

The Actor-Oriented Transfer Perspective and Its Contributions to Educational Research and Practice

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Although any mainstream thought is subject to theoretical challenges, the challenges to the mainstream cognitive perspective on transfer have had an unfortunate divisive effect. This article takes a pragmatic view that transfer perspectives are simply *designed objects* (Plomp & Nieveen, 2007), which provide different information for different purposes. Specifically, this paper compares one alternative approach—the *actor-oriented transfer perspective*—with the mainstream cognitive perspective on transfer, by examining the points of compatibility and tension across 5 dimensions. As a result, a space is opened up to explore 3 issues that are particularly well suited to an actor-oriented transfer approach: (a) how students interpret transfer situations, (b) the socially situated nature of transfer processes in classrooms, and (c) how contextual-sensitivity can play a productive role in the transfer of learning. Exploring the benefits and trade-offs of various approaches allows for greater understanding of the contributions of each perspective to educational research and practice.

People often notice the transfer of learning when it *doesn't* happen. For example, a mother is disappointed when her 3-year-old son fails to use his enumeration skills to count out the number of placemats she lays on the table. A calculus professor wonders how to help his college students when they are unable to solve a straightforward physics application using skills of integration they already have. A high school mathematics teacher is disheartened when students who had performed well on a test of linear functions and slope respond to a novel task by treating slope as a difference rather than a ratio. Our expectation for transfer in each case may be an indication of our everyday experience of the world having order and regularity: Past experiences carry over from one context to the next. Furthermore, nearly all learning theories presume that prior knowledge influences the comprehension of any new situation (Anderson, 1996; Bereiter, 1995; Booker, 1996; Bransford & McCarrell, 1974; Hatano & Greeno, 1999). For example, according to Dewey's (1938) principle of the continuity of experience, "Every experience both takes up something from those which have

gone before and modifies in some way the quality of those which come after" (p. 34).

On the other hand, transfer has been notoriously illusive to produce consistently in laboratory studies (Barnett & Ceci, 2002; Detterman, 1993; Perkins & Salomon, 1989). Furthermore, there have been numerous critiques of the theoretical and methodological underpinnings of transfer research (Beach, 1999, 2003; Bransford & Schwartz, 1999; Evans, 1998; Greeno, 1997; Gruber, Law, Mandl, & Renkl, 1996; Lave, 1988; Packer, 2001; Tuomi-Gröhn & Engeström, 2003). As a result, some researchers have abandoned transfer as a research construct (Carraher & Schliemann, 2002; Hammer, Elby, Scherr, & Redish, 2005; Laboratory of Comparative Human Cognition [LCHC], 1983), whereas others have developed alternative transfer perspectives (Beach, 1999, 2003; Bransford & Schwartz, 1999; Greeno, Smith, & Moore, 1993; Nemirovsky, 2011; Tuomi-Gröhn & Engeström, 2003; Wagner, 2006, 2010). This article focuses on one such alternative approach—the *actor-oriented transfer perspective* (AOT; Lobato, 2003, 2006, 2008a, 2008b). Among other points, I argue that AOT can be used as a lens to detect instances of the generalization of learning experiences (meaning the expansion of instructional or everyday experiences beyond the conditions of initial learning), even when there is a lack of transfer according to traditional definitions, as is the case in the three opening scenarios.

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The AOT perspective emerged in response to critiques of the mainstream cognitive approach to transfer, which often challenged its epistemological assumptions. However, I take a more pragmatic view that models of transfer are simply *designed objects* (Plomp & Nieveen, 2007), which provide different information for different purposes. Rather than conceiving of a particular perspective as being flawed and in need of replacement, points of compatibility and tension between models of transfer are explored, thus allowing for greater understanding of the contributions to educational research and practice by each perspective. Specifically, I begin by examining the benefits and trade-offs of both mainstream cognitive and actor-oriented perspectives on the transfer of learning, with the goal of fleshing out the tenets of AOT and opening up a space to explore transfer issues that are particularly well suited to an AOT approach. After all, there is no point in presenting an alternative approach if the dominant perspective can be used to satisfactorily explore the broad array of phenomena that interest transfer researchers. The goal of this article is not to supplant one perspective but rather to articulate specific issues that can benefit from an alternative approach. To that end, I draw upon empirical studies from a variety of researchers operating from an AOT perspective to articulate some specific ways in which AOT can afford new insights into understanding: (a) how students interpret transfer situations, (b) the socially situated nature of transfer processes in classrooms, and (c) how contextual-sensitivity can play a productive role in the transfer of learning.

DIFFERENTIATING BETWEEN THE MAINSTREAM COGNITIVE AND ACTOR-ORIENTED PERSPECTIVES ON TRANSFER: POINTS OF COMPATIBILITY AND TENSION

Mainstream Cognitive Perspective on Transfer

By the mainstream cognitive perspective on transfer, I refer broadly to the family of approaches that emerged during the last half of the 20th century as part of the cognitive revolution (offered by a variety of researchers including but not limited to Bassok & Holyoak, 1993; Gentner, 1983, 1989; Nokes, 2009; Novick, 1988; Reed, 1993; Ross, 1984; Singley & Anderson, 1989; Sternberg & Frensch, 1993). Researchers formulated explanations for transfer based on relationships between a learner's mental representations, as opposed to the theories posited by associationists and behaviorists based on environmental similarity and observable stimuli (Royer, Mestre, & Dufresne, 2005). A hallmark of this general approach is a commitment to a cognitive architecture comprising (a) short-term, long-term, and sensory memories; (b) representations as symbolic mental symbol structures that encode, process, and store one's experiences; and (c) a con-

trol mechanism to oversee the retrieval and utilization of information (Bruer, 2001; Ericsson & Simon, 1993).

Under this general umbrella exist different strands of research. One prominent approach is the cognitive descent to Thorndike's (1906) theory of identical elements. In their seminal cognitive account of transfer, Singley and Anderson (1989) explained that they "resurrected Thorndike's theory by redefining his identical elements as the units of declarative and procedural knowledge" (p. 248), thus addressing Thorndike's lack of an explicit representational language, which can allow for the flexible reconstruction of knowledge. A second influential theory is the structure-mapping approach of Gentner and colleagues, developed as an account of analogical reasoning but readily adapted to examine lateral transfer (Gentner, 1983, 1989; Gentner & Kurtz, 2006; Gentner, Loewenstein, & Thompson, 2003; Gentner & Markman, 1997). From this perspective, transfer involves a mapping between mental representations of relations among objects and their attributes in initial learning and transfer situations. Other points of diversity within the mainstream cognitive approach include (a) the use of different subprocesses to explain the occurrence of transfer, such as constraint violation (Ohlsson & Rees, 1991) or analogical systematicity (Markman & Gentner, 2000); (b) a focus on different types of transfer (as summarized in Barnett & Ceci, 2002), and (c) disagreements regarding whether the mental representation of a transfer situation is constructed during engagement with that situation (e.g., Gentner, 1983) or in the initial learning situation (Gick & Holyoak, 1983).

