

Math 150 Exam 2**Spring 2006**Name: Key Blue

For credit on a problem, all work must be shown with a brief explanation of each step.

1. For the given function y compute the derivative y' (Do not simplify)

1a. $y = \frac{x^2 + 41x - 10}{x + 1}$

$$y' = \frac{(x+1)(2x+41) - (x^2+41x-10)}{(x+1)^2}$$

1b. $y = \frac{1 + \cos(5x)}{x + \tan(2x)}$

$$y' = \frac{(x + \tan(2x))(-5 \sin(5x)) - (1 + \cos(5x))(1 + 2 \sec^2(2x))}{(x + \tan(2x))^2}$$

1c. $y = e^{-2x} \sin(4x)$

$$y' = -2e^{-2x} \sin(4x) + 4e^{-2x} \cos(4x)$$

EXAM SCORES

1a	5pts	
1b	5pts	
1c	5pts	
1d	5pts	
1e	5pts	
1f	5pts	
2a	10pts	
2b	10pts	
3	25pts	
4	25pts	
Total		

$$1d. y = \ln\left(\frac{2x+1}{x-2}\right) = \ln(2x+1) - \ln(x-2)$$

$$y' = \frac{2}{2x+1} - \frac{1}{x-2}$$

$$1e. y = \left((x^3+1)^2 + 1\right)^4$$

$$y' = 4 \left((x^3+1)^2 + 1\right)^3 \cdot 2(x^3+1)^1 \cdot 3x^2$$

$$1f. y = \arctan(\sqrt{x})$$

$$y' = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$$

$$2a. \text{ Let } y = \frac{x}{2x-1} \text{ compute } \frac{d^3y}{dx^3}.$$

$$y' = \frac{2x-1-x}{(2x-1)^2} = -1(2x-1)^{-2}$$

$$y'' = (-1)(-2)(2x-1)^{-3}(2)$$

$$y''' = (-1)(-2)(-3)(2x-1)^{-4}(2)(2)$$

$$2b. \text{ Let } y = x^{\cos(x)} \text{ compute } y'.$$

$$\ln y = \ln x \cos(x)$$

$$\frac{y'}{y} = \frac{1}{x} \cos x - \sin(x) \ln(x)$$

$$y' = x^{\cos(x)} \left[\frac{1}{x} \cos x - \sin x \ln(x) \right]$$

3. Find an equation for the tangent line to the curve $y^2(y^2 - 4) = x^2(x^2 - 5)$ at the point $P = (0, -2)$.

Compute the slope $\Rightarrow 2yY'(y^2-4) + y^2 \cdot 2y \cdot y' = 2x(x^2-5) + x^2 \cdot 2x$

$$\text{OR } y' = \frac{2x(x^2-5) + 2x^3}{2y(y^2-4) + 2y^3} \Rightarrow y'|_P = 0$$

$$y + 2 = 0(x - 0)$$

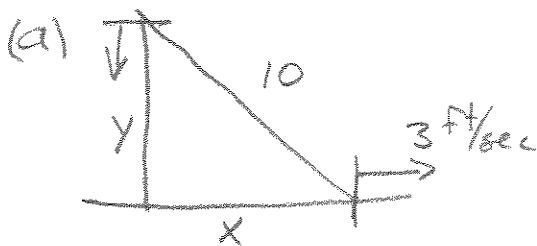
$$\text{OR } y = -2$$

4. A ladder 10ft long rests against a vertical wall. If the bottom of the ladder slides horizontally away from the wall at a speed of 3ft/sec:

a. Sketch and label a diagram for this problem.

b. How fast is the top of the ladder sliding toward the ground when the distance between the ground and the top of the ladder is 8ft?

c. How fast is the area of the triangle changing when the distance between the ground and the top of the ladder is 8ft?



(b) $\frac{dy}{dt} = ?$ $x^2 + y^2 = 100$
 $y = 8$ $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

OR $\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$



(c) $A = \frac{1}{2}xy$

$$\frac{dA}{dt} = \frac{1}{2}x \frac{dy}{dt} + \frac{1}{2}y \frac{dx}{dt}$$

$$\frac{dy}{dt} \Big|_{y=8} = -\frac{6}{8} \cdot 3 = -\frac{9}{4} \text{ ft/sec}$$

$$\frac{dA}{dt} \Big|_{y=8} = \frac{1}{2} \cdot 6 \left(-\frac{9}{4} \right) + \frac{1}{2} \cdot 8 \cdot 3 = 5\frac{1}{4} \text{ ft}^2/\text{sec}$$