Teaching in ways that support the vision of successful classrooms put forth in national reform documents requires new ways of noticing in classroom settings, with special attention to children's ways of thinking. We define the construct of professional noticing of children's mathematical thinking as a complex set of interrelated skills including (a) identifying, (b) describing, (c) interpreting, and (d) responding. Using a cross-sectional design, we assessed the professional noticing of children’s mathematical thinking for 132 prospective and practicing teachers who differed in terms of the number of years they had engaged in sustained professional development focused on children's mathematical thinking. We found that expertise related to the professional noticing of children's mathematical thinking can be learned, but the development of this expertise takes time. We also found that teasing out the 4 component skills of professional noticing was useful for understanding what expertise entails and for developing snapshots of participants who are in the process of gaining expertise. Implications for professional development efforts are also shared.

Classrooms are complex environments in which teachers cannot be aware of or respond to everything that is occurring. Choices must be made, and as a result, teachers often differ in terms of which aspects of classroom practice are salient, how they talk about and make sense of what they see, and, finally, in what ways they choose to respond. Teachers’ choices and their improvement through professional development are important issues for researchers and educators. In this paper, we discuss these ideas in relation to elementary school teachers of mathematics and their efforts to teach in ways consistent with the vision of successful classrooms put forth by national reform documents (National Council of Teachers of Mathematics [NCTM], 2000; National Research Council [NRC], 2001).

In national reform documents, students' ways of thinking, their flexibility with solution strategies, and their discussion of mathematics are all considered critical. Furthermore, teachers are expected to

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1 This research was supported in part by a grant from the National Science Foundation (ESI0455785). The opinions expressed in this paper do not necessarily reflect the position, policy, or endorsement of the supporting agency.
elicit and respond to students’ ideas in the midst of instruction. This instructional approach has been described in several ways, for example, as an adaptive style of teaching (Sherin, 2002), discovery teaching (Hammer, 1997), and teaching with an inquiry stance (Ball & Cohen, 1999; Cochran-Smith & Lytle, 1999). On the basis of this complex view of teaching, "sizing up students' ideas and responding" have been identified as core activities of teaching (Ball, Lubienski, & Mewborn, 2001, p. 453), but this expertise has proven challenging to develop. In response, the goal for professional development has been shifting from how to help teachers implement particular teaching strategies to how to help them develop new ways of reasoning over extended periods of time (Darling-Hammond & McLaughlin, 1996; Hawley & Valli, 1999; Richardson & Placier, 2001).

We argue that these new ways of reasoning should include new ways of noticing classroom environments, with special attention to children's mathematical thinking. In this paper, we elaborate on what we mean by new ways of noticing by defining the construct of professional noticing of children's mathematical thinking. We then examine teachers' professional noticing of children's mathematical thinking in a cross-sectional sample that provides evidence for the idea that professional noticing expertise can and should be learned. For us, professional noticing of children's mathematical thinking entails more than attending to children’s ideas during other simultaneous classroom events. Teachers not only must detect children’s ideas that are embedded in comments, questions, notations, and actions but must also make sense of what they observe in meaningful ways to use that information in deciding how to respond.

**Children’s Mathematical Thinking**

With their unique ways of making sense of mathematics, children often use problem-solving strategies that differ from adults’ strategies. During the past few decades, mathematics educators have gained substantial knowledge about both children’s mathematical thinking within specific
content domains and the power of teachers’ regularly eliciting and building on children’s thinking (Grouws, 1992; Lester, 2007; NRC, 2001). In response, Schifter (2001) and others have argued that the goal of professional development projects should include helping teachers "learn to attend to the mathematics in what children say and do" (p. 71). In their review, Wilson and Berne (1999) found that professional development based on children’s thinking helped teachers create rich instructional environments that promoted mathematical inquiry and understanding, leading to documented improvement in student achievement. They highlighted as exemplary one particular project, Cognitively Guided Instruction [CGI], a research and professional development project based on the idea that instruction can be improved by providing teachers access to research-based knowledge about children’s thinking and by helping them explore instruction that builds on children’s informal knowledge.

CGI has proven successful (Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Fennema et al., 1996), including in urban classrooms with high minority populations (Jacobs, Franke, Carpenter, Levi, & Battey, in press; Villaseñor & Kepner, 1993). CGI has also provided evidence for the importance of using individual children’s mathematical thinking as the basis for instructional decision making. CGI researchers identified four levels of engagement with children’s mathematical thinking, ranging from minimal engagement to extensive engagement in which children’s thinking served as the basis of instruction (Fennema et al., 1996). At the highest level, teachers used what they learned about individual children’s mathematical thinking to drive their instruction.

Attention to children's mathematical thinking has important benefits for teachers' growth as well. Using children’s mathematical thinking in professional development contexts can motivate and promote growth in teachers’ content knowledge of mathematics. Most elementary school teachers care, fundamentally, about children, not mathematics (Darling-Hammond & Sclan, 1996). Starting with children’s thinking can therefore motivate teachers to engage in mathematical discussions by
helping them realize that to understand and nurture the depth and variety in this thinking, they
themselves must grapple with the mathematics inherent in the strategies that children use to solve
problems. Finally, when professional development has helped teachers learn how to learn from the
thinking of children in their classrooms, research has shown that teachers can continue learning even
after formal professional development support ends (Franke, Carpenter, Levi, & Fennema, 2001).
Thus children’s mathematical thinking can provide a coherent and constant source of professional
development throughout teachers’ careers.

In summary, we recognize that teachers must attend to many features of classrooms such as
classroom management, physical layout of the room, and equity issues. However, because focusing
on children's mathematical thinking has documented benefits for teachers and children, we have
chosen to explore how teachers notice children's thinking in instructional situations.

Professional Noticing of Children's Mathematical Thinking

For many years, psychologists have studied how individuals notice or attend to stimuli in their
environment. More recently, researchers have begun describing the ways professionals notice.
Goodwin (1994) claimed that every profession develops perceptual frameworks that enable members
to view complex situations in particular ways. For example, archeologists develop sensitivities to
variations in color, texture, and consistencies of sand, and attending to these details is a critical
component of their abilities to reason about a landscape. Goodwin used the term *professional vision*
to describe these “socially organized ways of seeing and understanding events that are answerable to
the distinctive interests of a particular social group” (p. 606). Similarly, Stevens and Hall (1998)
described the visual practices characteristic of particular professions (or disciplines) as *disciplined
perception*. One component of disciplined perception is the “capacity of experts to quickly register
perceptual features that are relevant to their particular practice, features invisible at a glance to non-
experts” (p. 109). Stevens and Hall argued, however, that disciplined perception goes beyond
registering relevant perceptual features to also include the ability to coordinate aspects of a situation in meaningful ways. Finally, Mason (2002) focused on the idea of intentional noticing, contrasting this type of noticing that is characteristic of a profession with everyday noticing (what everyone does). He argued that the heightened sensitivities necessary for intentional noticing are reflective of professional expertise, and individuals with this expertise can move beyond rote reactions so that they instead respond by reasoning from professionally meaningful aspects of situations.

Drawing on these constructs, we use the term professional noticing to refer to how professionals view and make sense of complex situations, and we argue that learning to notice in particular ways is part of the development of expertise in a profession. For us, this expertise reflects a set of four interrelated skills: (a) identifying, (b) describing, (c) interpreting, and (d) responding. In the following sections, we describe each skill in relation to the professional noticing of children's mathematical thinking by elementary school teachers of mathematics.

Skill 1: Identifying Noteworthy Aspects of Instructional Situations

Terms such as highlighting (Goodwin, 1994) or making call-outs (Frederiksen, Sipusic, Sherin, & Wolfe, 1998) have been used to describe how professionals identify noteworthy aspects of complex situations. Researchers have also found that the aspects experts identify differ from those identified by novices. Because experts have developed sensitivity to meaningful patterns of information, they focus their attention on the underlying constructs of a situation rather than being distracted by surface features. They can also chunk information in meaningful ways that can assist recall (NRC, 2000). In short, professional noticing involves identifying noteworthy aspects of a situation, and experts are more likely than novices to focus on and remember aspects that are relevant to future decision making.

Van Es (2004) investigated the development of elementary school teachers’ noticing in the context of a video club. She found that over time teachers in the video club identified more aspects
of classroom practice important to reform mathematics teaching. Specifically, over the course of the professional development, teachers’ comments became more specific and more focused on students (instead of on teachers) and on mathematical thinking (instead of on classroom climate or pedagogy).

In summary, we are interested in what aspects of instructional situations teachers identify as noteworthy and how they talk about those aspects.

**Skill 2: Describing Instructional Situations**

No description of an event or a situation is entirely objective, but researchers have argued that professionals benefit by developing the ability to separate their descriptions of situations from their interpretations. Mason (2002) distinguished an *accounting-of* a situation (description) from an *accounting-for* (interpretation of) a situation. He argued that description should be as objective as possible because in delaying subjective comments, one maintains the complexity of a situation and leaves open the possibility of multiple interpretations for the same situation. In addition, individuals focused on describing are more likely to attend to critical details of a situation than are individuals focused on interpretation.

Blythe and her colleagues (Blythe, Allen, & Powell, 1999) also underscored the distinction between description and interpretation in a study of professional development involving the analysis of written student work. They cautioned that teachers might initially consider conversations centered on description to be trivial, even though these conversations can help teachers develop sensitivity to the complexity of children’s thinking. Rodgers (2002) similarly learned that teachers found description to be the hardest part of the reflective cycle she designed to help teachers better attend and respond to students’ learning. Specifically, she asked teachers to slow their thinking and describe, in detail as an artist might, particular situations within classroom practice. Only after completing this description phase were teachers encouraged to consider the meaning of what they saw and the ways they might respond. Finally, Sherin (2001) compared the professional vision of
teachers with that of researchers, for whom description is central. She found that teachers tended to immediately consider their next move and that helping teachers adopt some elements of researchers’ attention to details and description resulted in instructional changes consistent with the goals of reform.

