

The video clips are intended to be used together with some background and questions for reflection and discussion, as can be found in Appendix B of *Reconceptualizing Mathematics*. That appendix is copied here, so that the videos can be used by others not using this text.

Appendix B: Video Clips Illustrating Children's Mathematical Thinking

The six video clips associated with *Reconceptualizing Mathematics: Reasoning About Numbers and Quantities* were developed during a federally funded project at San Diego State University called *Integrating Mathematics and Pedagogy (IMAP)*. The clips are all from interviews of elementary school children. All of the teachers and students involved with the making of these videos have given their permission for the video clips to be used to help teachers better understand students' reasoning about, and understanding of, a variety of mathematical topics. As a viewer of these clips, we ask that you in turn respect these students and their teachers. Thus, if a student does not understand a topic, neither the student nor their teacher should be regarded in a negative fashion. There are many reasons why what you see in these clips could have come about, and you will be asked to explore some of these reasons. The purpose of each interview was to assess student knowledge, not to teach.

Although you may encounter some similar types of activities in a mathematics methods course, these videos are included with this mathematics content material because we have learned from experience that many prospective teachers come to view their own mathematics learning differently after watching these children. It will become clear to you that you will need to have a deep understanding of mathematics if you are to successfully teach children such as those you see here. We hope that seeing these children will motivate you to take seriously the study of the mathematics of elementary school.

Video Clip 1: Strategies

In this clip there are three first graders being asked to add and subtract small numbers. Strategies children often use for adding, in a relative order of sophistication, include (a) counting all, using manipulatives of some sort (in this case small blocks); (b) starting with the first number and then counting on in some fashion, such as using fingers; (c) counting on from the larger addend rather than the first addend; (d) compensating by using a known fact to produce a new fact. Counting with manipulatives such as in (a) also has different levels. Children sometimes try to count without moving the objects, and thus count too

many or too few. To subtract, children use similar strategies, using manipulatives, counting on fingers, or using known facts to arrive at a new answer.

Questions for reflection and discussion:

1. In the first interview, a girl is asked to take 5 objects away from 14 objects, and then 6 from 14. Describe the counting strategies she used. Were there any surprises here? If you were helping her, what problem would you give her next, and why?
2. The second girl is asked to add 7 to 4. How did she do this? Notice how she tapped her head when starting from 7. Her teacher taught students to “Put the big number in your head, then count on.” Do you think this child actually undertook the process she describes here? Or was she just trying to help the interviewers understand how to do the problem?
3. How would you describe the strategy used by the boy in the third part of this clip? How did he decompose one of the numbers? What problem would you give him next to solve for you and why?
4. These children do not yet appear to know their basic addition and subtraction facts. Are any of them ready to learn them? As a teacher or parent, do you think that children should think about problems like these before being asked to develop instant recall of facts?

Video Clip 2: Javier

Javier, a fifth-grade Limited English Proficient student, uses mental strategies to multiply 6 times 12 and 12 times 12. Before viewing the Javier video clip, tell how you could mentally find 6×12 and 12×12 .

Questions for reflection and discussion:

1. In each case, Javier appears to first do the problem mentally, then explains his reasoning to the interviewer. In each case, do you think his explanations matched his answers?
2. Were you surprised by his strategy for finding 12×12 ? Why or why not? How did Javier display good number sense?
3. Use the distributive property, as Javier did, to find 8×15 and 16×15 .
4. Javier is learning English. Does he explain his thinking clearly to the interviewer?

Video Clip 3: Rachel

The purpose of this clip is to contrast the effectiveness of teaching only procedural rules with teaching for understanding. Doing so is difficult because one cannot select and video a “bad teacher” to be shown in this type of forum. We therefore asked a teacher who does teach for understanding to first teach a lesson (in this case changing mixed numbers to improper fractions and vice versa) very procedurally, and then to teach the lesson about a month later in a way that focused on understanding. Rachel, a student in this class, is interviewed twice, once after each lesson.

