Using Video and Student Work Focused on Children’s Thinking to Help Professional Developers Support K–3 Teachers in Transforming Their Teaching

Presentation at the NCSM Annual Meeting
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NSF-funded project to study the effects of sustained professional development focused on children’s mathematical thinking

**Long-Term Sustained Professional Development Matters for Teachers’…**

- Mathematical content knowledge
- Beliefs
- Responsiveness to children’s thinking in 1-on-1 interviews
- Professional noticing of children’s mathematical thinking

You know who we are. Who are you?
How do teachers benefit from engaging with video or student work of children’s mathematical thinking?
Session Outline

• Discuss the use of video and written student work
• Share resources and frameworks that have been helpful to us

Note, today we will ask you to wear two hats (teachers’ and professional developers’), sometimes simultaneously, sometimes not.
At the time of this interview, Javier had been in the United States about one year, and he did not speak English before coming to this country.

Javier, IMAP Searchable CD, Clip #158, 0:00 - 1:10; Select CD, Clip #6.

Comments about video clip?
Planning To Use Video

(Professional Developer’s hat)

Imagine that you have the video and the written student work and you are planning to use them with teachers. One of your goals is to consider the distributive property.
Planning to Use Video

Think about how you would use the video and student work with teachers.

What guides your thinking? Are there principles that you find you are drawing upon as you plan?
Your Conjectured Sequence for Using Written Student Work and Video With Teachers*

Take one minute to write a sequence you might follow. Consider your use of the written work and the video.

*We also use the conjectured sequence with PSTs.
A Conjectured Sequence for Using Written Student Work and Video With Teachers*

(Handout)

1) Teachers solve the problem 2 ways: How many eggs in six dozen? Share.
2) Teachers consider students’ thinking:
   How might a student solve this problem without using the standard algorithm?
3) Distribute student’s written work. Teachers describe, then interpret, the work.
4) View video. Teachers describe, then interpret, the video.
5) Teachers consider implications for mathematics, teaching, and learning.
   • Describe how this student used the distributive and associative properties.
   • Describe the interviewer’s role in revealing the student’s thinking?
   • What problem would you pose next to this child? Why choose that problem?
   • What do we as teachers do when faced with a child who is more mathematically creative and innovative than we are?
   • What would a teacher need to know to understand Javier’s thinking?

*We also use the conjectured sequence with PSTs.
One Representation of Javier’s Thinking

\[ 6 \times 12 = (5 \times 12) + (1 \times 12) \]

\[ = \left( \left( \frac{1}{2} \times 10 \right) \times 12 \right) + 12 \]

\[ = \left( \frac{1}{2} \times (10 \times 12) \right) + 12 \]

\[ = \left( \frac{1}{2} \times 120 \right) + 12 \]

\[ = 60 + 12 \]

\[ = 72 \]
One Representation of Javier’s Thinking

6 x 12

= \((5 \times 12) + (1 \times 12)\)  
  (Distributive property of \(x\) over \(+\))

= \([((\frac{1}{2} \times 10) \times 12] + 12\)  
  (Substitution property)

= \([\frac{1}{2} \times (10 \times 12)] + 12\)  
  (Associative property of \(x\))

= \([\frac{1}{2} \times (120)] + 12\)  
  Place value

= 60 + 12

= 72

What would a teacher need to know to understand Javier’s thinking, and where do teachers learn this mathematics?
1) Questions to prepare teachers to understand the child’s thinking
How could you solve this problem using two different strategies?
How might a child solve this problem?

2) Questions to encourage teachers to explore the child’s thinking in depth
How did the child solve this problem?
Why might the child have thought this way?
What is the mathematics embedded in this student’s thinking?

3) Questions to help teachers identify instructional next steps to extend the child’s thinking
What questions could you ask to help the child reflect on the strategy?
What questions might encourage the child to consider a more efficient strategy?
On the basis of the child’s existing understandings, what task might you pose next?
Four Principles of Mathematics and Mathematics Teaching and Learning Addressed by Focusing Upon Children’s Mathematical Thinking

Principle 1. The way most students are learning mathematics in the United States is problematic.

Principle 2. Learning concepts is more powerful and more generative than learning procedures.

Principle 3. Students’ reasoning is varied and complex, and generally it is different from adults’ thinking.

Principle 4. Elementary school mathematics is not elementary.
Four Principles of Mathematics and Mathematics Teaching and Learning Addressed by Focusing Upon Children’s Mathematical Thinking

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Principle 4  Elementary school mathematics is not elementary.
On a sheet of paper, respond to the following.

1) Describe the video.
2) Interpret the video.

Discuss the differences between description and interpretation.

(Even after many years in the classroom, professional development led to dramatic changes in teachers’ abilities to describe, interpret, and respond to children’s mathematical thinking.)
Video Example:
Compare 4.7 and 4.70.
(Professional developer’s hat)

For what purpose might you use this video when working with teachers?