In summary, we are interested in teachers’ abilities to describe instructional situations. Of particular interest is their expertise in describing, in detail, children's strategies.

Skill 3: Interpreting Instructional Situations

In addition to description of instructional situations, professional noticing requires interpretation. When interpreting, individuals must make sense of what they see by linking it to other relevant knowledge. Mason (2002) captured this idea of explaining and theorizing about what was seen with his notion of accounting-for a situation.² Similarly, Rodgers (2002) argued that teachers need to learn to interpret classroom situations, support their interpretations with evidence, and generate multiple ways to interpret the same evidence. Van Es and Sherin (2002) encouraged interpretation by engaging prospective teachers with software specifically designed to foster interpretation based on evidence, and they found that prospective teachers who used the software became substantially more interpretive in their analyses of classroom video than a control group.

In summary, we are interested in teachers’ abilities to make sense of instructional situations. Of particular interest is how teachers use children's thinking to determine what children do and do not understand.

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² Mason (2002) also included evaluation as a possible component of accounting-for a situation, but he distinguished productive forms of accounting-for (e.g., detailed analyses of what was observed) with unproductive forms (e.g., snap evaluations based on minimal evidence). Similarly, Seidel (1998) argued that evaluation can be done thoughtfully, on the basis of evidence, or quickly, almost as an automatic reaction regardless of whether supporting evidence exists. We have chosen to exclude snap evaluations from our construct of interpretation, thereby focusing only on productive forms of interpretation. Our decision to separate interpretation and evaluation is consistent with the work of other researchers (Blythe et al., 1999; van Es, 2003; 2004; van Es & Sherin, 2002).
**Skill 4: Responding in Instructional Situations**

Sassi (2001), drawing on the Aristotelian idea of *practical judgment*, argued that “learning to deliberate about the actions one should take is inseparable from cultivating perception of the salient features of one’s situation” (p. 15). In other words, what one notices cannot be separated from one’s goal for noticing. Thus professional noticing involves an act of responding.

Some may argue that responding should occur after noticing but we feel that it is an integral part of professional noticing of children's mathematical thinking. Specifically, a teacher’s goal in attending to and making sense of children's thinking is to use that information to make effective instructional responses. Because of the nature of teachers' work, these responses must generally be made quickly, often, and in the midst of instruction. Thus we argue that responding should be considered a component skill of professional noticing of children's mathematical thinking because identifying, describing, interpreting, and responding are all interrelated.

**Investigation of Professional Noticing of Children's Mathematical Thinking**

A premise of our study is that professional noticing of children's mathematical thinking is a construct worthy of attention by teachers, professional developers, and researchers. We were specifically interested in documenting the professional noticing of children's mathematical thinking at the elementary school level, including the development of this expertise over time. In this paper, we share results from a cross-sectional study in which we compared the professional noticing of children's mathematical thinking of prospective and practicing elementary school teachers. The data are drawn from a larger study entitled "Studying Teachers’ Evolving Perspectives" (STEP), in which our overall goal is to map a trajectory for the changing needs and perspectives of teachers engaged in sustained professional development.
METHODS

In our cross-sectional study, we explored the professional noticing of 132 prospective and practicing teachers. Professional noticing was measured with a written assessment that was designed to capture reactions to a classroom video clip along the four dimensions of identifying, describing, interpreting, and responding. We compared the reactions of four groups of participants who differed in terms of the number of years they had engaged in sustained professional development focused on children's mathematical thinking.

Participants

We used a cross-sectional design to investigate the professional noticing of three groups of practicing teachers and a group of prospective teachers who were just beginning their studies to become elementary school teachers (see Table 1).

Table 1

<table>
<thead>
<tr>
<th>Participant Groups</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prospective Teachers</td>
<td>Undergraduates enrolled in a first mathematics-for-teachers content course</td>
</tr>
<tr>
<td>Practicing Teachers</td>
<td></td>
</tr>
<tr>
<td>Initial Participants</td>
<td>Practicing elementary school teachers who were about to begin sustained professional development focused on children’s mathematical thinking</td>
</tr>
<tr>
<td>Advancing Participants</td>
<td>Practicing elementary school teachers who had been engaged with sustained professional development focused on children’s mathematical thinking for 2 years</td>
</tr>
<tr>
<td>Emerging Teacher Leaders</td>
<td>Practicing elementary school teachers who had been engaged with sustained professional development focused on children’s mathematical thinking for at least 4 years and who were beginning to engage in formal or informal leadership activities that supported the development of other teachers (e.g., inviting teachers into their classrooms, sharing resources, etc.)</td>
</tr>
</tbody>
</table>

A major decision we made in our study was to group teachers according to the length of their engagement in sustained professional development focused on children’s thinking instead of by the
number of years they had been teaching. We recognize that many important aspects of teaching improve with experience, but we argue that teaching experience alone is generally insufficient for teachers to develop the type of expertise necessary to teach mathematics in ways consistent with current recommendations (NCTM, 2000; NRC, 2001). Instead, teachers need opportunities to participate in sustained professional development, and one of our goals is to better understand the evolution of professional noticing expertise by considering the expertise reflected in our cross-sectional groups.

We worked with 132 participants: 36 Prospective Teachers, 32 Initial Participants, 31 Advancing Participants, and 33 Emerging Teacher Leaders. The teacher groups were all relatively experienced; only 3 teachers had less than 5 years of teaching experience and the mean number of years of teaching was 15 years (ranging from 2–31 years) for the Initial Participants, 15 years (ranging from 5–33 years) for the Advancing Participants, and 16 years (ranging from 4–30 years) for the Emerging Teachers Leaders. Consistent with the elementary school teacher population, our participants were overwhelmingly female (120 females and 12 males).

Prospective Teachers were volunteers enrolled in a first mathematics-for-teachers content course at a large urban comprehensive university in Southern California. Practicing teachers were volunteers who met our cross-sectional criteria: (a) enrolled but not yet begun professional development (Initial Participants), (b) completed 2 years of professional development (Advancing Participants), and (c) completed at least 4 years of professional development and begun to look toward supporting the growth of other teachers (Emerging Teacher Leaders). Participants also needed to have at least 2 years of teaching experience as a classroom teacher at a K–3 level.

Practicing teachers were drawn from 3 districts in Southern California that were similar in demographics, with 1/3 to 1/2 of the students classified as Hispanic, about 1/4 classified as English Language Learners, and 1/4 to 1/2 receiving free or reduced-cost lunch. The majority of the teachers
were drawn from one district in which professional development efforts specifically focused on children's mathematical thinking had been underway for 5 years. The professional development sessions were all facilitated by the same experienced mathematics program specialist, and the professional development was delivered in a cohort model such that teachers who started together in one year continued working with the same group of teachers for multiple years.

**Professional Development**

The Advancing Participants and Emerging Teacher Leaders were all volunteer participants in sustained professional development focused on children's mathematical thinking. Participants engaged in professional development for an equivalent of about 5 full-days per year (in either half- or full-day increments). This professional development drew heavily from the research and professional development project Cognitively Guided Instruction [CGI] (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, & Levi, 2003). The overarching goals were to help teachers learn how children think about and develop understandings in particular mathematical domains and how teachers can elicit and respond to children’s ideas in ways that support those understandings.

Children's mathematical thinking served as the focus for interactions in the professional development. The conversations were informed by research on children’s thinking, and the teachers worked together to make explicit the similarities and differences in children's strategies. Teachers were given opportunities to recognize the power of attending to the subtle details in children's strategies—details that reflected mathematically relevant differences in the understandings children bring to their problem solutions. Teachers also engaged in conversations about how mathematical tasks, classroom interactions, and classroom norms could be used to support and extend children's understandings within particular mathematical domains. Special attention was paid to what children could do rather than to what they could not do. Teachers discussed how to reconceptualize children's
struggles as opportunities to identify understandings (not misunderstandings) children held and then questions teachers could pose to elicit and build on those understandings.

During professional development sessions, teachers engaged by solving mathematics problems, reading research, and analyzing video and written student work derived from their own classrooms as well as from artifacts provided by the facilitator. At times, teachers also had opportunities to work collaboratively with other teachers while conducting problem-solving interviews with individual children. Between professional development sessions, teachers were asked to pose problems to their students and to bring the written student work to the next session. With the help of their colleagues, they then worked to make sense of their students' thinking in ways that highlighted core mathematical ideas and children's understandings related to those ideas.

**Measures**

We developed a written assessment to capture participants' professional noticing of children's mathematical thinking. Participants were asked to watch a classroom video clip and then respond, in writing, to sequenced prompts about identifying, describing, interpreting, and responding.

**Lunch-Count Video Clip**

The participants watched an edited 9-minute video clip of a 40-minute lesson taught in February in a combination class with Grades 1 and 2. This lesson was selected because of its complexity. There were on-task and off-task behaviors, extensive teacher questioning, and sharing of various strategies. Additionally, a child (rather than the teacher) posed the problem for all children to solve. Overall, this lesson included many characteristics of instruction recommended in reform documents.

Specifically, in the video clip presented to the participants, a child posed the problem “We have 19 children, and 7 are hot lunch. How many are cold lunch?” The teacher then allowed the children

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3 *Hot lunch* was the classroom terminology used to refer to students who were buying their lunches, and *cold lunch* referred to students who had brought their lunches that day.
to work on the problem individually or in pairs, solving the problem in any way that made sense to them. A variety of tools were available. While the children were solving the problem, the classroom was noisy. When the children came together to share their solutions, the teacher reminded them to set their tools down and listen to their classmates. Three strategies were shared. One pair of children shared that they counted back 7 from 19 on their fingers. Another pair of children drew 19 tally marks and erased 7 to find the number of cold lunches. The third strategy was shared by Sunny, a boy who used a counting frame (10 rods with 10 beads on each rod) and built 19 beads by twos before removing 7. During the sharing, the teacher questioned the children about their reasoning and corrected some off-task behavior. All children in the video clip arrived at a correct answer. (A partial transcript of the video clip in Appendix A includes a more detailed description of the children's strategies and the teacher's questioning.)

