Connecting Fraction Reasoning to Algebra Success

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The Big Questions

- How can we help students reason algebraically?
- How can developing fraction reasoning help students reason algebraically?
- What do students need to know and be able to do so they can reason with fractions?
"If students genuinely understand arithmetic at a level at which they can explain and justify the properties they are using as they carry out calculations, they have learned some critical foundations of algebra."

Carpenter, Franke, and Levi, 2003, p. 2
Laying the Foundation for Algebra

Encourage young students to make algebraic generalizations without necessarily using algebraic notation.

NCTM Algebra Research Brief

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From the NCTM Focal Points: Relating Fractions and Algebra

- Grade 3 - Foundational fraction concepts, comparing, ordering, and equivalence. . . They understand and use models, including the number line, to identify equivalent fractions.
- Grade 4 - Decimals and fraction equivalents
- Grade 5 - Addition and subtraction of fractions
- Grade 6 - Multiplication and division of fractions
- Grade 7 - Negative integers
- Grade 8 - Linear functions and equations
Types of Models for Fractions

- **Area/region**
  - Fraction circles, pattern blocks, paper folding, geoboards, fraction bars, fraction strips/kits

- **Set/discrete**
  - Chips, counters, painted beans

- **Linear**
  - Number lines, rulers, fraction bars, fraction strips/kits
Most children need to use concrete models over extended periods of time to develop mental images needed to think conceptually about fractions.

Children benefit from opportunities to talk to one another and with their teacher about fraction ideas as they construct their own understandings of fraction as a number.

Students who don’t have mental images for fractions often resort to whole number strategies.

(Post et al, 1985; Cramer et al, 1997)
Ordering Fractions

Fractions with the same denominator can be compared by their numerators.
Ordering Fractions

Fractions with the same numerator can be compared by their denominators.

\[\frac{4}{10}, \frac{4}{37}, \frac{4}{8}\]
Ordering Fractions

Fractions close to a benchmark can be compared by finding their distance from the benchmark.
Ordering Fractions

\[
\begin{align*}
4 \quad & \frac{1}{2} \\
\frac{2}{9} \\
\frac{x^2 - 1}{2} \quad & \frac{x^2 - 1}{x^2 - 1}
\end{align*}
\]

The denominator is twice the value of the numerator.
Ordering Fractions

Fractions close to one can be compared by finding their distance from one.
Strategies for Ordering Fractions

- Same denominator
- Same numerator
- Benchmarks: close to 0, 1, 1/2
- Same number of missing parts from the whole ("Residual strategy")

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Introducing the “Clothesline” Activity

- **Task:** Order fraction tents on a clothesline.
- **Mathematically justify the reasons for your ordering.**
- **Materials:** fraction tents and clothesline, string, yarn, etc.
“Clothesline” Fractions Activity

\[ \frac{1}{2}, \frac{3}{4}, 1 \]
“Clothesline” Fractions Activity

1 \frac{2}{3}, \quad \frac{7}{4}
“Clothesline” Fractions Activity

\[
\frac{8}{15} \quad \frac{4}{9} \quad \frac{15}{29}
\]
“Clothesline” Fractions Activity

\[
\frac{1}{4}, \quad \frac{3}{13}, \quad \frac{6}{27}
\]

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“Clothesline” Fractions Activity

\[ \frac{1}{8}, \frac{5}{6}, \frac{15}{16} \]
“Clothesline” Fractions Activity

\[ x, 2x, -x \]
“Clothesline” Fractions Activity

\[
\frac{1}{x}, \quad \frac{x}{2}, \quad \frac{1}{2x},
\]

(\text{where } x \neq 0)
“Clothesline” Fractions Activity

\[ x + 2, \ x - 1, \ 1 - x \]
“Clothesline” Fractions Activity

\[ \frac{1}{x}, \frac{2}{x}, \frac{3}{x} \]

(where \( x \neq 0 \))

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“Clothesline” Fractions Activity

\[
\frac{x + 1}{x}, \quad \frac{x - 1}{x}, \quad \frac{x}{x}
\]

(where \(x \neq 0\))

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“Clothesline” Fractions Activity

\[
\frac{x - 1}{x + 1}
\]

(where \(x \neq -1\))

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The Number Line Helps Develop

- Fraction sense
- Benchmarks
- Relative magnitude of fractions
- Algebraic connections

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What Should “Kids” Know?

- Fractions aren’t just between zero and one; they live between all the numbers on the number line;
- A fraction can have many different names;
- There are more strategies than just cross-multiplying to find a common denominator for comparing and ordering fractions;
- Fractions can be ordered on a number line just as whole numbers.
- The thinking involved when ordering fractions on a number line supports **algebraic reasoning**.
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Slides and Fraction Tents Master are available at:
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