Despite these differences, there are many common features among family members of the mainstream cognitive perspective on transfer. First, the formation of sufficiently abstract representations is a necessary condition for transfer (so that properties and relations can be recognized in both initial and transfer situations), where abstraction is conceived as a process of decontextualization (Fuchs et al., 2003; Gentner, 1983; Reeves & Weisberg, 1994; Singley & Anderson, 1989). Second, explanations for the occurrence of transfer are based on the psychological invariance of symbolic mental representations (Bassok & Holyoak, 1993; Nokes, 2009; Sternberg & Frensch, 1993). Finally, transfer occurs if the representations that people construct of initial learning and transfer situations are identical, overlap, or can be related via mapping (Anderson, Corbett, Koedinger, & Pelletier, 1995; Gentner et al., 2003; Gick & Holyoak, 1983, 1987; Novick, 1988; Reed, 1993).

From a mainstream cognitive perspective, transfer is characterized as "how knowledge acquired from one task or situation can be applied to a different one" (Nokes, 2009, p. 2). From the AOT perspective, transfer is defined as the generalization of learning, which also can be understood as the influence of a learner's prior activities on her activity in novel situations (Lobato, 2008a). The differences between perspectives may not be apparent from these definitions, especially because there are instances in which researchers operating

from the mainstream cognitive perspective have described transfer as the influence of prior learning experiences on attempts to solve problems in new situations (see Marini & Genereux, 1995; Reed, Ernst, & Banerji, 1974). Thus, I explore five dimensions across which the two perspectives differ: (a) the nature of knowing and representing, (b) point of view, (c) what transfers, (d) methods, and (e) goals. For each dimension, I discuss points of contact and tensions as well as the benefits and trade-offs of each perspective.

Nature of Knowing and Representing

Both actor-oriented and mainstream cognitive perspectives on transfer share the view that the basis for transfer is psychological similarity rather than similar features of physical or task environments (à la Thorndike, 1906). However, the AOT perspective places greater emphasis on the interpretative nature of knowing than is present in many studies conducted from a mainstream cognitive perspective. This means that researchers operating from an AOT perspective look for the ways in which students appear to treat transfer situations as instances of something they have already thought about, based on their interpretation and construal of meaning of the activities and events in the initial learning situation. That is, knowing and representing arise as a product of interpretive engagement with the experiential world, through an interaction of prior learning experiences, task and artifactual affordances, discursive interplay with others, and personal goals.

Within the mainstream cognitive perspective on transfer, there appears to be a much closer correspondence between events/objects in the world and mental representations. In principle this relationship is problematized, as evidenced in the following quote from Anderson, Reder, and Simon (2000): “The representational view of mind, as practiced in cognitive psychology, certainly makes no claims that the mind represents the world accurately or completely” (p. 14). Similarly, Gentner and Markman (1997) acknowledged that mental representations are informed by an individual’s goals and prior knowledge. However, after analyzing many empirical studies conducted within both the identical elements and structure-mapping strands, Wagner (2010) concluded that in practicality both models tend to treat representations as unproblematic, “as if situational structure could be directly perceived in the world” (p. 447). This stance is also reflected in other empirical and theoretical papers, for example, by English and Halford (1995), when they stated that “cognitive processes entail operations on mental representations, which are internal mental structures that correspond to the structure of a segment of the world” (p. 21); by Rittle-Johnson, Siegler, and Alibali (2001), when they coded mental presentations of learning and transfer situations as either correct or incorrect; and by Anderson (1996), when he claimed that “declarative knowledge is a fairly direct encoding of things in our environment; procedural knowledge is a fairly direct encoding of observed transformations” (p. 364). A notable

exception is the work by Day and Goldstone (2011), who attempted to disambiguate external similarity from similar mental representations.

These observations are not intended as criticism; rather, the differences between perspectives suggest implications regarding optimal research domains for each perspective. For example, the research venues that can most benefit from the use of an AOT perspective are ones with semantically rich content that is open to a variety of often idiosyncratic ways of comprehending and interpreting. The AOT approach was not developed to address areas that have typically been the focus of research from the mainstream cognitive perspective, such as the transfer of procedural skill (e.g., transfer across text editors, Singley & Anderson, 1989; transfer of LOGO debugging skills, Klahr & Carver, 1988), puzzle-type problems (e.g., Tower of Hanoi/Monster problems, Kotovsky & Fallside, 1989), procedural elements of semantically rich domains (e.g., algebraic skills in word problems, Koedinger & Anderson, 1998), and tasks with a rule-oriented or syntactic focus (e.g., continuing sequences of letters in a pattern, Nokes, 2009).

To illustrate what is meant by focusing on the interpretative nature of knowing within a transfer study, I briefly turn to an example from my previous research with colleagues (Lobato, Ellis, & Muñoz, 2003). The study occurred in the ninth-grade algebra classroom of a teacher using a reform-oriented curricular unit on linear functions. We expected that the development of linear functions as dependency relationships in multiple real-world situations would increase the likelihood that students would successfully negotiate the quantitative complexity of novel transfer situations. Furthermore, the unit spent much longer investigating slope than is typical in traditional algebra classrooms, and it linked informal explorations with the presentation of the slope formula ($m = \frac{y_2 - y_1}{x_2 - x_1}$). Thus, we were surprised when qualitative analysis revealed that the interview participants interpreted the slope of a linear function, not as a *ratio* of the changes in the dependent variable for each 1-unit change in the corresponding independent variable, but incorrectly as a *difference* (in *y*-values, *x*-values, or in the scale of the *x*-axis). For example, in an interview task in which the slope represented the ratio of the amount of water collected from a leaking faucet over time (i.e., 8 oz per hour), a common response was to identify the difference in amounts of water presented in a table of data, without regard for the corresponding difference in time (e.g., using 10 as the slope when 10 oz of water leaked in 1.25 hr rather than 1 hr). We found this surprising because these students had been selected as higher performers in the class and had been able to correctly find the slope of linear functions in unit quizzes prepared by their teacher. Perhaps their understanding was bound to the context of the learning situation.

However, closer examination revealed that all of the students who produced an equation for a given line or table wrote “ $y = \square \pm \square x$ ” (rather than the more standard $y = mx + b$) and referred to the two boxes respectively as the

“starting point” and “what it goes up by.” The fact that this language and these inscriptions were used in class suggested that students were *generalizing* or expanding their instructional experiences beyond the conditions of initial learning. This was confirmed during qualitative analysis of videotaped classroom episodes. Specifically, we identified several features of the classroom practices that regularly directed students’ attention to various differences in a single quantity rather than to the coordination of quantities. In each case, the classroom practices were understandable, despite the unforeseen and undesirable consequences. For example, the teacher regularly used the phrase “goes up by” when talking about slope, perhaps in an effort to connect with students through initial use of more accessible language before moving to formal symbolization. Although the teacher used the phrase to speak about ratios (e.g., “the y ’s go up by 3 when the x ’s go up by 1”), the students apparently interpreted the teacher’s utterances in terms of differences in a single quantity. In sum, this example demonstrates the unexpected connections that learners can make between their personal interpretations of learning experiences and transfer situations.