Sequenced Prompts

After viewing the video clip once (additional viewings were not permitted), participants were asked to write in response to the following prompts:

1. Everyone views instructional situations differently. What 3 aspects of this video did you find noteworthy?
   1)  
   2)  
   3)  

2. Pretend that you are the teacher of these children. What problem or problems might you pose next?  
   (We are interested in how you think about selecting next problems, but we do not believe that there is ever a single best problem, and we also recognize that as the teacher of these children you would have additional information to inform your selection.)
   Problem or Problems:
   Rationale:

Participants could take as much time and writing space as they needed to respond to these prompts. The first prompt was specifically designed to assess participants' approaches to identifying, and the
second prompt was designed to assess how they approached responding. After completing (and submitting) their written work, participants were asked to write in response to the following prompts:

1. Please describe in detail what the children said and did in response to this problem. (We recognize that you had the opportunity to view this video only one time, so please just do the best you can.)
   - Pair 1:
   - Pair 2:
   - Sunny:

2. Please explain what you learned about these children’s understandings.

The first prompt was specifically designed to assess participants' approaches to describing, and the second prompt was designed to assess how they approached interpreting.

We sequenced the tasks in two sets because we did not want the second set of prompts to bias the participants' reasoning about the first set. Specifically, we were interested in what teachers identified as noteworthy and their reasoning about how they might respond in a classroom setting without exposure to the ideas embedded in our second set of prompts.

Connections to Earlier Expert-Novice Studies

In creating this assessment, we extended the methodology used in a series of expert-novice studies conducted in the 1980s (for a summary, see Berliner, 1994). In these studies, researchers used scenario methodology to assess teachers' recall and analyses of classroom situations. Specifically, participants were shown slides or video and were asked, in a structured interview, to recall as much as possible of what they had seen and then to comment on the situation. Of most interest to researchers was the participants’ recall of general pedagogical techniques such as management and organization rather than subject-specific instructional issues. Results about teaching expertise confirmed findings about expertise in other areas.

We extended the methodology of these earlier studies by moving beyond a general pedagogical approach to instead focus on the specific expertise needed to teach mathematics effectively by eliciting and building on children’s mathematical thinking. Furthermore, most earlier studies
compared only two groups: experts versus novices. This approach limits the conclusions that can be drawn about the acquisition of skills (Peterson & Comeaux, 1987). We have included four groups of participants, ranging from Prospective Teachers to Emerging Teacher Leaders, in an attempt to begin to map a trajectory for teachers’ development of expertise related to professional noticing of children's mathematical thinking.

Analyses

Our goal was to characterize expert professional noticing of children's mathematical thinking and to use our cross-sectional design to capture intermediate levels in the development of this expertise. We began our analyses by creating rubrics for the four professional-noticing skills of identifying, describing, interpreting, and responding, with particular attention paid to how and when participants used individual children's thinking in their reasoning. Analysis was iterative: After the data had been blinded, we double coded all data; used discussion to resolve disagreements and revise rubrics; and, when necessary, recoded previously coded data. Interrater reliability for final coding was 80% or higher, and all discrepancies were resolved through discussion.

Chi-square tests of homogeneity were used to compare the professional noticing of our four participant groups, and for all tests, we used a Type I error rate of .05. Post hoc comparisons were used to explore patterns of interest. We were particularly interested in (a) how groups who had participated in sustained professional development focused on children's thinking (Advancing Participants and Emerging Teacher Leaders) compared with those who had not participated (Prospective Teachers and Initial Participants) and (b) whether performance improved with more experience with children's thinking. We considered Prospective Teachers to have the least

4 The Goodman critical value for post hoc comparisons was calculated as the square root of the critical value for the omnibus test (Marascuilo & Serlin, 1988). Specifically, the critical value was 3.55 for post hoc comparisons related to Table 7 and 2.79 for post hoc comparisons related to all other tables.
experience with children's thinking followed by Initial Participants (who had experience from teaching) and then Advancing Participants (who had experience from teaching and 2 years of professional development) and finally, Emerging Teacher Leaders (who had experience from teaching and at least 4 years of professional development).

**FINDINGS**

We examined the professional noticing of children's mathematical thinking across our four participant groups. In this section, we share our findings for each of the component skills of professional noticing: identifying, describing, interpreting, and responding.

**Identifying**

Because many elements of a complex classroom environment are available for comment, we sought to examine what participants identified as noteworthy in the video vignette. Did participants focus on management issues, the mathematical task, the teacher’s questioning, the children’s success, the children’s strategies, or something else? Thus, in our first task, the participants watched the Lunch Count video clip and were asked, "What 3 aspects of this video did you find noteworthy?" Space was explicitly provided for the participants to identify 3 aspects.

**Topics Identified As Noteworthy**

We grouped the aspects participants identified according to 12 topics, including a category of *Other* (see Table 2 for a list of the topics, their frequencies, and sample responses).

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5 The *Other* category included topics that fewer than 5% of the teachers identified as noteworthy.
### Table 2

**Topics Identified as Noteworthy by Participants**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Frequency</th>
<th>Example</th>
</tr>
</thead>
</table>
| Multiple student-generated strategies | 77%       | *Example 1.* Students were given a chance to show multiple strategies. (#8761)*  
|                                    |           | *Example 2.* Each child was able to choose from a variety of tools in order to solve the problem. (#2384)* |
| Teacher’s questioning              | 34%       | The teacher questioned the students about their answer and the operation. She encouraged them by her repeated question to tell why adding didn’t make sense. (#4711) |
| Student-authored problem           | 32%       | A student generated the problem to be solved and it was something familiar that the children could relate to. (#4711) |
| Sharing of students' strategies    | 30%       | I found it noteworthy that the teacher made it a point to give each child their own time in the spotlight to explain how they understood the problem and the process to find the answer. (#4222) |
| Complexities of classroom management | 25%       | I thought the aspect of management during the problem solving was noteworthy. The viewer is able to see the difficulty of keeping the students focused when they have tools in front of them to play with during sharing. It can be distracting. (#5273) |
| Opportunities to work with a partner | 15%       | Some students worked as partners to share their strategies. This may have been to boost self-esteem or to obtain a response for a more reluctant participant. (#3281) |
| Flow of the classroom and set-up of the problem | 14%       | The teacher repeated the question and asked students to say the question. She provided time for students to work together or individually to solve problems. The teacher was flexible and provided opportunities for students to work using their best learning styles. (#8870) |
| Specific aspects of the students' strategies | 11%       | I particularly noted the student who reasoned that 26 would not make sense because there were only 19 students in the class. She really understood what the problem asked. (#5456) |
| Students' understanding of the task | 8%        | The kid in red who explained what the problem was. It was clear that he, as well as the rest of the kids, completely understood what the task (problem) was. The teacher did a good job. (#5142) |
| Students' affect                   | 7%        | I found it noteworthy that the children seemed eager to solve problems/to share their thinking. (#6209) |
| Realistic video                    | 6%        | It was a real classroom with real situations. All too often we are shown videos and the class is too good to believe. This one had a space problem (most rooms do) [and] a problem with students not listening (some [students] don’t listen). It felt real. (#2846) |
| Other                              | 10%       | ‘Consensus’ about the answer was a good way to put it. (#3808) |

*Note.* The percentages indicate the number of participants who identified a topic as one of their 3 noteworthy aspects. Because some participants identified 2 (and at times 3) of their noteworthy aspects related to the same topic, the percentages do not sum to 300%.
The topic mentioned by the largest percentage of participants (79%) was the idea that this classroom depicted a variety of student-generated strategies. Participants' responses with this focus included comments about multiple ways to solve the problem, allowing children to choose their own strategies or tools, and not seeing the teacher provide a particular strategy or answer. We hypothesize two reasons this topic was identified so frequently: First, the majority of the video was devoted to showing children solving the problem in different ways and talking about their strategies. Second, many participants may have had little experience with classrooms in which children solve problems in multiple ways or choose their own tools.

Identification of atypical classroom practices was also prevalent in other topics identified in the participants' responses. For example, about one third of the participants found noteworthy that the child created the problem from a daily routine, in this case, the lunch count. One participant’s response captured a common sentiment in reflecting on the difference between a student-generated problem and teachers’ typical routines: “The students were offered the chance to pose a problem from data collected regarding lunches. Often times we pose the problem to be shared without kids’ input”(#6902).

Other atypical practices that are now being recommended in reform documents and were depicted in the video clip included children explaining their strategies and discussing their reasoning. Many participants identified as noteworthy the discussion of students' strategies and the teacher's role in making that discussion possible. About one third of all participants identified as noteworthy just the idea that the children were encouraged to share their strategies. Furthermore, in the responses addressing the complexity of classroom management, offered by one fourth of participants, many acknowledged the challenges of keeping children focused while other students share their strategies. Finally, the responses about teacher's questioning, a topic mentioned by about one third of the participants, often highlighted how the teacher posed questions to press her students for justification.
in ways that went beyond checking and reviewing (more typical classroom-questioning practices). We hypothesize that sharing strategies and pressing students for justification were identified as noteworthy because they may be opportunities that the teachers rarely provided their students and that the Prospective Teachers may not have experienced as K–3 students.