Questions for reflection and discussion:

1. In the first 50 seconds of the video clip the teacher describes how she taught the first lesson. Do you think this type of teaching is typical of most teachers? How does this match up with your own introduction to work with fractions in elementary school?
2. Rachel is interviewed after this lesson. She is asked to change $\frac{9}{5}$ to a mixed number. She does not appear to be clear about what to do. Why did this happen, do you suppose?
3. The teacher is then interviewed about the second lesson she taught. What do you think she did differently?
4. Rachel is interviewed after the second lesson and asked to change $3\frac{3}{8}$ to an improper fraction. She begins by trying to apply the rule. What goes wrong here?
5. Rachel immediately tries to make a drawing to explain her reasoning. What happens then, and why? How does her drawing help her?
6. Many times teachers teach a lesson procedurally first because they have a lot of content to cover before standardized testing. They then go back and try to supply reasons for the procedures they’ve taught. Would you have any suggestions for these teachers about the advisability of this strategy of ordering lessons?

Video Clip 4: Ally

Ally is fifth grader whose teacher has been teaching fractions to her class. This clip is an excerpt from a much longer clip in which the interviewer first tried to diagnose what Ally knew about fractions, he next taught a lesson based on what was learned about Ally’s knowledge of fractions, and finally he assessed what Ally had learned from the lesson. You are seeing only some of the first part of the

interview with Ally followed by an interview of her teacher. Ally is a student at a school that has regularly had very high standardized text scores.

Notice how the interviewer gives Ally opportunities to work and tell about how she reasoned, without interrupting her or giving any indication about whether she was correct or not.

Ally is first asked to compare the following pairs of fractions by circling the large in each pair:

$$\frac{1}{6} \text{ and } \frac{1}{3} \quad 1 \text{ and } \frac{4}{3} \quad \frac{3}{6} \text{ and } \frac{1}{2} \quad \frac{1}{7} \text{ and } \frac{2}{7} \quad \frac{3}{10} \text{ and } \frac{1}{2}$$

She is also asked to change $1\frac{1}{3}$ to an improper fraction and $\frac{13}{6}$ to a mixed number.

Before viewing the video clip, do each of the above problems.

Questions for reflection and discussion:

1. If possible, stop the video and first compare the fractions that Ally is being asked to compare.
2. For each of the pairs of fractions Ally was asked to compare, describe how Ally reasoned about the pair.
3. What was the interviewer thinking when he chose a new pair of fractions for Ally to compare?
4. How well do you think Ally understands what a mixed number is? Why do you say this?
5. What did you learn about Ally from her teacher? Were you surprised about how much the teacher knew about Ally, and probably the other thirty-some students in the class?
6. Why did students in this teacher's class think multiplication and division of fractions was easier than adding and subtracting fractions?
7. If you were asked to tutor Ally, where would you begin, and why?

Video Clip 5: Felisha

During the summer after second grade, Felisha and three other students were given opportunities to explore basic fraction concepts over several days with a skilled teacher. In a post interview, Felisha was asked to add two simple fractions, $\frac{3}{4}$ and $\frac{1}{2}$, even though addition of fractions had not been taught during

the prior sessions. She works silently for a few minutes before being asked about her reasoning.

Questions for reflection and discussion:

1. Knowing that this child had not had any instruction on adding fractions, what knowledge do you think she did possess to undertake this work?
2. Was her explanation clear? Correct?
3. Were her drawings sufficient for solving this problem?
4. What problem would you pose next to Felisha? Why?
5. Do you think students can solve a lot of problems even though they've not been taught procedures for doing so? Why or why not?

Video Clip 6: Elliot

Elliot, a sixth-grade student, solves two division-of-fractions problems using his understanding of division. Before viewing this video-clip, do the following division problems: $1 \div \frac{1}{3}$ and $1\frac{1}{2} \div \frac{1}{3}$.

Questions for reflection and discussion:

1. Describe Elliot's reasoning for the first problem. Are his drawings correct? Are they the drawings you would use? Do you think Elliot has a good understanding of division? Why or why not?
2. Do you think Elliot has a good understanding of fractions? Why do you say that?
3. Is your answer the same as Elliot's for the second problem? What went wrong? (Hint: Attend to the last comment by Elliot. Was he thinking of the reference unit for the answer, or the reference unit 1? What was confusing here?)
4. How do you think you could help Elliot understand his error?