A critical reader may question the value of identifying generalizing activity that results in incorrect performance. However, the AOT perspective responds to the following challenge by Bransford and Schwarz (1999):

Prevailing theories and methods of measuring transfer work well for studying full-blown expertise, but they represent too blunt an instrument for studying the smaller changes in learning that lead to the development of expertise. New theories and measures of transfer are required. (p. 66)

Novices are likely to demonstrate greater variety in their interpretations of learning environments than experts; thus, making them a desirable object of research from the AOT perspective. Furthermore, I revisit this study later to demonstrate how information from an AOT study can inform revisions to curriculum and pedagogy during iterative cycles of instructional design, leading to improvements in the nature of what is learned and transferred.

Point of View

Central to the AOT perspective is the distinction between an “actor’s” and an “observer’s” point of view. Taking an observer’s point of view entails predetermining the particular strategy, principle, or heuristic that learners need to demonstrate in order for their work on a novel task to count as transfer. It is also in operation when learners perform correctly on tasks that an observer sees as structurally similar to initial learning tasks, and inferences are made that the learner sees the same similarity as the observer. When taking an actor’s point of view, the researcher does not measure transfer against a particular cognitive or behavioral target but rather investigates instances in which the stu-

dents’ prior experiences shaped their activity in the transfer situation, even if the result is non-normative or incorrect performance.

Transfer research from the mainstream cognitive perspective is typically conducted from an observer’s point of view. Consider the study by Bassok and Holyoak (1989) examining analogical transfer across the domains of algebra and physics. In one of the experiments, half of the ninth-grade students were taught formulas for arithmetic progression methods with practice on a variety of algebraic word problems, such as $a_n = a_1 + (n - 1)d$, where a_1 and a_n are the initial and n th terms in the sequence, respectively, and d is the constant difference of successive terms. Similarly, the other half were taught formulas related to constant acceleration with practice on physics problems (i.e., $v_f = v_i + at$, where v_i and v_f are the initial and final velocities of an object traveling in a straight line, a the constant acceleration, and t the time taken to move from the initial to the final state). The physics students were then asked to solve algebra transfer tasks (such as the tree diagram task shown below), and the algebra students received physics transfer tasks (such as the acceleration task that follows):

During a laboratory observation period it is found that the diameter of a tree increases the same amount each month. If the diameter was 8 mm at the beginning of the first month, and 56 mm at the end of the 24th month, by how much does the diameter increase each month?

What is the acceleration (= increase in speed each second) of a racing car if its speed increased uniformly from 44 meters per second (44 m/s) at the beginning of the first second, to 55 m/s at the end of the 11th second? (p. 156)

The measure of transfer was “whether the learned method had been applied to structurally isomorphic but unfamiliar problems” (p. 157), meaning that transfer was dependent on explicit evidence of the formula and notation taught in class. Using this standard, the algebra students successfully mapped the arithmetic-progression methods onto the physics problems 72% of the time. Additional verbal protocol evidence supported the claim that the algebra students spontaneously recognized that the kinematics problems could be addressed using the arithmetic-progression formulas. In contrast, the physics students used the constant acceleration formulas on only 10% of the algebra problems. The researchers concluded that arithmetic-progression procedures transferred better than the kinematics procedures, due to the greater content specificity of the physics representations.

One benefit of the observer’s point of view is the emergence of a yardstick to illuminate differences in reasoning. In this study, it helped distinguish the effect of training differences in learners’ ability to map equations across domains. The observer’s point of view can also be used effectively

in the summative assessment of an instructional treatment. A trade-off is that the generalization of learning can be underestimated. For example, the physics students in the Basok and Holyoak (1989) study did, in fact, solve 94% of algebra transfer tasks correctly (despite using the taught equations and notations on only 10% of the tasks). Their use of the same methods to solve both the algebra transfer tasks and the pretest items suggests that they were generalizing but that they generalized some experiences gained prior to the study rather than generalizing their experiences with the targeted techniques. A second trade-off is that dimensions related to learners' comprehension of situations can be overlooked as the basis for task isomorphism. For example, although the two transfer tasks presented above are isomorphic along the dimension of mapping values of terms in the same positions in two sequences, they are not isomorphic along the dimension of measurable attributes to be conceived. Using the physics formula to solve the tree diagram task entails mapping an *extensive quantity* (a directly measurable quantity—the tree diameter) onto a *first order ratio* (the ratio of two extensive quantities, here, the ratio of distance to time, or velocity) and a *first order ratio* (the growth rate of diameter) onto a *second order ratio* (the ratio of a ratio to an extensive quantity, here the ratio of velocity to time, or acceleration; J. L. Schwartz, 1988). Thus, according to the dimension of measurable attributes, the tree problem is isomorphic to a constant velocity rather than a constant acceleration problem. Research suggests that forming these different relationships among measurable attributes of a situation is a central yet challenging aspect of understanding this domain for novices (Lobato, Hohensee, Rhodehamel, & Diamond, 2012; Stroup, 2002; Thompson, 1994).

The definition of transfer from an AOT perspective as the generalization of learning signifies a research interest in the expansion of experiences beyond the conditions of initial learning, rather than the formation of particular highly valued generalizations alone. Both represent legitimate but somewhat different avenues of research. To further illustrate the differences, consider a brief example of data analyzed from both observer and actor points of view. The data were collected from exams given to 139 high school introductory algebra students participating in a 6-week unit on slope and linear functions (Lobato, 1996). The students performed well on tasks encountered in the experimental curriculum, such as finding the slope of staircases (87% correct) and lines (80% correct). The transfer tasks asked students to find the slope of a playground slide and the roof of a house. In each case, the slope can be determined by identifying and then measuring the “rise” or vertical change of the object (by fitting a staircase or stairstep to the object—a method taught in class), identifying and measuring the “run” or horizontal change of the object, and then dividing the rise by the run. Strong transfer findings were predicted, in part because the situation aligned with Singley and Anderson's (1989) finding

of nearly total transfer for related-rates calculus problems given the conditions that the tasks shared the same production rules and solution methods and the students had extensive practice. Contrary to expectations, transfer was poor—40% to the slide task and 33% to the roof task.