Other topics were mentioned with less frequency, but each topic provides insight into participants’ focus when watching classroom video clips. Of particular interest were the comments that reflected appreciation for the realistic nature of the video. Although this comment was offered by fewer than 10% of the participants, we recognize that this sentiment may arise from participants’ experience with a lack of video clips with an authentic feel. Mathematics educators, in a quest to use video to make particular points (about a child’s thinking, a teacher’s set up of a problem, and so on), may unwittingly present sterile classroom environments that seem unrealistic.

In addition to tracking which topics participants identified, we were interested in whether participant groups identified topics with different frequencies. For topics mentioned by more than 20% of the participants, we compared the distribution across participant groups with chi-square tests of homogeneity. Significant differences were found for the topics of multiple student-generated strategies ($\chi^2(3, N = 132) = 16.61, p < .05$) and teacher's questioning ($\chi^2(3, N = 132) = 14.70, p < .05$) but not for student-authored problem, sharing of students' strategies, or the complexities of classroom management. In the following sections, we explore the topics that showed differences across participant groups.

**Multiple student-generated strategies.** More than three fourths of all participants identified the children's generation of multiple strategies as noteworthy, and we found differences among participant groups. Post hoc comparisons indicated that the percentage of individuals who mentioned this topic was significantly higher in the participant groups who had not yet begun professional development than in the groups who had participated ($Z = 4.14, p < .05$), and in fact there was a
monotonic decreasing trend across all four participant groups ($Z = -3.79$, $p<.05$). Specifically, almost all the Prospective Teachers (89%) and Initial Participants (94%) mentioned this topic whereas some, but fewer, Advancing Participants (68%) and Emerging Teacher Leaders (58%) did so. We hypothesize that a smaller percentage of the Advancing Participants and Emerging Teacher Leaders addressed this topic because many already provide opportunities for students to generate multiple strategies in their classrooms. Thus seeing multiple strategies or a choice of tools was less noteworthy to them than to participants in the other two groups in which these practices may be more atypical.

**Teacher's questioning.** About one third of all participants identified the teacher's questioning as noteworthy, but this topic was most salient for the Emerging Teacher Leaders. Post hoc comparisons indicated that the percentage of Emerging Teacher Leaders (61%) who identified this topic as noteworthy was significantly higher than the percentage (25%) in the other three participant groups combined ($Z = 3.70$, $p < .05$). We suspect that the Emerging Teacher Leaders identified the teacher’s questioning as noteworthy because they recognized the challenges of crafting questions in response to their students’ reasoning, and they appreciated this teacher’s expertise in questioning.

**Describing**

Professional noticing of children's mathematical thinking requires the ability to accurately describe the details of children's strategies. We examined whether the ability to describe was consistent across participant groups or instead was a skill learned through teaching or professional development activities. To elicit the level of detail of the children’s strategies participants could provide, we gave the following prompt: "*Please describe in detail what the children said and did in response to this problem.*"

We were interested in whether participants' descriptions tended toward general features of the strategies, for example, stating that the children in Pair 1 used their fingers; that Pair 2 made tally
marks; or that the last boy, Sunny, used a counting frame. In contrast, we noted descriptions that included mathematically significant details about the strategies, such as how the girl in Pair 1 counted back on her fingers (and knew when to stop) and that Sunny initially counted by 2s but later subitized 4 of the beads before counting on by 1s to 12. We compared the attention to strategy details demonstrated by participants at different points in sustained professional development.

**Overall Strategy Descriptions**

We categorized descriptions for each strategy on the basis of whether participants provided most, or only a few, of the strategy details. See Table 3 for sample descriptions for the strategies of Pair 1, Pair 2, and Sunny.

Table 3

*Examples of Participants' Descriptions of the Children’s Strategies*

<table>
<thead>
<tr>
<th>Level of detail</th>
<th>Pair 1</th>
<th>Pair 2</th>
<th>Sunny</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Most details</strong></td>
<td>They wrote out a number sentence: $19 - 7 =$ Then counted backwards using their fingers to end up with 12. 19, [then] 18, 17, 16, 15, 14, 13, 12. She also made a great point that they couldn’t count up because they didn’t have 26 kids. (#9515)</td>
<td>They made 19 individual tally marks in a row. Then, they counted out 7 tally marks and erased them. They counted the remaining tally marks by ones to get 12. (#7505)</td>
<td>He used an abacus and counted by 2s until he got to 18. Then he said if he added two more that would make 20 so he only added 1 more to equal 19. He also said that he knew five 2s make 10. Then he moved 7 beads over or away and counted on starting at 4 and counting what was left up to 12. (#9045)</td>
</tr>
<tr>
<td><strong>Few details</strong></td>
<td>Katie and Sam worked more as a team. They both wrote the problem on the board and both explained it together to the class. (#9529)</td>
<td>Annette and Maureen drew tally marks on the board. Tally marks are fundamental and basic. (#8870)</td>
<td>He used a maths tool. Using counting and a visual aid he was able to solve the problem by counting and then taking away the amount of hot lunches. (#7985)</td>
</tr>
</tbody>
</table>

Responses reflecting most details tended to capture the majority of the features of the strategy with some level of specificity. In contrast, responses reflecting few details tended to omit features of the strategy and were generally written with minimal specificity.
Comparison of Strategy Descriptions Across Participant Groups

We recognize that during any 9-minute video, just as in any live classroom context, participants will be less focused at some times than at others. We thus share only the general patterns of each participant group's responses; instead of sharing the percentage of participants in each group who identified most details for every strategy, we provide the percentage of participants within each group who were able to describe most details on at least two of the three strategies (see Table 4).6

Table 4

Level of Detail for At Least Two of Three Strategies Across Participant Groups

<table>
<thead>
<tr>
<th>Level of detail</th>
<th>Prospective Teachers</th>
<th>Initial Participants</th>
<th>Advancing Participants</th>
<th>Emerging Teacher Leaders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Most details</td>
<td>31%</td>
<td>44%</td>
<td>65%</td>
<td>79%</td>
</tr>
<tr>
<td>Few details</td>
<td>69%</td>
<td>56%</td>
<td>35%</td>
<td>21%</td>
</tr>
</tbody>
</table>

Performance differed significantly across participant groups ($\chi^2 (3, N = 132) = 18.85, p < .05$), and the differences were compelling. Whereas fewer than one third of the Prospective Teachers and fewer than half of the Initial Participants described most details for two of the three strategies, almost two thirds of the Advancing Participants and more than three fourths of the Emerging Teacher Leaders did so. Post hoc comparisons indicated that describing the details of children's strategies is a learned skill. We found that the percentage of individuals who could describe most details was significantly higher in the groups who had participated in professional development than in the groups who had not ($Z = 4.38, p < .05$), and there was a monotonic increasing trend across all four groups ($Z = 4.91, p < .05$), ranging from 31% to 79%.

For the conceptual reason discussed, we chose to share results for participants' level of detail in 2 (of 3) strategies. The same patterns, however, emerged in the data for those able to share most details on all three strategies.

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6 For the conceptual reason discussed, we chose to share results for participants' level of detail in 2 (of 3) strategies. The same patterns, however, emerged in the data for those able to share most details on all three strategies.
Describing Subtle Strategy Details

When analyzing the children's strategies, we recognized that the presentation of some details was more subtle than of others and thus more challenging to remember. We identified three subtle, yet mathematically significant, details in the strategies shared by the children: (a) Katie's (girl in Pair 1) ability to count back using her fingers, beginning with 18 and knowing to end at 12 (because 7 fingers were raised); (b) Sunny's comment that he knew that 5 twos is 10; and (c) Sunny's final counting strategy in which he subitized 4 beads as a group before counting on by 1s to 12. These subtle details occupied little time in the video clip, and the teacher did not highlight them by repeating or discussing the children's reasoning.

In examining participants' descriptions for inclusion of these three subtle strategy details, which, because of their relative obscurity, participants might not include at all, we tracked whether the participants described at least one of the details (see Table 5). Performance differed significantly across participant groups ($\chi^2 (3, N = 132) = 18.39, p < .05$), and, again, the differences were compelling and revealed the same pattern found in other data. Whereas fewer than one fourth of the Prospective Teachers and only a little more than one third of the Initial Participants described at least one of the three subtle details, more than half of the Advancing Participants and almost three fourths of the Emerging Teacher Leaders demonstrated this skill. Post hoc comparisons again revealed higher performance in the groups who had participated in professional development than in the groups who had not ($Z = 4.29, p < .05$), and there was a monotonic increasing trend across all four groups ($Z = 4.79, p < .05$), ranging from 22% to 70%.
Table 5

Description of Subtle Details Across Participant Groups

<table>
<thead>
<tr>
<th>Number of subtle details described</th>
<th>Prospective Teachers</th>
<th>Initial Participants</th>
<th>Advancing Participants</th>
<th>Emerging Teacher Leaders</th>
</tr>
</thead>
<tbody>
<tr>
<td>At least one detail</td>
<td>22%</td>
<td>38%</td>
<td>58%</td>
<td>70%</td>
</tr>
<tr>
<td>No details</td>
<td>78%</td>
<td>63%</td>
<td>42%</td>
<td>30%</td>
</tr>
</tbody>
</table>

On the basis of these findings, we suggest that the ability to attend to and describe children's strategies is an acquired skill that takes time to develop. Our results also corroborate the findings in expert/novice studies in that experts tend to chunk information in ways that significantly enhance later recall (NRC, 2000). Not only did the Advancing Participants and Emerging Teacher Leaders notice more details than the other groups, but they were also more likely to notice the subtle details in children's strategies. Similar to the expert chess players who can re-create meaningful chessboards because their chessboard schemas are rich and well connected (NRC, 2000), they could more easily re-create the mathematical details in children’s strategies, perhaps because their schemas related to children’s strategies were richer and better connected.

