = \frac{2}{2x+1} - \frac{3}{x-2}$

1e. $y = \left((x^3+5)^6+3\right)^4$

$y' = 4\left((x^3+5)^6+3\right)^3 \cdot 6(x^3+5)^5 \cdot 3x^2$

1f. $y = 3\arctan(\sqrt{x+1})$

$y = \frac{3}{2+x} \cdot \frac{1}{2} \frac{1}{\sqrt{x+1}}$

2a. Let $y = \frac{x}{2x-1}$ compute $\frac{d^3y}{dx^3}$.

$y' = \frac{2x-1-x\cdot 2}{(2x-1)^2} = \frac{-1}{(2x-1)^2} = (-1)(2x-1)^{-2}$

$y'' = 4(2x-1)^{-3}$

$y''' = -24(2x-1)^{-4}$

2b. Let $y = x^{\cos(2x)}$ compute y'.

$\ln y = \cos(2x) \cdot \ln x$

$\frac{y'}{y} = -2\sin(2x) \cdot \ln x + \frac{\cos(2x)}{x}$

$y' = x^{\cos(2x)}\left[-2\sin(2x) \cdot \ln x + \frac{\cos(2x)}{x}\right]$

3. Find an equation for the tangent line to the curve $y^2 - 3y = x^3 + 3x^2$ at the point $P = (-2, -1)$.

Calc Derivative $2yy' - 3y' = 3x^2 + 6x$

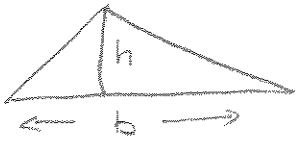
$$y' = \frac{3x^2+6x}{2y-3} \Rightarrow y'|_P = \frac{12-12}{-5} = 0$$

$\boxed{y = -1}$

4. The altitude of a triangle is increasing at a rate of 1cm/min while the area of the triangle is increasing at a rate of $2cm^2/min$:

a. Sketch and label a diagram for this problem.

b. At what rate is the base of the triangle changing when the altitude is 10cm and the area is $100cm^2$?

a. 

$\frac{dh}{dt} = 1\ cm/min \quad \frac{dA}{dt} = 2\ cm^2/min$

b. $A = \frac{1}{2}bh \Rightarrow \frac{dA}{dt} = \frac{1}{2}b\frac{dh}{dt} + \frac{1}{2}h\frac{db}{dt}$

$100cm^2 = \frac{1}{2}b\,10cm \Rightarrow b = 20cm$

Thus $2\ cm^2/min = \frac{1}{2}\cdot 20cm \cdot 1\ cm/min + \frac{1}{2}\,10cm\,\frac{db}{dt}$

$\boxed{\frac{db}{dt} = -1.6\ cm/min}$