Follow-up interviews using an AOT perspective presented a different picture, namely, that all interview participants demonstrated evidence of the generalization of their learning experiences. While working on the playground slide task (see Figure 1a), the students correctly recalled the slope formula as “rise divided by run” and treated the slope formula as relevant in the novel situation. However, they made incorrect rise and run choices. Jarek's response is particularly striking because the rise and run seem disconnected from the part of the apparatus that is steep (see Figure 1b). However, his interpretation of experiences with staircases during instruction was conceptually related to his reasoning on the transfer task. Specifically he appeared to look for a stair step in the slide setting (e.g., an object with connected “up” and “over” components that visually affords climbing in an imagined state of affairs), which he found on the right side of the slide (Figure 1c). The platform as the run may have held appeal because it was the only visible “tread” or “over” affordance. In sum, taking an actor's point of view helped illuminate ways in which learners' unanticipated interpretations of instructional experiences were connected to their comprehension of transfer situations. Furthermore, it helped identify elements of mathematical understanding (e.g., the constraint that mathematical “staircases” need to be connected to the part of the object that is steep), which can remain implicit in an expert model until their absence is surprisingly demonstrated in student work.

By revealing unexpected ways in which people generalize their learning experiences, the use of an actor's point of view can help guard against conclusions that reasoning is hopelessly context-bound or that transfer failures “are an inevitable consequence of the limited power and generality of human knowledge” (Singley & Anderson, 1989, p. 2; see also Detterman, 1993; Hatano, 1996; Perkins & Salomon, 1989). One trade-off of the AOT perspective is the time-intensive nature of the qualitative data analysis (especially because *what* is transferring is typically not known in advance) and the need of a research design that affords access to the influencing experience. Furthermore, a critical reader may interpret the slope study as simply juxtaposing novice with expert representations, which has been addressed from a mainstream cognitive perspective. For example, Novick (1988) found that when training and transfer tasks shared structural features but not surface features, experts were more likely to use the training-taught procedure to solve the transfer problems than novices were. On the other hand, when the two problems shared surface but not structural features, novices were more likely to continue using the taught procedure inappropriately (resulting in negative transfer). By similarly adopting an observer's perspective to interpret the

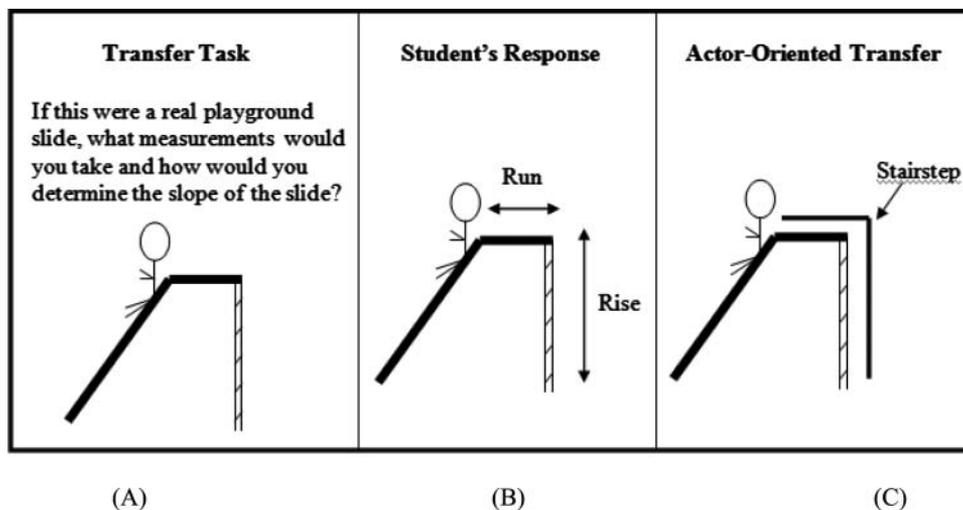


FIGURE 1 From Lobato's (1996) study: (a) the Transfer Task, (b) one student's response, which identified the *rise* in $slope = rise/run$ as the height of the ladder and the *run* as the length of the platform, and (c) possible image connecting transfer and learning tasks from the student's point of view.

results of the slope study, one can conclude that the students used a taught procedure ($slope = rise/run$) inappropriately. However, this leaves unexplored the issue that the students' interpretation of the instruction did not match what was intended. Thus, the actor's point of view allows an investigation of the particular ways in which students interpret the meaning of slope, staircases, steepness, and so on. This opens up the issue of *what* transfers—a strategy versus one's comprehension of a situation—which I address next.

What Transfers?

In their review of transfer research during the 20th century, Singley and Anderson (1989) concluded that there is little evidence for the transfer of general problem-solving faculties across a broad range of domains. Instead, much of the recent research from both mainstream cognitive and AOT perspectives has focused on the transfer of specific content knowledge. However, there is one important distinction between the nature of knowledge studied in mainstream cognitive accounts (particularly the common elements approach) and AOT, namely, the transfer of well-defined actions and strategies versus a more holistic conceptualization. This distinction is elaborated through the following example.

Thompson (2011) interpreted a case study from his own research (Thompson, 1994) from both mainstream cognitive and AOT perspectives. The study began with a sixth-grader called JJ answering questions such as the following, "How much time will it take Turtle (a computer character) to travel 200 cm if he goes 25 cm/sec?" JJ drew successive line segments, each representing 25 cm, until she reached a total of 200 cm and then counted the number of segments—in this case, 8—for an answer of 8 s. She could also answer these questions using division (e.g., with $200 \div 25$). JJ was then

asked questions such as the following, "At what speed must Rabbit (a second computer character) travel so that it will travel 200 cm in 7 sec?" She initially experienced a dilemma because she wanted to make a speed segment but didn't know the length of the segment to draw. Consequently, she adopted a guess-and-test strategy, trying first one "speed" and then another, without the use of division.

According to Thompson, this episode would unlikely be considered an instance of transfer from the cognitive common elements perspective. When transfer involves comparing two productions (condition-action pairs) for different tasks, then the individual performs some well-defined mental or physical operation (often a strategy or calculation in a mathematical context), when the task representation meets particular conditions (Anderson, 1996, 2005; Anderson et al., 1995; Muldner & Conati, 2010; Singley & Anderson, 1989). Thus, Thompson concluded that JJ's work on the second type of task would not count as transfer under this perspective because the student did not use the same solution method.

However, Thompson claimed that this would count as complete transfer from an AOT perspective, where what transferred was JJ's conceptualization of speed, distance, and time, rather than the reuse of a well-defined action. Specifically, speed was not a ratio for JJ but rather a distance (or what Thompson calls a "speed-length"). She comprehended both settings as essentially the same—that of traveling a distance in successive speed-lengths. In the first case, JJ imagined measuring the given distance in units of a speed-length, and the number of speed-lengths contained in the given distance told her the amount of time the character traveled. In the second situation, she again imagined measuring the given distance in units of a speed-length—this time a guessed speed-length guided by an estimate of how many speed-lengths it would take to create the given number of seconds. In a sense,

she was searching for a ruler of the right length by which to measure the to-be-traveled distance. Thus, JJ appeared to comprehend the second situation as being pretty much the same as another that she had already thought about.