Interpreting

Professional noticing of children's mathematical thinking requires not only description of children's strategies but also interpretation of the understandings reflected in those strategies. Therefore, we examined participants' interpretations of the children's understandings in the Lunch Count video. Specifically, after being asked to describe what the children said and did in response to the problem, participants were prompted, "Please explain what you learned about these children's understandings."

We compared the interpretations of participants at different points in sustained professional development in terms of the focus of their analyses, the depth of their analyses, and their general orientations to the task of interpreting. Note that we did not seek a single best interpretation but were instead interested in how participants reasoned about children's understandings.
Focus and Depth of Interpretation

Although we asked participants to analyze the video in terms of children's understandings, we found that only 61% adopted our requested focus and 39% chose alternate foci. For each focus, we share sample responses and the prevalence of that focus in the participant groups.

Analyses focused on children's understandings. Participants who provided analyses focused on children's understandings shared some insight into the children's understandings about the mathematics related to the Lunch Count problem or to mathematical problem solving in general. These responses varied in depth, however, and we identified 17% of responses as elaborated analysis in which the comments were detailed and extensive compared with 43% of responses that reflected more general or nonelaborated analysis. Table 6 provides sample responses of analyses of children's understandings with varying depth.
### Examples of Participants' Analyses Focused on Children's Understandings

<table>
<thead>
<tr>
<th>Analysis of children's understandings</th>
<th>Elaborated analysis of children's understandings</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first set that shared had computational and representational understanding of the problem. They knew what the algorithm would be to solve the word problem. The second set was very one sided. They were at the one-to-one correspondence picture stage and could have easily miscounted or made a computational mistake. The third boy [Sunny] seemed to have very good number sense. He was able to group beads, skip count, and explain his thinking very clearly. I would say that he would be able to solve much more complex problems. (#4665)</td>
<td>The first pair understands the problem is a [subtraction problem] by writing a number sentence that showed (19 - 7 = _). They did not need to count out 19 and take away 7 to get 12. They simply used their fingers to count backwards from 19. They seem to have good number sense. The second pair has a simpler strategy than the first because they have to count out 19 tallies and then take away 7. They still need to make the amount. They can't hold it in their head yet like the first pair. Also they did not group their tallies into 5's which [would] allow them to keep better track of their numbers. The last boy [Sunny] has good number sense and understands different amounts. He was able to count by groups of 2s and switch to a group of 1 to make 19. He then took away 7 and counted what remained. He was able to start with 4 and count on by 1s which shows he has some understanding of amounts. He still needs to make 19 and so I think the 1st pair has the best number sense because they were able to start right at 19 and count down. (#5273)</td>
</tr>
</tbody>
</table>

In the first (nonelaborated) analysis, the participant drew broad conclusions about the children's understandings such as Pair 2’s being at the "one-to-one correspondence picture stage" and Sunny’s being able to "skip count." However, details were limited and some terms, such as "computational and representational understanding," were left undefined. In contrast, in the elaborated analysis, the participant made elaborations in several ways.

- The participant made sense of the details of each strategy and how these details reflected what the children did understand. For example, when discussing Sunny's understandings, this participant recognized Sunny's ability to count by 2s, his ability to switch between counting...
by 2s and 1s, and his ability to subitize an amount of 4 and count on from that quantity. These comments all point to mathematically relevant details that reflect Sunny's understandings.

- The participant recognized what strategies and understandings the children did not yet demonstrate. For example, when discussing Pair 2’s understandings, this participant recognized that they did not group their tallies into 5s, which would have been a more efficient strategy, and perhaps less prone to error.

- The participant compared the relative sophistication of the understandings needed for each of the strategies. Specifically, this participant recognized that the ability to mentally abstract a quantity was required for Pair 1’s counting back strategy and was more sophisticated than the understandings needed for the strategies of Pair 2 and Sunny.

This example of an elaborated analysis consisted of three of the four types of elaborations participants tended to provide. The fourth type of elaboration related to children's understandings of the problem structure. Research has shown that because no action is specified in problems like the Lunch Count problem, children's strategies may be more varied than for other problems with an explicit joining or separating action (Carpenter et al., 1999). Some participants recognized the curiosity that, despite this problem structure, all children in the video interpreted the problem as subtraction, and participants wrote responses such as “All of them used subtraction to solve this even though there was no action in this problem” (#9996).

We share one final note about analyses focused on children's understandings. The elaborated analyses were generally consistent with the video and with the research base on children's mathematical development. In contrast, in the nonelaborated analyses, participants often misinterpreted or went beyond the evidence and overgeneralized their conclusions. For example, one participant wrote, "These children understand subtraction and addition—and which to choose when
presented with a problem. … They know how to write a number sentence" (#3906), concluding that the children understood the operations of addition and subtraction on the basis of watching their performance on a single problem. Furthermore, only Pair 1 wrote a number sentence, but this participant seemed to assume that Pair 1’s understanding was necessarily shared by the rest of the class. Such comments cannot be supported by evidence in the video, but overgeneralization could be an artifact of participants’ use of informal (and thus imprecise) language on a written instrument. However, we found overgeneralizations disproportionately linked to nonelaborated analyses of children's understandings; we found overgeneralizations in 53% of nonelaborated analysis responses whereas they were present in only 16% of all other responses. The overgeneralizations may be due to both a lack of knowledge and the pressures and habits of the teaching profession that reinforce our general cultural tendencies to evaluate (too) quickly and (too) often (Blythe et al., 1999).

**Analyses with alternate foci.** Some participants chose not to focus their analyses on children's understandings at all, even though the prompt explicitly asked them to do so. Among the alternate foci, participants often shared something learned about mathematics teaching and learning in general. For example, one participant wrote

> I learned that it's important to allow students to use different tools to come up with mathematical problem solutions. Of course with this, it's vital to provide lessons on how to use several different tools. Only after that, can students decide what's easiest for them, and in turn choose tools which best work for the individual. I also learned that a math lesson can be so much more than just math. This teacher invited the students to a lesson in communication, listening, and respect in addition to subtraction (no pun intended). (#6925)

Other participants' responses included positive evaluations of the teaching in the video (e.g., "I was glad that the teacher allowed her students to use multiple ways of arriving to the correct answer" (#6220)) or suggestions for improving that teaching (e.g., "I would have liked to hear the word difference and would have liked to have seen a way of checking or proving answers were correct" (#8480)). Finally, some participants offered commentary on the children, but not on their
understandings (e.g., "The two girls didn't seem to be communicating about what they were doing. Annette wanted to be in charge. Maureen withdrew from explaining. Katie and Sam worked together nicely" (#7303)).

Without posing follow-up questions, we cannot determine whether participants chose alternate foci because they interpreted our prompt differently than we had intended or because they were unable to make sense of the children's understandings. However, given the distribution of these alternate-foci responses across our participant groups, we suspect that an inability to make sense of children's understandings may have played a role.

**Comparison across participant groups.** We compared the focus and depth of interpretation across participant groups (see Table 7), and interpretation differed significantly across groups ($\chi^2 (6, N = 132) = 59.49, p < .05$). Post hoc comparisons indicated that the percentage of individuals who provided elaborated analyses was significantly higher in the participant groups who had engaged in professional development than in the groups who had not ($Z = 5.54, p < .05$). We also found a monotonic increasing trend across all four groups in terms of elaborated analysis ($Z = 5.72, p < .05$) and a monotonic decreasing trend in terms of analysis with alternate foci ($Z = -9.86, p < .05$).

Strikingly, no Prospective Teacher and only one Initial Participant provided elaborated analyses of children's understandings. Furthermore, Prospective Teachers overwhelmingly (81%) provided analyses with alternate foci whereas only one Emerging Teacher Leader offered an analysis not focused on children's understandings. These results indicate that the professional development helped teachers develop a focus on children's understandings and the ability to provide elaborated analyses of those understandings. However, given that less than 50% of any participant group generated elaborated analyses, these results also show that generating elaborated analyses is challenging and that this expertise takes years to develop.
Table 7

Focus and Depth of Interpretation Across Participant Groups

<table>
<thead>
<tr>
<th>Analysis focus</th>
<th>Prospective Teachers</th>
<th>Initial Participants</th>
<th>Advancing Participants</th>
<th>Emerging Teacher Leaders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elaborated analysis of children's understandings</td>
<td>0%</td>
<td>3%</td>
<td>23%</td>
<td>46%</td>
</tr>
<tr>
<td>Analysis of children's understandings</td>
<td>19%</td>
<td>56%</td>
<td>48%</td>
<td>52%</td>
</tr>
<tr>
<td>Analysis with alternate foci</td>
<td>81%</td>
<td>41%</td>
<td>29%</td>
<td>3%</td>
</tr>
</tbody>
</table>

Orientation to Interpretation

We tracked how participants approached responding to our request that they explain what they learned about children's understandings. Participants oriented their responses in one of two ways: (a) including separate comments about Pair 1, Pair 2, or Sunny or (b) exclusively addressing the children as a group (e.g., "The children in this class have…” (#3281)). We compared the orientation to interpretation across participant groups (see Table 8).

Table 8

Orientation to Interpretation Across Participant Groups

<table>
<thead>
<tr>
<th>Interpretation orientation</th>
<th>Prospective Teachers</th>
<th>Initial Participants</th>
<th>Advancing Participants</th>
<th>Emerging Teacher Leaders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpretations that included separate comments about Pair 1, Pair 2, or Sunny</td>
<td>17%</td>
<td>34%</td>
<td>58%</td>
<td>79%</td>
</tr>
<tr>
<td>Interpretations that exclusively addressed the children as a group</td>
<td>83%</td>
<td>66%</td>
<td>42%</td>
<td>21%</td>
</tr>
</tbody>
</table>

Orientation differed significantly across participant groups ($\chi^2 (3, N = 132) = 30.29, p < .05$), and the percentages indicated that an orientation toward individual children may be learned. Post hoc comparisons revealed that the percentage of individuals demonstrating an orientation toward individual children was significantly higher in the participant groups who had engaged in
professional development than in the groups who had not ($Z = 5.73$, $p < .05$), and again, a monotonic increasing trend was evident ($Z = 6.81$, $p < .05$), ranging from 17% of Prospective Teachers who demonstrated an orientation toward individual children to 79% of the Emerging Teacher Leaders. This attention to the understandings of individual children is critical for teachers to effectively differentiate instruction so that all children are challenged and supported, and our results provide evidence that sustained professional development can help teachers develop an orientation that is foundational for this skill.

