How do teachers benefit from video of children’s mathematical thinking?

• Objective viewpoint—one can stand back and see differently;
• Confronts perceived notions/expectations of what children can do (for practicing and prospective teachers);
• Raises teachers’ expectations of what students can do;
• Video may be stopped, rewatched, edited, and so on;
• Provides opportunities to be critically reflective—helps us see our actions in others, without watching ourselves;
• Allows us to see that children approach things and think differently from adults;
• Enables us to watch the same person over time;
• Allows for articulation/collaboration and discussion of next steps;
• Is less threatening than entering one another’s classrooms;
• Enables us to model questioning techniques in a safe way;
• Video is efficient; it can be used after school; it can provide opportunities to quickly raise complex issues.
What common errors might third graders make when using the standard algorithm to subtract the following?

\[
\begin{array}{c}
400 \\
- 150 \\
\end{array}
\]

Clip #148, Freddie, Grade 3, 0:00 - 0:50 (400 – 150 procedural, incorrect)

On the basis of the child’s existing understandings, what task might you pose next?
Consider this problem.

I have 150 sea shells and I went to the beach and collected some more. By the time I was done, I had 400 sea shells. How many sea shells did I collect?

Solve this problem 2 ways.
Clip #148, Freddie, Grade 3, 0:50–2:34
(JCU, 150 + ___ = 400, solving mentally)

What issues might you raise with teachers around this video clip?
Four principles of mathematics and mathematics teaching and learning addressed by focusing upon children’s mathematical thinking.

**Principle 1.** The way most students are learning mathematics in the United States is problematic.

**Principle 2.** Learning concepts is more powerful and more generative than learning procedures.

**Principle 3.** Students’ reasoning is varied and complex, and generally it is different from adults’ thinking.

**Principle 4.** Elementary mathematics is not elementary.
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Video Example
(Megan and Donna, Clip #388)

Compare 4.7 and 4.70.

How could you solve this problem using two different strategies?

How might a child solve this problem?
Video Example
(Megan and Donna, Clip #388)

Compare 4.7 and 4.70.
How did the children think about this problem?
1) Questions to prepare teachers to understand the child’s thinking
How could you solve this problem using two different strategies?
How might a child solve this problem?

2) Questions to encourage teachers to explore the child’s thinking in depth
How did the child solve this problem?
Why might the child have thought this way?
What is the mathematics embedded in this student’s thinking?

3) Questions to help teachers identify instructional next steps to extend the child’s thinking
What questions could you ask to help the child reflect on the strategy?
What questions might encourage the child to consider a more efficient strategy?
On the basis of the child’s existing understandings, what task might you pose next?
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**Principle 4** Elementary mathematics is not elementary.
Learning concepts is more powerful and more generative than learning procedures.

Search for Wow! in the database for clips to illustrate Principle 2.

- Shannon (Clip #198), Grade 3, 1:58 ($140 \div 7$)
- Rachel (Clip #367), Grade 5, 1:20
- Rachel (Clip #366), Grade 5, 3:59
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Students’ reasoning is varied and complex, and generally it is different from adults’ thinking.

Search the database for examples illustrating Principle 3 for JCU problems.

• Look at some of those with $6 + \_ = 13$.
• 1) #120, Arriel, Grade 2, 2:00
• 2) #193, Petrisha, Grade 2, JCU; then JRU, 1:10
• 3) #134, Conner, Grade 1 (Derived fact), 0:27
• 4) #166, Jocelyn, Grade 1 (Derived fact), 0:53
Four principles of mathematics and mathematics teaching and learning addressed by focusing upon children’s mathematical thinking.

**Principle 1**  The way most students are learning mathematics in the United States is problematic.

**Principle 2**  Learning concepts is more powerful and more generative than learning procedures.

**Principle 3**  Students’ reasoning is varied and complex, and generally it is different from adults’ thinking.

**Principle 4**  Elementary mathematics is not elementary.
At the time of this interview, Javier had been in the United States about one year, and he did not speak English before coming to this country.
Javier*, Grade 5

How many eggs are in six cartons (of one dozen)?

Javier thinks for 15 seconds, and says, “72.”

I: How did you get that?

Javier writes and says, “5 times 12 is 60, and 12 more is 72.”

I: “How did you know 12 x 5 is 60?”

J: “Because 12 times 10 equals 120. If I take the [sic] half of 120, that would be 60.”

*At the time of this interview, Javier had been in the United States about one year, and he did not speak English before coming to this country.
One Representation of Javier’s Thinking

\[ 6 \times 12 \]

\[ = (5 \times 12) + (1 \times 12) \]

\[ = \left[ \left( \frac{1}{2} \times 10 \right) \times 12 \right] + 12 \]

\[ = \left[ \frac{1}{2} \times (10 \times 12) \right] + 12 \]

\[ = \left[ \frac{1}{2} \times (120) \right] + 12 \]

\[ = 60 + 12 \]

\[ = 72 \]
One Representation of Javier’s Thinking

\[6 \times 12\]

\[= (5 \times 12) + (1 \times 12) \quad \text{(Distributive prop. of } x \text{ over } +)\]

\[= \left[\left(\frac{1}{2} \times 10\right) \times 12\right] + 12 \quad \text{(Substitution property)}\]

\[= \left[\frac{1}{2} \times (10 \times 12)\right] + 12 \quad \text{(Associative property of } x\text{)}\]

\[= \left[\frac{1}{2} \times (120)\right] + 12\]

\[= 60 + 12\]

\[= 72\]
One Representation of Javier’s Thinking

\[ 6 \times 12 \]

\[ = (5 \times 12) + (1 \times 12) \quad \text{(Distributive prop. of x over +)} \]

\[ = [(\frac{1}{2} \times 10) \times 12] + 12 \quad \text{(Substitution property)} \]

\[ = \left[ \frac{1}{2} \times (10 \times 12) \right] + 12 \quad \text{(Associative property of x)} \]

\[ = \left[ \frac{1}{2} \times (120) \right] + 12 \]

\[ = 60 + 12 \]

\[ = 72 \]

What would a teacher need to know to understand Javier’s thinking, and where do teachers learn this?
Examples illustrating Principle 4 but not shared in the session because of time constraints

• Felisha, #325, 1 shared with 4, then 3; 3:25

• Elliot, #321, Fraction division with undistributed remainder
Thank you!

Questions?
Comments?
Critiques?
Compliments?