A benefit of targeting well-defined actions is that they translate well into the if/then statements of computer programs, which can then be used to build intelligent tutoring systems. The trade-off is that such an approach may not account for an underlying conceptualization that can give rise to multiple strategies or behavioral actions. That said, a reader may wonder, as Reed (2012) did, if the notion of mapping—the formulation of a set of systematic correspondences—could be used to establish commonalities between mainstream cognitive and AOT perspectives. For example, Reed interpreted the slope example with Jarek (from Figure 1) as an instance of a partial mapping (meaning partially successful because the rise but not the run component was correct) from the symbolic representation of the slope formula to the diagrammatic representation of the playground slide. In contrast, from an AOT perspective, I consider Jarek's work—much like JJ's—to indicate complete transfer of his conceptualization of slope. Jarek appeared to comprehend slope situations as linked with staircases, which in turn, brought to mind images of steps, with up and over components that afford climbing.

How one diagnoses the problem—as related to discrete actions, partial mappings, or an underlying conceptualization—has important implications for instructional responses. For example, Reed diagnosed Jarek's problem as failing to construct the auxiliary line segment that would allow a correct mapping to the run component. In response a teacher could present worked examples that include the critical parts of a diagram before asking students to construct them. From an AOT perspective, Jarek's conceptualization was problematic. He and the other students appeared to conceive of slope as two whole numbers—a rise and run value—which were not compared multiplicatively to form a ratio. Furthermore, Jarek's rise choice was correct only in a calculational sense, not a conceptual one, because it was disconnected from the part of the apparatus that was steep. Our subsequent design-based instructional approach focused on isolating the attribute to be measured and constructing slope as a ratio to measure the particular attribute (Lobato & Siebert, 2002; Lobato & Thanheiser, 2002; Olive & Lobato, 2008).

Methods

Singley and Anderson (1989) described the methods often used to establish transfer in both historical and mainstream cognitive approaches. Specifically, subjects are typically taught a solution, response, or principle in an initial learning situation and then solve a transfer task(s). The initial learning and transfer tasks share some structural features (e.g., a common solution approach) but have different surface forms (e.g., different word problem contexts or domain-specific

details). The performance of the experimental group is compared with that of a control group, which is given the transfer tasks but receives no practice on the learning tasks. If the performance of the experimental group on the transfer tasks is better than the control group, then transfer is said to occur.

Some researchers have made adaptations to this basic approach by using multiple measures to capture the transfer of learning. For example, Chen and Klahr (1999) investigated the transfer of a “control of variables” strategy to design unconfounded experiments by using transfer tasks set in two contexts, a “strategy similarity awareness” measure, and a delayed remote transfer measure. Other studies have used verbal protocol methods to examine solution procedures (e.g., Bassok & Holyoak, 1989; Gentner, 1989; Nokes, 2009), though, according to Novick (1988), most transfer studies from a mainstream cognitive perspective rely primarily on performance measures. In addition, accounts of transfer found in ACT-R studies (Anderson, 1996, 2005; Koedinger & Terao, 2002; Singley & Anderson, 1989) demonstrate a care for “what” transfers in their articulation of fine-grained production rules (though in practicality, accuracy of performance or time to complete a task is often used, and the object of transfer is inferred). However, the use of a predetermined standard or a cognitive model based on an observer's perspective leaves an opening for more information to be gathered regarding unexpected ways in which people may construe learning and transfer situations as connected.

To provide this type of information, the AOT perspective relies on qualitative methods to identify the nature of students' reasoning in transfer situations and their comprehension of previous learning activities, allowing researchers to identify what transfers from an actor's point of view (Lobato, 2008a). Often inductive codes emerging from the data are used rather than a priori codes (Miles & Huberman, 1994), because the nature of reasoning in the transfer situation and the particular meanings students develop during instruction are often unanticipated. A typical AOT design (e.g., as used in Karakok, 2009, or Lobato & Siebert, 2002) relies on extended, conceptually oriented classroom instruction, followed by the use of transfer tasks in clinical interviews (Ginsburg, 1997), but one could use a series of interviews or examine the use of novel tasks during instruction (e.g., as illustrated by Ellis, 2007, and Sinha et al., 2010). Within a classroom/interview design, conducting preinstructional interviews or relying on instructional settings where participants have limited knowledge of the content to be learned can help isolate the experience that is influencing participants' reasoning on the transfer tasks. Typically the interview data are analyzed using open coding from grounded theory (Strauss & Corbin, 1990) to categorize students' inferred ways of thinking, comprehending, and meaning-making related to the transfer tasks. The classroom data are then analyzed qualitatively to identify any plausible conceptual connections between the students' reasoning on the transfer tasks and the instructional activities.

There are benefits, as well as trade-offs, associated with the methodological approach of each transfer perspective. Specifically, the reliance on transfer as a performance measure allows researchers from a mainstream cognitive perspective to investigate the relationship between transfer and other factors such as motivation, achievement goals, metacognition, and learning disabilities (Belenky & Nokes-Malach, 2012; Brownell, Mellard, & Deshler, 1993; Butterfield & Nelson, 1991; Pugh & Bergin, 2006). On the other hand, performance alone is a limited basis on which to infer an underlying cognitive model, as multiple models can lead to the same performance. This can be offset when qualitative methods are used, but the additional use of the observer's point of view can constrain the generalizing that is captured. On the other hand, the reliance on ethnographic methods in the AOT perspective constrains researchers to small sample sizes and brings with it the associated difficulties in generalizing claims and accounting for selection bias (Sloane & Gorard, 2003). However, a benefit associated with this trade-off is the ability of AOT methods to capture the often unexpected nature of reasoning on transfer tasks, interpretative meanings of learning activities, and personal connections constructed between learning and transfer situations.

When mainstream cognitive transfer studies are grounded in an experimental design, they can capitalize on the logic of stochastic causality to make claims about the effectiveness of both preparatory and learning activities on students' ability to perform on transfer tasks. This type of information may be of greater use to policymakers than the results from AOT approaches regarding the particular nature and quality of individuals' reasoning. In contrast, AOT studies are typically supported by Maxwell's (2004) articulation of a type of scientific explanation that identifies processes that connect events conceptually and that can help explain later events, qualitatively. This approach helps capture explanatory accounts of reasoning over extended periods of time, which can be useful in addressing questions of *how* or *why* something is happening.

Goals

A major goal of mainstream cognitive transfer research is to document the occurrence of transfer (or explain the failure of transfer), which includes investigating the types of knowledge that transfer better, the conditions that promote or hinder transfer, and the instructional methods that support transfer (e.g., Butler, 2010; Butterfield & Nelson, 1991; Chen & Mo, 2004; Gentner et al., 2003; Rittle-Johnson, 2006). In contrast, AOT research assumes that people regularly generalize their learning experiences and finds the lack of transfer from the mainstream cognitive perspective understandable, given the large research base demonstrating that novices rarely make the same connections as experts (Bransford, Brown, & Cocking, 2000). Therefore, the goal of AOT studies is not to obtain transfer (as it is already

assumed to occur) but rather to understand the interpretative nature of the connections that people construct between learning and transfer situations, as well as the socially situated processes that give rise to those connections.