What questions would you pose to teachers to support the goals you have for using the video?
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**Principle 4** Elementary school mathematics is not elementary.
Felisha, End of Grade 2

Felisha learned fraction concepts using equal-sharing tasks in a small group over 14 sessions (7 days). She had not been taught any procedures for operating on fractions.

\[ \frac{3}{4} + \frac{1}{2} = 1\frac{1}{4} \]
Consider two problems

• A problem for secondary-school algebra students

• A problem for primary-grade students (K–2)
A Secondary-School Algebra Problem

Nineteen children are taking a minibus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 7 seats. How many children will have to sit three to a seat, and how many can sit two to a seat?

\[ x = \# \text{ of seats with 2 children}; \quad y = \# \text{ of seats with 3 children} \]

\[ x + y = 7 \quad \text{and} \quad 2x + 3y = 19 \]

\[
\begin{align*}
2x + 3y &= 19 \quad \text{and} \quad y = 7 - x \\
2x + 3(7 - x) &= 19 \\
2x + 21 - 3x &= 19 \\
-x &= -2 \\
x &= 2 \quad \text{and} \quad y = 5
\end{align*}
\]
A Primary-Grade Problem

Nineteen children are taking a minibus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 7 seats. How many children will have to sit three to a seat, and how many can sit two to a seat?

In six classes, 51% of kindergarten students (36/70), correctly solved this problem in May.

How do you think they did that?

“Throughout the year children solved a variety of different problems. The teachers generally presented the problems and provided the children with counters, . . . but the teachers typically did not show the children how to solve a particular problem. Children regularly shared their strategies. —Carpenter et al., 1993, p. 433. (JRME, Vol. 24)
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**Principle 4** Elementary mathematics is not elementary.
At the time of this interview, Javier had been in the United States about one year, and he did not speak English before coming to this country.
For what purpose might you use this video when working with teachers?

What questions would you pose to teachers to support the goals you have for using the video?
Ones Task

Below is the work of Terry, a second grader, who solved this addition problem and this subtraction problem in May.

Problem A

\[
\begin{array}{c}
2 & 5 & 9 \\
+ & 3 & 8 \\
\hline 
2 & 9 & 7 \\
\end{array}
\]

Problem B

\[
\begin{array}{c}
3 & 1 & 2 & 9 \\
- & 3 & 4 \\
\hline 
2 & 9 & 5 \\
\end{array}
\]

- Does the 1 in each of these problems represent the same amount? Please explain your answer.
- Explain why in addition (as in Problem A) the 1 is added to the 5, but in subtraction (as in Problem B) 10 is added to the 2.

Andrew Task

In March, Andrew, a second grader, solved 63 - 25 = □ as shown below.

\[
\begin{array}{c}
6 & 3 \\
- & 2 & 5 \\
\hline 
4 & 0 \\
\end{array}
\]

- Explain why Andrew’s strategy makes mathematical sense.
- Please solve 432 - 162 = □ by applying Andrew’s reasoning.
Teacher Groups
\[ N = 131 \ (30+ \text{ per group}) \]

<table>
<thead>
<tr>
<th>Practicing Teachers</th>
<th>(average of 15–16 years of teaching experience per group)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Emerging Teacher Leaders</strong> (ETLs)</td>
<td>At least 4 years of sustained professional development</td>
</tr>
<tr>
<td><strong>Advancing Participants</strong> (APs)</td>
<td>2 years of sustained professional development</td>
</tr>
<tr>
<td><strong>Initial Participants</strong> (IPs)</td>
<td>0 years of sustained professional development</td>
</tr>
<tr>
<td><strong>Prospective Teachers</strong> (PSTs)</td>
<td>Undergraduates enrolled in a first mathematics–for–teachers content course</td>
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# Group Means by Task

(0–4 scale)

<table>
<thead>
<tr>
<th></th>
<th>Preservice Teachers</th>
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<tr>
<td>Andrew</td>
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* **Strong Mathematics Students** are advanced undergraduates or graduate students who have completed several upper-division mathematics courses and who have no particular experience or interest in teaching.*
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Elliot, Grade 6: \[1 \div \frac{1}{3}\]?

(*IMAP Select CD, Clip #16)

“One third goes into 1 three times because there is three pieces in one whole.”
Follow-up: $1 \frac{1}{2} \div \frac{1}{3}$?

“There are three one-thirds in 1, and another $\frac{1}{3}$ in $\frac{1}{2}$, and there is $\frac{1}{6}$ left over.”

Elliot does not realize that the remaining $\frac{1}{6}$ needs to be seen as $\frac{1}{2}$ of $\frac{1}{3}$.
Elliot, Grade 6

Follow-up
Explain Elliot’s reasoning. What does one need to understand to be in the position to support Elliot?

Issue
Understanding is seldom total or nonexistent. Everybody understands something about anything, and no one understands everything about anything.
Selecting Video

(As opposed to *using* video)

What principles guide your *choice* of video to use with teachers?

1)

2)

3)

4)

5)

6)

7)
Searchable Set of CDs: Operationalizing Selection Criteria

• Student gender, grade, ethnicity
• Content
• Teaching/Interview
• Strategy
• Miscellaneous
Discussion

Questions?      Comments?