**Responding**

Professional noticing of children's mathematical thinking includes responding to a situation because teachers' work typically requires in-the-moment decision making about what to do next; making sense of an instructional situation is linked to responding to that situation. Therefore, we examined participants' ideas about instructional decision making in response to the Lunch Count video. Specifically, participants were asked, "Pretend that you are the teacher of these children. What problem or problems might you pose next?" Space was explicitly provided for a next problem (or problems) and a rationale. Note that we were not seeking a particular next problem or rationale. Instead, we compared the responses of participants at different points in sustained professional development in terms of their mathematical goals and how, if at all, their reasoning involved explicit use of children's mathematical thinking.

Before sharing these comparisons, we provide a sense of the types of problems participants suggested. They drew heavily on the original part-whole problem structure and lunch context to construct their new problems. We found that 67% used a part-whole problem structure in which the whole or part was unknown and 61% used the lunch context. No other problem structure was used by more than 5% of the participants, but suggestions included situations involving joining, separating, comparing, and multiplying. Only 3% of participants chose to engage children with a
symbolic number sentence (without context), and 5% chose not to provide a specific problem, writing instead about their general instructional moves.

**Mathematical Goals**

We found that 86% of the participants articulated a mathematical goal in one of four categories: practicing the same concept or skill, exploring fact-family relationships, increasing the problem difficulty, or encouraging more sophisticated strategies. See Table 9 for responses reflecting each of these mathematical goals.
Table 9

Participants' Mathematical Goals for Responding

<table>
<thead>
<tr>
<th>Mathematical goal</th>
<th>Problem(s)</th>
<th>Sample response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practice the same concept or skill</td>
<td>How many more cold lunches are there than hot lunches?</td>
<td>They have already solved how many cold lunches so now they will have to take that answer and apply it to the next problem. It will further help their subtraction skills and stay on the same subject. (#3459)</td>
</tr>
<tr>
<td>Explore fact-family relationships</td>
<td>We have 7 hot lunches and 12 cold lunches. How many lunches do we have altogether? We have 19 children and 12 eat hot lunches. How many eat cold lunches?</td>
<td>To show by manipulating 2 of the same numbers you can come up with the third number (i.e., fact families). (#1412)</td>
</tr>
<tr>
<td>Increase the problem difficulty</td>
<td>You could use the same problem but increase the numbers. For example: We have 28 children, and 14 are hot lunch. How many are cold lunch?</td>
<td>To challenge the higher students especially those in 2nd grade—give students practice working with larger numbers. (#6131)</td>
</tr>
<tr>
<td>Encourage more sophisticated strategies</td>
<td>We have 20 children. 8 are wearing velcro shoes. How many children are wearing tie or other types of shoes?</td>
<td>I'd write a similar problem with different number choices to see if kids can be moved from direct counting by ones. The first group counted back from 19, the 2nd group direct modeled with tally marks and erased 7 and counted the remaining by ones. The third boy counted out by twos, removed 7 by ones and counted the remainder by ones. I selected 20 as a friendly number to see if kids could use their knowledge of tens combinations in solving this problem with any of the tools they may select. (#2572)</td>
</tr>
</tbody>
</table>

The 14% of participants who did not articulate any of these four goals instead mentioned a variety of others such as graphing, looking for patterns, discussing how to check answers, and motivating children by choosing problems that related to their lives. Each of these goals was given by fewer than 5% of the participants.
Of particular interest was whether the participants' suggestions increased complexity for the children. The two mathematical goals that, by definition, explicitly increased the complexity, were increasing problem difficulty and encouraging more sophisticated strategies. Table 10 shows the emphasis on complexity and specific mathematical goals offered by each participant group. We found that emphasis on complexity differed significantly across groups ($\chi^2 (3, N = 132) = 41.09$, $p < .05$) and post hoc comparisons revealed that the percentage of individuals who demonstrated an intention to increase complexity was significantly higher in the participant groups who had engaged in professional development than in the groups who had not ($Z = 7.52$, $p < .05$). Specifically, 68% of the Advancing Participants and 79% of the Emerging Teacher Leaders articulated a mathematical goal of either increasing problem difficulty or encouraging more sophisticated strategies compared to 25% of the Prospective Teachers and 12% of the Initial Participants. Numerous comments about preserving children's comfort level led us to believe that challenging students may provide some discomfort for participants, and our results indicated that this discomfort may lessen as participants learn more about children's mathematical development.

The absence of a mathematical goal within participant groups was also noteworthy. For example, the goal of encouraging more sophisticated strategies was given by only one Prospective Teacher and one Initial Participant, yet it was the most prevalent goal for the Advancing Participants and Emerging Teacher Leaders. In contrast, the goal of practicing the same concept or skill was the most prevalent goal for the Prospective Teachers and Initial Participants but was given by only three Teacher Leaders. In short, although we recognize value in all the mathematical goals discussed, the distribution suggested that the instructional reasoning varied across participant groups.
### Table 10

**Mathematical Goals Across Participant Groups**

<table>
<thead>
<tr>
<th>Mathematical goals</th>
<th>Prospective Teachers</th>
<th>Initial Participants</th>
<th>Advancing Participants</th>
<th>Emerging Teacher Leaders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emphasis on increased complexity</td>
<td>25%</td>
<td>12%</td>
<td>68%</td>
<td>79%</td>
</tr>
<tr>
<td><em>Increase the problem difficulty</em></td>
<td>22%</td>
<td>9%</td>
<td>29%</td>
<td>21%</td>
</tr>
<tr>
<td><em>Encourage more sophisticated strategies</em></td>
<td>3%</td>
<td>3%</td>
<td>39%</td>
<td>58%</td>
</tr>
<tr>
<td>No emphasis on increased complexity</td>
<td>75%</td>
<td>88%</td>
<td>31%</td>
<td>21%</td>
</tr>
<tr>
<td><em>Practice the same concept or skill</em></td>
<td>42%</td>
<td>50%</td>
<td>19%</td>
<td>9%</td>
</tr>
<tr>
<td><em>Explore fact-family relationships</em></td>
<td>14%</td>
<td>13%</td>
<td>6%</td>
<td>6%</td>
</tr>
<tr>
<td><em>Other goals</em></td>
<td>19%</td>
<td>25%</td>
<td>6%</td>
<td>6%</td>
</tr>
</tbody>
</table>

### Explicit Use of Children's Mathematical Thinking in Responding

Because research has shown the value of eliciting and building on children's mathematical thinking, we examined whether and how the participants' reasoning about next steps related to children's strategies and understandings. Only 27% of participants shared reasoning that explicitly used children's mathematical thinking. When their design of problems was informed by children's thinking, participants mentioned (a) building on the specific strategies or understandings of the children in the video, (b) anticipating specific strategies, or (c) both.

The reasoning of participants who used children's thinking varied in depth. Specifically, we identified 11% of the responses as *elaborated use of children's mathematical thinking* in which problems were explicitly designed both to build on the children's thinking in the video and in anticipation of specific strategies. The nonelaborated responses (16%) included only one of these uses of children's thinking and typically were presented with less precision. In the Table 11 examples of responses with explicit use of children's thinking, participants focused on the idea that children could use a counting-up strategy with this problem structure. As noted previously, children are likely to use either addition or subtraction on problems with this structure (Carpenter et al., 1999), and the
participant with the elaborated response took this idea one step further in recognizing the power of using this problem structure to promote classroom discussion of the relationship between addition and subtraction.

Table 11
Participants' Responding Examples That Explicitly Used Children's Mathematical Thinking

<table>
<thead>
<tr>
<th>Use of children's mathematical thinking</th>
<th>Elaborated use of children's mathematical thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>More of the same type [of problem], but I'd change it so the unknown is the smaller number. I'd like to see if the kids would count up.</td>
<td>We have ____ children, and ____ are hot lunch. How many are cold lunch? (20, 14) — With these numbers, I'd be curious if students would use their same strategies of building the whole, then counting back 14 or removing 14 by ones. These numbers might encourage them to find the other &quot;part&quot; of 20 by counting on by ones or even counting on by 1s and 5s — 14 +[1] +[5] 15, 20 If that didn't come up, I'd make the numbers even more obvious, such as using (20, 19). I'd really want to press kids toward using this [counting-up] strategy so we could have a class discussion about how some students arrived at their solution by adding on, and others arrived at it by subtracting or counting back. It's important to have this kind of mathematical discussion so students see the relationship between addition and subtraction. (#7505)</td>
</tr>
</tbody>
</table>

The participant with the elaborated response targeted the discussion of relating addition to subtraction. Recognizing that all children in the video used a separating action (counting back or removing beads or tallies), this participant designed two problems to encourage (but not force) children to use a joining strategy (e.g. counting up) to create this discussion opportunity. In short, this participant not only identified and built on the separating strategies of the children in the video but also anticipated joining strategies in response to some carefully designed problems. In contrast, the participant with the nonelaborated response also anticipated a counting up strategy but there was no explicit link to the children's strategies in the video, and overall, the suggested problem and rationale were much less detailed.
A limitation of any written assessment is that some participants may naturally choose to write more than others. However, given the striking distribution of the responses across our participant groups, we suspect that other factors also played a role in participants' use (or lack of use) of children's thinking. Table 12 provides a comparison of the use of children's mathematical thinking across participant groups.