Investigating the nature of how people generalize their learning experiences, even when such generalizing results in incorrect performance, should not be misinterpreted as a lack of interest in the goal of ultimately having students achieve mathematical correctness or expertise. An important aim of many AOT studies is to improve the nature of students' generalizing activity. Therefore, AOT is often situated within design-based research, where information regarding how students generalize their learning experiences informs and improves the next cycle of instruction (Kelly, Lesh, & Baek, 2008; Lobato, 2003, 2008a). In fact, mainstream cognitive and AOT perspectives may overlap in the final stages of design-based research when the ultimate goal of the instructional innovation should be met, namely, to support the formation of connections between learning and transfer situations that are more expert in nature. (To see an overlap, qualitative measures would need to be used in the mainstream cognitive approach and the focus would need to be on identifying underlying conceptualizations rather than strategies.) However, in practicality, the goal of research conducted by my colleagues and myself has been to identify increasing levels of sophistication in displays of transfer, much like Minstrell's (2001) facets of students' understanding of physics, where one facet may be indicative of more sophisticated understanding than another, even when both facets represent non-normative or incorrect reasoning. This is because, even after several iterations, we often do not achieve full-blown expertise (perhaps because of limits in the length of instruction or the age of the participants).

To illustrate how AOT research can meet the goal of leading to substantive improvements in both instruction and in the ways students generalize their learning experiences, we revisit the classroom study in which students had generalized slope to novel situations as a difference rather than a ratio (Lobato, Ellis, & Muñoz, 2003). Because the analysis of the transfer interviews revealed that students' conceptualization of slope focused on differences in a single quantity, one goal of the instructional redesign was to necessitate the coordination of two quantities (Lobato, 2005; Lobato, Rhodehamel, & Hohensee, 2012a, 2012b; Lobato & Siebert, 2002). Because the classroom analysis revealed that the use of tables in which the x -values increased by 1 in successive rows focused attention on the y -values, a goal of the instructional revision was to promote multiplicative reasoning between x and y -values with data not presented in unit intervals. Because the classroom analysis suggested that the language of numbers and recursive number patterns (e.g., "goes up by") focused attention on single quantities, the next iteration asked students to speak of measurable attributes (e.g., distance, time, speed) and covarying quantities (e.g., 5 s for 7 cm). These and other principles were intended to direct attention

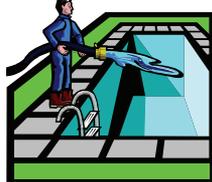
toward covarying quantities and away from single quantities changing.

As a result, a later iteration in the design-based research resulted in more productive generalizing about slope and linear functions (Lobato, Rhodehamel, et al., 2012a, 2012b). The design consisted of an adaptation for younger students—seventh graders—and took place in a context in which it (Class 1) could be compared to another class that addressed the same content goals with a different instructional approach (Class 2). Students from both classes participated in postinstructional clinical interviews (Ginsburg, 1997) using transfer tasks set in contexts not covered in either class. Qualitative analysis revealed distinct differences in how students reasoned with a table of linear data in a water pumping situation (see Figure 2) (Lobato, Rhodehamel, et al., 2012a). In Class 1, 88% of the students coordinated the two quantities in a way that preserved the multiplicative relationship between the quantities and correctly determined the pumping rate (which corresponds to the slope of the function). In contrast, only 33% of students in Class 2 reasoned similarly, with the rest of the students engaging in nonmultiplicative reasoning on the task, including reasoning with differences in only one quantity and reasoning additively.

A critical reader may wonder if we could have shifted to an observer's point of view for this later iteration of the design-based study and achieved the same goal. After all, it was surely a goal of the instruction in Class 1 to coordinate two quantities in a way that preserved the multiplicative relationship between them. In actuality, there had been a more sophisticated goal for instruction in Class 1, namely, to form a ratio as a multiplicative comparison (e.g., Kaput & Maxwell-West, 1994), and this goal would likely have been used as the standard by which to judge whether or not

students' reasoning counted as transfer, from an observer's point of view. For example, in the Pool Task, forming a ratio as a multiplicative comparison entails noticing that the water values are twice as large as the corresponding time values, obtaining 2 as the ratio, and interpreting it in context as 2 gal/min. If we had restricted transfer to this expert goal, we would have missed the way that many students thought about the task. For example, one student used the information from the second and third rows, concluded that 4 gal were pumped in 2 min, formed a unit of these two amounts, halved the unit to produce 2 gal in 1 min, and then built up both amounts to check (2 gal in 1 min, 4 gal in 2 min, 6 gal in 3 min, etc.). Although many researchers call this *pre-ratio* reasoning (Lesh, Post, & Behr, 1988), we believe it demonstrates an advance over the reasoning from the original study and represents a generalization of the students' learning experiences. Thus, using the AOT perspective in our design-based research helped us meet our goals of uncovering the nature of the connections that students made between learning and transfer situations at each iteration and using this information to make incremental and productive changes in how students generalized their learning experiences in each successive iteration.

By differentiating AOT from the mainstream cognitive perspective on transfer along five dimensions, I have argued that the AOT perspective (a) emphasizes the interpretative nature of knowing; (b) operates from an actor's point of view; (c) focuses on the transfer of conceptualizations rather than strategies, solution methods, or well-defined actions; (d) is grounded in the use of inductive qualitative methods; and (e) was developed to explore and iteratively improve the nature of novices' generalization of their learning activities in semantically rich content domains. These features make the perspective well suited for investigating particular aspects



Water is being pumped through a hose into a large swimming pool. The table shows the amount of water in the pool over time. The amount of water is measured in gallons. The time is measured in minutes.

Do you think the water is being pumped equally fast over time or is it being pumped faster at certain times? How do you know? How fast is the water being pumped into the swimming pool?

Time in minutes	Amount of Water in gallons
0	0
3	6
5	10
9	18

FIGURE 2 The Pool Task (color figure available online).

of the broad array of issues and questions that interest transfer researchers—three of which are explored next.

HOW STUDENTS INTERPRET TRANSFER SITUATIONS

Just as taking an actor's perspective entails setting aside a pre-determined standard for judging the occurrence of transfer, the AOT perspective also sets aside observer assumptions regarding the surface/structure distinction. At the heart of this distinction is a presumption that initial learning and transfer situations share a similar level of complexity. However, what constitutes a surface feature for an expert may introduce a structural complexity for the novice, along a dimension that was unforeseen during the task design process. The use of an AOT perspective can foreground students' comprehension of transfer situations as an object of inspection, which in turn can make explicit particular understandings that are implicit in the researcher's own expertise and can provide useful information for an instructional response.

For example, Rebello and colleagues conducted an AOT study to gain insight into the connections that students make between concepts and techniques learned in a calculus class and physics problems that utilize these ideas (Cui, 2006; Cui, Rebello, & Bennett, 2006; Rebello, Cui, Bennett, Zollman, & Ozimek, 2007). To maximize the chances that students would form productive connections, the researchers used straightforward physics tasks and paired each task with an isomorphic calculus problem. However, the physics transfer problems were much more difficult for students than anticipated. In response, the researchers investigated how students experienced the transfer problems. They discovered that students had no trouble carrying out the calculus procedures but found it challenging to decide which variables in the physics situations needed to be integrated or differentiated and to determine the limits of integration. The researchers concluded that what may be conceived, from the perspective of an expert, as a straightforward instance of transfer involving the activation and mapping of new information onto an existing knowledge structure, may in fact involve the creation of new knowledge or knowledge reorganization for students.

The information gained from an investigation of learners' construal of transfer situations can reveal surprising complexities, which can then productively inform an instructional response. For example, when researchers were surprised by the failure of young children to use their counting skills in everyday situations, they used an AOT approach to investigate the children's comprehension of the transfer situations (Hannula & Lehtinen, 2004, 2005; Lehtinen & Hannula, 2006). They discovered that young children often have difficulty structuring the physical world in such a way that the feature of cardinality becomes prominent, especially in naturalistic settings where so many other features compete for their attention (such as the color or shape of objects or the physical

movements of the adults who they are mimicking). Once children's ability to focus on numerosity was identified as crucial, the researchers were able to demonstrate that successful transfer of enumeration skills was related to this propensity (Lehtinen & Hannula, 2006). In addition, subsequent interventions capitalized on the insight that what appears to be an obvious and surface feature for an adult (namely, the ability to isolate and attend to cardinality) is a significant structural feature for young children—one that needs explicit development. Consequently, the researchers designed an effective intervention by training Finnish daycare providers to notice and follow up on the moments when children spontaneously paid attention to numerosity in everyday situations, such as cleaning up or free play (Hannula, Mattinen, & Lehtinen, 2005). The intervention led to a long-term effect on the children's tendency to focus on cardinality and to use their counting skills in new situations.

These studies demonstrate that transfer situations may be isomorphic to initial learning situations along a particular dimension, yet may include a dimension of complexity that is hidden from the view of an expert until one investigates students' understanding of the transfer situations more closely. Partly, this is because what is challenging for students to understand early on in their development of an idea is often no longer apparent to an adult looking through the lens of sophisticated understanding (Simon, 2006). An actor's point of view, along with the use of qualitative analysis of students' reasoning in transfer situations, can help researchers understand what it takes for students to successfully tackle such conceptual complexities.

THE SOCIALLY SITUATED NATURE OF TRANSFER PROCESSES IN CLASSROOMS

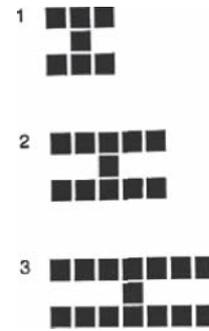
A number of researchers have called for the expansion of transfer processes in order to acknowledge the contribution of social interactions, language, cultural artifacts, and normed practices in the occurrence of transfer (Guberman & Greenfield, 1991; Lave, 1988; Pea, 1989). In response, some researchers have shifted away from attributing transfer to cognitive mechanisms (Beach, 1999, 2003; Bereiter, 1995; LCHC, 1983; Tuomi-Gröhn & Engeström, 2003). However, this puts the field in danger of losing important insights gained from cognitive models of transfer. Consequently, our recent work from the AOT perspective has offered an explanatory account of the occurrence of transfer in a classroom-based study, by coordinating individual cognitive processes with socially situated processes via the construct of "noticing" (Lobato, Rhodehamel, et al., 2012a, 2012b). This is in keeping with the AOT position that transfer is a distributed phenomenon across individual cognition, social interactions, material resources, and normed practices.

To illustrate, we briefly outline our explanatory account of the previously described finding that two classes of

seventh graders reasoned differentially on a transfer task (see Figure 2). Eighty-eight percent of the students in Class 1 coordinated two quantities in a way that preserved the multiplicative relationship, whereas two thirds of the Class 2 students engaged in non-multiplicative reasoning on the task, including reasoning with differences in only one quantity and additive reasoning (Lobato, Rhodehamel, et al., 2012a). To explain why this result occurred, we first analyzed the classroom data to identify what individual students noticed mathematically. By noticing, we do not mean simply “paying attention” but rather the selecting and processing of particular properties, features, or conceptual objects, when multiple sources of information compete for one’s attention. Specifically, students in Class 1 shifted from initially noticing a single quantity to noticing a joined or composed unit of two quantities. In Class 2, students initially noticed differences in a single quantity (the *additive growth* of the function). Then two thirds of the students discovered a relationship between two quantities, which had the potential of becoming a multiplicative relationship. Unfortunately, during the next lesson, students’ attention returned to additive growth and stayed there for the rest of the unit. This is problematic for the development of slope, because slope is multiplicative in nature, not additive.

To understand how these differences in what students noticed mathematically emerged in each class, we examined the role of both students and teachers in the co-constitution of what was noticed through discursive practices (conceived broadly to include gesture, diagrams, and talk). This approach acknowledges Goodwin’s (1994) contention that what people notice “is not a transparent, psychological process, but is instead a socially situated activity” (p. 606). To illustrate the approach, consider the discursive practices that occurred close in time to the shift in noticing back to additive growth in Class 2, as this appeared to be a pivotal event.

The class had been investigating the visual pattern shown in Figure 3. The teacher validated the relationship that two thirds of the students had noticed, namely that the ordinal position of a figure in the pattern (called the “step number”) is the same as the number of squares in each “arm” in the associated figure. As a result, students could determine the number of squares in any figure by multiplying the step number by 4 (the number of “arms”) and adding 3 (the number of squares in the middle of each figure). The teacher demonstrated how students used the step number to calculate the number of squares in the first three figures of the pattern (as shown in Figure 4). For the statement associated with the third figure ($3 \cdot 4 + 3 = 15$), she labeled the *step number*, the *number of arms*, and the *middle* (see Figure 4). In an important move, a student directed attention back to additive growth and to a single quantity by asking why they couldn’t just add $11 + 4$ (from Step 2 to Step 3), as they knew the growth was 4. The teacher validated the student’s idea, and in a crucial move, *renamed* the 4 in “ $3 \cdot 4 + 3 = 15$ ” as the *growth* and wrote “growth” beneath “# of arms.” However, the 4 is not



1. Study the pattern.
2. Build the fourth term.
3. Make a sketch of the fourth and fifth terms.
4. Use the pattern you discovered to sketch the tenth term.
5. How many units make up the tenth term?
6. Describe any number patterns you notice.