Table 12

<table>
<thead>
<tr>
<th></th>
<th>Prospective Teachers</th>
<th>Initial Participants</th>
<th>Advancing Participants</th>
<th>Emerging Teacher Leaders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explicit use of children's thinking</td>
<td>3%</td>
<td>3%</td>
<td>45%</td>
<td>57%</td>
</tr>
<tr>
<td>Elaborated use of children's thinking in responding</td>
<td>0%</td>
<td>0%</td>
<td>10%</td>
<td>33%</td>
</tr>
<tr>
<td>Use of children's thinking in responding</td>
<td>3%</td>
<td>3%</td>
<td>35%</td>
<td>24%</td>
</tr>
<tr>
<td>No explicit use of children's thinking</td>
<td>97%</td>
<td>97%</td>
<td>55%</td>
<td>43%</td>
</tr>
</tbody>
</table>

Performance across participant groups differed significantly on whether individuals made any explicit use (elaborated or nonelaborated) of children's mathematical thinking in responding ($\chi^2(3, N = 132) = 41.27, p < .05$). Post hoc comparisons revealed that the percentage of individuals who made use of children's mathematical thinking in responding was significantly higher in the participant groups who had engaged in professional development than in the groups who had not ($Z = 7.45, p < .05$), and there was a monotonic increasing trend ($Z = 7.20, p < .05$), ranging from 3% to 57%. The percentages are striking. Specifically, only one Prospective Teacher and one Initial Participant made explicit use of children's thinking. In contrast, almost half of the Advancing Participants and more than half of the Emerging Teacher Leaders made explicit reference to building on or anticipating children's thinking in their rationales. These results again indicated that the professional development helped teachers develop strong orientations toward considering children's
mathematical thinking in their reasoning, but we again found that this skill took years to develop. Furthermore, with only 33% of the Emerging Teacher Leaders demonstrating an elaborated use of children's mathematical thinking, we learned that even the group with the most expertise showed room for growth.

CONCLUSIONS

We began this study with the idea that the construct of professional noticing of children’s mathematical thinking merited attention from teachers, professional developers, and researchers working toward the vision of successful classrooms put forth by national reform documents (NCTM, 2000; NRC, 2001). One of our major assumptions was that studying teachers at different points in time defined by length of engagement in sustained professional development focused on children's thinking is meaningful. In our cross-sectional study of 132 prospective and practicing teachers, we found that this assumption was validated. Our participant groups differed significantly in terms of the component skills of professional noticing, and because all three teacher groups had similar number of years of teaching experience, these differences can more likely be attributed to the number of years of engagement in sustained professional development than to years of teaching experience. By identifying these differences, we not only better understand what expertise entails but also gain snapshots of groups of participants who are in the process of developing this expertise. In the following sections, we summarize what we learned about the development of expertise related to professional noticing of children's mathematical thinking, and we consider how, because of connections among the component skills of professional noticing, teachers need expertise in all four skills to teach in ways consistent with reform recommendations. We conclude with some practical implications for the professional development of teachers.
Development of Expertise in Professional Noticing of Children's Mathematical Thinking

Our results were robust in showing that children's mathematical thinking can play an important role in teachers' reasoning but that the development of this expertise takes time. Specifically, the Emerging Teacher Leaders exhibited the most expertise in taking into account children’s thinking, followed respectively by the Advancing Participants, the Initial Participants, and the Prospective Teachers. Table 15 provides a summary of the results for our four participant groups.

Table 15
Summary of Professional Noticing of Children's Mathematical Thinking Across Participant Groups

<table>
<thead>
<tr>
<th>Professional noticing component</th>
<th>Prospective Teachers</th>
<th>Initial Participants</th>
<th>Advancing Participants</th>
<th>Emerging Teacher Leaders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe</td>
<td>few</td>
<td>some</td>
<td>most</td>
<td>Most</td>
</tr>
<tr>
<td><em>How many were generally able to describe most details of children's strategies?</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interpret</td>
<td>few</td>
<td>some</td>
<td>most</td>
<td>almost all</td>
</tr>
<tr>
<td><em>How many analyzed children's understandings when asked?</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Did they generally consider the understandings of individual children or children as a group?</td>
<td>children as a group</td>
<td>children as a group</td>
<td>individual children</td>
<td>Individual children</td>
</tr>
<tr>
<td>Respond</td>
<td>few</td>
<td>few</td>
<td>most</td>
<td>Most</td>
</tr>
<tr>
<td><em>How many targeted mathematical goals that increased complexity?</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many explicitly used children's thinking when reasoning about how to respond?</td>
<td>almost none</td>
<td>almost none</td>
<td>some (minimal elaborated use)</td>
<td>Some (some elaborated use)</td>
</tr>
</tbody>
</table>

We can use these results to develop a profile of the professional noticing of children's mathematical thinking for each of the participant groups.
• Prospective Teachers typically could not describe the children's strategies and did not show evidence of using children's mathematical thinking when interpreting and responding. A distinguishing feature of this group was that they did not focus on children's understandings even when they were specifically asked to do so; more than three fourths of the Prospective Teachers focused on other topics when asked to explain the children's understandings.

• Initial Participants were better able to describe the children's strategies than the Prospective Teachers, but many still struggled. When interpreting children's understandings, they typically considered the children as a group and almost exclusively provided general analyses. Their instructional responses were characterized by goals that did not increase the complexity for students and reasoning that did not show evidence of drawing on children's thinking.

• Advancing Participants were typically able to describe the details of children's strategies, and most of their interpretations focused on children's understandings. Similar to the Initial Participants’ interpretations, the majority of these interpretations were general, but, in contrast, about one fourth of the Advancing Participants provided elaborated analyses. Most of the Advancing Participants also articulated instructional goals that increased complexity, and the reasoning behind these responses drew on children's thinking in general ways.

• Emerging Teacher Leaders were characterized by explicit attention to individual children’s thinking. They were typically able to describe the details of children's strategies and focused their interpretations on individual children’s understandings. The majority of the Emerging Teacher Leaders articulated instructional goals that increased complexity, and compared with other participant groups, their interpretations and reasoning for their instructional responses were more elaborated.

Because of the cross-sectional nature of our design, these profiles can be used to consider the developmental trajectory of teachers who are learning to use children's mathematical thinking to
inform their teaching.⁷ We hypothesize that many individuals enter the teaching profession with little focus on children's mathematical thinking when viewing instructional situations or when making instructional decisions about how to respond. Through years of teaching experience, teachers seem to gain some focus on children's strategies and understandings but typically consider children as a group. Furthermore, children's mathematical thinking does not seem to inform their decision making about how to respond. By engaging in sustained professional development focused on children's mathematical thinking, teachers can gain a stronger focus on children's strategies and understandings, learn to give more attention to details, and move toward considering individual children's understandings. Teachers can also begin to use children's thinking to inform their instructional responses. This movement toward using the details of individual children's thinking to inform instruction is central to the recommendations of reform documents, and, according to our results, gains strength with more years of professional development. Notably, the Emerging Teacher Leaders, who had engaged in at least 4 years of professional development, demonstrated substantial expertise, but we also recognize that growth was still possible. In the next section, we further explore the development of professional-noticing expertise by considering how the skills of describing, interpreting, and responding are intertwined in important ways.

**Connections Among Professional Noticing Skills**

At the outset of this study, we hypothesized a hierarchy wherein one must be able to describe children’s strategies before being able to interpret the understandings reflected in those strategies and interpret the children's understandings before being able to respond on the basis of those understandings. (See Figure 1 for a flow chart of our hypothesized hierarchy.) In other words, we

⁷ We recognize that not all teachers who demonstrate use of children's mathematical thinking when determining how to respond in the context of video will be able to do so when they are teaching in actual classroom situations. We would argue, however, that teachers who do not demonstrate use of children's thinking in video tasks are unlikely to do so when teaching in actual classroom situations.
believe that being able to describe and interpret the details of children's strategies is foundational for responding on the basis of children's thinking and thus for teaching in reform-minded ways.

We tested the existence of our hypothesized hierarchy with our 132 participants, examining whether participants followed expected trajectories. For example, we expected that participants who could not describe the strategies would be able neither to interpret the children's understandings nor to use the children's understandings when deciding how to respond. We also expected to find trajectories wherein participants could use children's thinking in some of the professional noticing skills but not in others, according to the hierarchy stated above. For example, we expected that some participants would be able to effectively describe and interpret children's thinking but not yet use children's thinking when determining their instructional responses.

We found that for 83% of the participants, our hypothesized hierarchy was validated (i.e., in only 17% of the cases, participants had an unexpected trajectory, such as not showing evidence of describing the strategies but being able to interpret the children's understandings). A chi-square goodness of fit test showed that this distribution was statistically significant ($\chi^2 (1, N = 132) = 111.23, p < .05$). Thus, not only are describing and interpreting important in and of themselves, but each skill is also foundational for the skill of responding on the basis of children’s thinking; only teachers who can describe children’s strategies have the necessary foundation for interpreting what those strategies mean in terms of children’s understandings, and only those who can interpret have the necessary foundation for responding on the basis of children's understandings.

Implications for the Professional Development of Teachers

We designed our study to help us understand professional noticing of children's mathematical thinking and the development of this expertise. Although variability existed within our participant groups, we found the consistent patterns across groups to be convincing and worthy of attention. Below we draw on our results to provide suggestions for the professional development of teachers.
Recognize the value of professional development focused on children's mathematical thinking. Reform documents call for attention to the details of children's mathematical thinking. We found that some of this expertise seems to develop from experience in the classroom given that we consistently found monotonic trends in which Initial Participants generally demonstrated greater expertise than Prospective Teachers in terms of describing and interpreting children's thinking. However, only about half of the Initial Participants demonstrated these describing and interpreting skills, and their interpretations were almost always general. In addition, only one Initial Participant showed evidence of using children's thinking when reasoning about how to respond.