FIGURE 3 The Visual Pattern Task.

the growth; rather it represents the number of arms, which does not change. The teacher conjoined these two constructs by saying that “they use arms for growth here . . . every time it’s growing by 4,” consequently bringing attention back to additive growth. This discursive interchange—beginning with the teacher responding to a relationship students had noticed, followed by a student’s attention-focusing response and an emergent *renaming* move from the teacher—signaled a turning point in the unit, shaping what students attended to mathematically in subsequent visual patterns for the remainder of the unit.

This study, along with subsequent research by Hohensee (2011), demonstrates that the particular mathematical features students notice are conceptually connected to the ways in which students transfer their learning experiences. Furthermore, noticing is socially organized by the joint participation of students and teachers in classroom discourse practices. This exploratory work suggests that it is unlikely for a teacher to simply say, “Look here!” and her students will notice what she targets. Instead, there is a system of elements (discourse

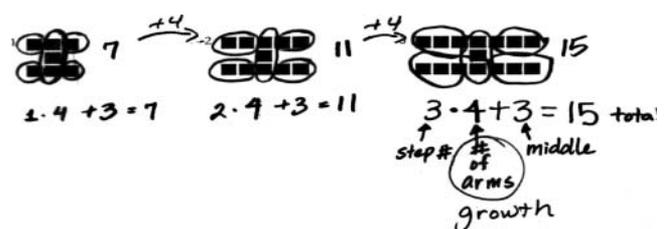


FIGURE 4 The Class 2 teacher’s annotations on figures from the Visual Pattern Task.

practices, mathematical tasks, and the nature of mathematical activity) that work together to bring forth the noticing of particular mathematical features in classrooms.

THE PRODUCTIVE ROLE OF CONTEXTUAL-SENSITIVITY IN TRANSFER

A common theme in the history of transfer research has been that transfer involves some experience of *similarity* or *sameness* across situations. As the locus of such similarity, the mainstream cognitive perspective has emphasized the encoding and recognition of abstract structures that “delete details across exemplars and avoid contextual specificity so that they can be applied to other instances or across situations” (Fuchs et al., 2003, p. 294). The importance of overcoming context is summarized in a report of the National Research Council: “Knowledge that is overly contextualized can reduce transfer; abstract representations of knowledge can help promote transfer” (Bransford et al., 2000, p. 53). Although the transfer of learning may occur via the formation of abstract representations, it need not be the only way in which transfer occurs. Wagner (2010), drawing upon both the AOT perspective and diSessa’s (1993) knowledge-in-pieces perspective, offers an alternative account in which transfer is supported through the incremental growth and organization of smaller elements of knowledge, which are *highly sensitive to context* and are only gradually refined to extend to a widening circle of situations. That is, sensitivity to context—rather than something to be overcome—can play an important role in the transfer of learning.

Specifically, Wagner argues that a concept may have associated with it multiple *concept projections*, which are particular knowledge resources that allow the knower to interpret the situation’s affordances in a meaningful way (diSessa & Wagner, 2005; Wagner, 2010). To illustrate, Wagner (2010) presented a case study of a college student, Jason, who formed two concept projections linked with the concept of the law of large numbers (i.e., the idea that larger samples are more likely than smaller samples to be representative of their parent population). In solving problems across a variety of settings, Jason explained some problems in the language of “more or less well,” revealing one concept projection that was particularly useful in contexts involving people’s physical skill (e.g., skiing or playing squash). In other problems, Jason spoke in terms of “small groups/large groups” and “more or less often,” revealing a second concept projection, which was useful in contexts associated with a statistical interpretation of repeated events (e.g., gender of births in various hospitals or the results of coin tosses). According to Wagner, forming and connecting the two concept projections were the means by which Jason saw the “same thing” across multiple problems. Thus, the case study demonstrates how a single mathematical principle (e.g., the law of large numbers) came to be recog-

nized through a variety of fine-grained interpretive cognitive resources that were influenced by contextual factors.

Wagner’s account of the productive role of context sensitivity in transfer is consistent with the AOT perspective’s emphasis on the interpretative nature of knowledge. From an AOT approach, *structuring* is an active process that occurs through an interaction of contextual affordances, personal goals, and prior learning experiences. *Structuring* is contrasted with the view of extracting a *structure* from a situation, where, as I argued previously, a closer correspondence between the external world and mental structures is often assumed. Relatedly, AOT is rooted in the notion of reflective abstraction (Campbell, 1977/2001; von Glasersfeld, 1990), which is a constructive rather than inductive formulation of abstraction. It focuses on the abstraction of regularities in records of experience in relationship to one’s goals and expectations, rather than on regularities inherent in a situation or the encoding of common properties across instances (Goodson-Espy, 2005).

In sum, one way in which a concept may become more robust and general is due to the abstractness of mental representations, which backgrounds contextual details. In an alternative account, generalizability is supported by the increasing complexity of a concept’s composition and the context-sensitivity of its parts to accommodate new situations (Wagner, 2006, 2010). From the former perspective, comparing multiple examples can promote the extraction of a common structure (Reeves & Weisberg, 1994). From the latter perspective, having more examples may not necessarily help unless they necessitate a new concept projection or help the learner construct connections among concept projections (diSessa & Wagner, 2005).

CONCLUDING REMARKS

According to Campione, Shapiro, and Brown (1995), “it is not clear that a single theory could exist to cover the range of phenomena to which the term [transfer] might be, and has been, applied” (p. 35). In this article, I have argued that the AOT perspective emphasizes the interpretative nature of knowing and the transfer of learners’ underlying conceptualizations, relinquishes a predetermined standard for judging what counts as transfer and draws upon inductive qualitative methods. These characteristics make the perspective well suited for investigating how learners construe meaning in transfer situations, understanding the often unexpected connections learners make between learning and transfer situations and then mining this information to improve instructional responses, accounting for the socially situated nature of transfer processes, and understanding how sensitivity to context can be useful in the generalization of learning. Correspondingly, there are many aspects of the phenomena of transfer for which other perspectives are better matched. For example, from a situated cognitive perspective, Engle,

Lam, Meyer, and Nix (2012/this issue) explore the role of social framing in the transfer of learning in classrooms. From a preparation for future learning perspective, D. Schwartz, Chase, and Bransford (2012/this issue) develop and explore the construct of adaptive transfer. And from the mainstream cognitive perspective, Nokes (2009) proposed a unified theory of how multiple transfer subprocesses (such as constraint violation, analogical reasoning, and knowledge compilation) interact with each other and with particular task conditions.

Viewing these transfer approaches as designed objects that provide different information for different purposes is analogous at a metalevel to the overarching message from the research on transfer-appropriate processing (Morris, Bransford, & Franks, 1977). Countering the accepted view that superficial levels of processing were always inferior to semantic processing, Morris et al. demonstrated that the nature and retention of memory depends not just on the level of processing but on how well the conditions of learning activities match the goals and purposes of the retrieval activities. Similarly, rather than judging any one transfer model in an absolute sense, there is value in differentiating various approaches to gain a better understanding of the features and methods of each approach relative to its goals and purposes.

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