In contrast, Advancing Participants and Emerging Teacher Leaders consistently demonstrated more expertise in describing, interpreting, and responding than groups who had not participated in the professional development. Thus on the basis of our results, we conclude that professional development matters, even for teachers who have many years of teaching experience. Our three groups of practicing teachers were relatively experienced, with almost all teachers having at least 5 years of experience and all groups reflecting an average of 15–16 years of experience. Nonetheless, groups who had participated in the professional development demonstrated more expertise.

Conceptualize professional development as sustained over years. Although teachers like the Initial Participants in this study may learn some professional noticing skills by teaching or attending short-term professional development, we found that these experiences were generally not sufficient for learning to teach in ways consistent with reform recommendations. Similarly, the two years of professional development experienced by the Advancing Participants was not sufficient. We consistently found monotonic increasing trends with the Emerging Teacher Leaders showing the most expertise, including more attention to details of children's strategies, more elaborated analyses of children's understandings, and more explicit use of children's thinking when deciding how to respond. However, these Emerging Teacher Leaders, who had already engaged in at least 4 years of
professional development, still had room to grow. In short, teaching according to reform recommendations is challenging. Therefore, to support this type of teaching, professional development must be conceptualized in terms of multiple years and, in fact, continued throughout teachers' professional lives.

**Be patient and celebrate growth benchmarks.** Given this long-term view of professional development, facilitators (and administrators) need to be patient and to conceptualize teachers' development similarly; teachers need years (not months) to develop the skills necessary to teach in ways that build on children's thinking. Our study also provides several benchmarks that may help to identify teachers' growth during this long-term process of development. Specifically, when developing expertise in the professional noticing of children's mathematical thinking, teachers are likely to (a) shift from focusing on children as a group to focusing on the understandings of individual children, (b) shift from providing general strategy descriptions and interpretations to more elaborated ones, and (c) shift from instructional decision making that does not consider children's thinking to determining how to respond on the basis of children's understandings.

**Start with attention to individual children's strategies.** Reform recommendations include teachers making instructional decisions on the basis of children's mathematical thinking. However, our findings related to the hierarchical relationship among describing, interpreting, and responding suggest that a focus on responding may not be fruitful unless teachers have already learned to attend to the details of individual children's strategies. We do not suggest that one must focus exclusively on describing before moving to issues of interpreting and responding. Instead, we argue only that even though the skill of describing strategies takes time to learn (and may be deemed unnecessary by some), time spent learning to describe children's strategies is time well spent to support future goals of interpreting and, most important, responding on the basis of children's understandings.
**Target the specific needs and perspectives of a particular audience.** The differences that we found among participant groups should be useful to professional developers in helping them more effectively target support for a particular audience. In particular, our results related to the professional-noticing skill of identifying should be useful in helping professional developers effectively select and use artifacts of practice, such as classroom video.

For example, Prospective Teachers and Initial Participants were more likely than the Advancing Participants and Emerging Teacher Leaders to identify as noteworthy the fact that students generated multiple strategies. Because classroom practices surrounding this type of problem-solving (including the fact that children can solve problems on their own in multiple ways and that teachers can provide opportunities for them to do so) may be novel for Prospective Teachers and Initial Participants, offering many opportunities to view video clips of classrooms and of individual children solving problems may be an important first step for promoting awareness of these possibilities. However, video clips shown only for these reasons may do little to further the thinking of Emerging Teacher Leaders who most likely already consider a variety of student-generated strategies to be a core element of mathematics classrooms.

Similarly, Emerging Teacher Leaders were more likely than the other participant groups to identify as noteworthy the details of the teacher’s questioning. Spending time focused on a single video clip highlighting features of a teacher’s questions that contributed to, for example, supporting children’s strategy explanations, may be an appropriate endeavor for working with Emerging Teacher Leaders. This type of discussion may be inappropriate, however, for participants from other groups who may still be unsure whether children can generate strategies on their own. In fact, it is conceivable that they may interpret teachers' questions to support children's explanations as teachers' attempts to show children how to solve the problem. In short, by understanding salient components
of classroom activity for a particular audience, professional developers should be poised to improve their professional development through more targeted video-clip selection and questioning.

**Final Thoughts**

Through this study, we have become further convinced that professional noticing of children's mathematical thinking is a critical, but often hidden, set of skills needed by elementary school teachers who are working to teach in ways consistent with reform recommendations. Our theoretical conceptualization of professional noticing proved useful for teasing apart differences among participant groups and thus for better understanding professional noticing of children's mathematical thinking and the development of this expertise.

We conclude with a caveat and an invitation. In the study reported here, we investigated participants' reactions to a single classroom video, and we recognize that every video, just as every classroom interaction, has idiosyncratic components. Would we find similar results with a different classroom video? What types of patterns would emerge with video of individual children solving problems or with the use of written student work? We will examine some of these issues as part of our larger study, Studying Teachers Evolving Perspectives (STEP), and we encourage others to use multiple contexts to explore the construct of professional noticing of children's mathematical thinking.
References


Appendix A: Lunch-Count Classroom-Video Excerpt

Teacher: Is there another math problem we could do from the attendance this morning or from the lunch count?

Russ's problem: We have 19 children, and 7 are hot lunch. How many are cold lunch?

Teacher: Why don't you guys go get a tool or go figure it out and come back in about 2 minutes.

Sharing by Pair 1 (Katie & Sam)

Katie writes 19 - 7 = [ ] on board

Katie then counts back from 19 on her fingers, stating, “19” then counting on her fingers, “18, 17, 16, 15, 14, 13, 12.” She stops counting after she has raised 7 fingers (5 on one hand and then 2 more on the same hand). Katie writes 12 below the pull down, and Sam writes 12 in the box.

19 - 7 = [12]

12

The teacher asks questions about their strategy:

Teacher: Why did you decide to write a number sentence that looked like that?
Katie: Because if we put plus, then it would be a different answer than Rusty's.
Child: It would be 26.
Teacher: Would that make sense?
Sam: No.
Teacher: Why not?
Sam: Well, if you put a plus it would be 19, (counting up on fingers) 20, 21, 22, 23, 24, 25, 26. It would be 26 instead of 12.
Teacher: Why wouldn't that work?
Sam: Because we want to count down, not up.
Teacher: Why?
Katie: So we can figure out the answer of cold lunches.
Sam: Yeah.
Teacher: Well, why wouldn't 26 work?
Sam: Because that's a too high of a number.
Teacher: Well, how do you know it is too high of a number?
Katie: We don't even have 26 kids in the class.
Teacher: Oh, so we don't even have 26 kids in the class, and then you said we're looking for the cold lunches, so that wouldn't make sense.
Sam: No, it wouldn't make sense.
Teacher: Okay, everybody understand what they are saying? Any questions? [no questions posed] Why don't you guys pick someone else?

Sharing by Pair 2 (Annette & Maureen)

Annette draws 19 tallies on the board to represent the total number of students:

```
| | | | | | | | | | | | | | | | | | |
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She counts 7 tallies by 1s from the right. She then recounts the 7 tallies from the right and erases them.

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| | | | | | | |
```

She counts the remaining tallies by 1s to get 12.
Sharing by Sunny

Sunny works with a counting frame [10 rods with 10 beads on each rod].

Sunny: Well, I started by putting out 2s.
Teacher: Why?
Sunny: Because I knew that if you do 2s, it would be kind of easier, because I'm really good at 2s.
Teacher: Okay.
Child: I'm also good at 2s.
Sunny: And I counted while I was doing it.
Teacher: Ok, can you count as you are doing it?
Sunny: And I knew this was 10.

Teacher: Why?
Sunny: Because I know that five 2s is 10.
Teacher: Ok.
Sunny: And I kept on putting them until the last one, because if I put two 2s at the end, that would make 20, but I only want 19 right now.
Teacher: Why would it make 20?
Sunny: Because [counting the pairs of beads on the left] 2, 4, 6, 8, 10, 12, 14, 16, 18, and then 19, but then 1 more would make 20.
Teacher: [To the other children in the class] Excuse me, I don't think you are listening. Sunny has waited a long time as everyone else was giving their way. You need to show him that you are also listening to his way please. Thank you.
Sunny: And then I counted out 7, and I counted by these ones, because those are closer [moves 7 beads to the side, starting with the third row]. And now, and after I finished with that, I counted out all the ones I had left.

Teacher: Okay, how did you count?
Sunny: I first did the 4 [points to the first 2 rows], and then [pointing to each single bead] 5, 6, 7, 8, 9, 10, 11, 12.
Teacher: Okay, so it sounds to me like we kind of came to a consensus about the answer. What number kept coming up as the answer?
Class: 12.
Is the participant able to describe the strategies?

- no
  - Because the participant cannot describe the strategies, he or she will not be in the position to interpret the understandings underlying the details of the strategies. Further, the participant will not be able to use the children's understandings when responding.

- yes
  - Is the participant able to interpret the children’s understandings?
    - no
      - Although the participant is able to describe the strategies, the ability to use those details to make conjectures about the children's understandings is still developing. Because this participant is unable to interpret the children’s understandings, he or she will not be able to use the children's understandings when responding.
    
    - yes
      - Is the participant able to respond on the basis of the children's understandings?
        - no
          - Although the participant is able to describe and interpret the strategies, the ability to use that information to respond on the basis of the children's understandings is still developing.
        
        - yes
          - This participant is able to respond on the basis of the children's strategies and mathematical understandings.

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Figure 1. Flow chart of professional noticing of children’s mathematical thinking: A hypothesized hierarchy.