BARELY IN STEP*
How Professional Development Affects
Teachers’ Perspectives on and Analysis of Student Work

by

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expressed in this paper do not necessarily reflect the position, policy, or endorsement of the supporting agency.
The purpose of this study was to determine how professional development affects teachers’ perspectives on and analysis of student work. To achieve this purpose, we examined, scored, and analyzed responses from four groups of teachers with varying levels of professional development experience. The results indicate that sustained professional development positively influenced teachers’ interpretations, understanding, and assessment of student work.

Purpose and Background
Recent research has shown that U.S. teachers lack the specific content knowledge necessary for teaching mathematics (Ball, Hill, & Bass, 2005). We examined whether involvement in sustained professional development affects teachers’ specific content knowledge required for teaching mathematics at the elementary school level. This study was a small part of the larger project Studying Teacher’s Evolving Perspectives (STEP), with principal investigators Randy Philipp and Vicki Jacobs (2004). STEP researchers are attempting to reveal the gains from professional development by studying four teacher groups with varied experience in professional development: prospective teachers (PSTs) and three groups of elementary school teachers teaching in Southern California. The overall goal of STEP is to improve professional development on a national level.

Therefore, stemming from the goals of STEP, our purpose in this study was to understand and analyze the mathematical views and specific content knowledge of teachers. More specifically, we investigated how participation in sustained professional development influences teachers’ perspectives on and analysis of student work. Recent research has indicated that good teachers need not only mathematics content and pedagogical knowledge but also specific content knowledge for the teaching of mathematics. Teachers need these special skills to, for example, design lessons, evaluate student work, and create lesson plans (Ball, Bass, Hill, & Hoover, 2004). Hill, Rowan, and Ball (2005) even used assessment materials to examine the correlation between student achievement and teachers’ specific mathematical content knowledge. These researchers reported that mathematical content knowledge positively affects student achievement. Thus, in particular, we examined the relationship between elementary school teachers’ mathematical content knowledge and participation in professional development programs. Randy Philipp, Bonnie Schappelle, and the authors of this paper analyzed two tasks from the STEP Content Assessment. The STEP participants completed this content assessment along with other assessments and interviews.

Mathematics education has always been a major concern in the United States, especially with the release of each new national report assessing students’ mathematical achievement. This growing concern causes changes in policies at the national level, requiring more specialized classes, certificates, and degrees for teachers. For example, educational reform policies, like No Child Left Behind, require that teachers attain more advanced degrees or take more advanced mathematics classes in an effort to improve students’ mathematical achievement and understanding (Ball et al., 2005). However, teachers may not need more advanced mathematics courses, especially with Begle’s (1979) alarming research that showed that a negative or insubstantial correlation exists between teachers with post-calculus courses or credits and higher student achievement (as cited in Ball et al., 2005). Therefore, attention now turns toward professional development. Perhaps increased professional development will improve student
achievement and understanding in mathematics and will change current teaching practices. Thus, this study is significant, even on a national level, because we attempted to address these concerns by examining how involvement in professional development affects teachers’ perspectives on and analysis of student work.

**Theoretical Framework**

We investigated the relationship between a teacher’s years of CGI professional development and the teacher’s understanding of grade-appropriate mathematical concepts. In particular, the initial impetus for this study was mathematical proficiency and the issue of transparency, as raised by Zazkis in *Divisibility and Transparency of Number Representation* (in press). Zazkis defined a property of a representation to be *transparent* “if this property ‘can be seen’ considering the representation.” In other words, if a person attends to the property that a given representation was supposed to bring forth, then the representation would be considered transparent to that person in respect to the given property. In this study, we hoped to find what properties the participants found transparent in the standard and nonstandard addition and subtraction algorithms and, furthermore, to probe whether professional development had an effect on the transparency of different mathematical properties.

To study these transparency issues, however, one must first define what constitutes *mathematical proficiency*. To align with the STEP project, we chose to consider the five strands of proficiency from *Adding It Up: Helping Children Learn Mathematics* by the National Research Council [NRC] (2001): adaptive reasoning, conceptual understanding, procedural fluency, productive disposition, and strategic competence. *Conceptual understanding* includes knowledge of mathematical concepts and ideas. *Procedural fluency* includes ability to use, manipulate, and apply mathematical algorithms. *Strategic competence* includes the ability to solve and reason through mathematical problems. *Adaptive reasoning* includes the ability to reason logically about mathematical relationships and concepts. *Productive disposition* includes one’s attitude or perception of mathematics as a logical, rational, and worthwhile subject. All these aspects of mathematical proficiency are important, as the authors emphasized in noting that “the five strands are interwoven and interdependent in the development of proficiency in mathematics” (NRC, 2001, p. 116). In particular, while developing the rubrics for items scored in this study, we continually discussed these five issues to ensure that all were addressed.

**Methods**

The participants in this study were three groups of practicing K–3 elementary school teachers from a single elementary school district in Southern California. The initial participants (IPs) were elementary school teachers enrolled in, but not yet attending, the professional development program, Cognitively Guided Instruction (CGI). The advancing participants (APs) were elementary school teachers with at least 2 years of CGI professional development. The teacher leaders (TLs) were elementary school teachers with at least 4 years of CGI professional development. Because these teachers varied in the number of years of experience in the classroom, a fourth category was added as a baseline—prospective elementary school teachers enrolled at a large university located in Southern California and about to start a course focused on children’s mathematical thinking. The participants were asked individually to examine written student work and to explain what mathematical issues the students may have been trying to address. For this study, we looked at two of those tasks, namely Andrew’s task (see Figure 1)
and Terry’s task (see Figure 2). Fifty responses were randomly chosen and blinded. The responses were fairly evenly distributed across the four categories.

Figure 1. Andrew’s task.  
Figure 2. Terry’s task.

Data analysis took place in two phases for each task. We first developed a rubric to score the participants’ responses to the task by organizing 5 randomly selected participants’ responses from least to most sophisticated. We then analyzed 5 more randomly selected responses and from these 10 responses, formed a rubric. We used an additional 10 responses to check the viability of the rubric and then used the rubric to score the remaining 30 responses. The categories defined in the rubrics were used to investigate what effect, if any, professional development in CGI has on teachers’ understandings. The second step of our data analysis was to relate each rubric score to characteristics of the responses.

<table>
<thead>
<tr>
<th>Score</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No effort, or explanation verifies an incorrect view</td>
</tr>
<tr>
<td>1</td>
<td>Responds only in terms of the algorithm, but knows the difference between the 1s</td>
</tr>
<tr>
<td>2</td>
<td>States that the 1s are “the same,” but explanation deals with place-value issues</td>
</tr>
<tr>
<td>3</td>
<td>Responds in terms of issues of regrouping, but only in a rudimentary way about place value</td>
</tr>
<tr>
<td>4</td>
<td>Provides evidence of understanding issues of place value but does not state it clearly</td>
</tr>
<tr>
<td>5</td>
<td>Exhibits good understanding of place-value issues</td>
</tr>
</tbody>
</table>

Figure 3. Response characteristics for Terry’s task.
<table>
<thead>
<tr>
<th>Score</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Failure to explain Andrew’s approach on top and failure to apply it on the bottom</td>
</tr>
<tr>
<td>1</td>
<td>(a) Correct explanation of Andrew on top without attention to place value and incorrect application on bottom; or (b) generally following explanation on top with attention to place value but something missing and incorrect application of Andrew on bottom; or (c) correct application on bottom with no or incorrect explanation of Andrew on top</td>
</tr>
<tr>
<td>2</td>
<td>(a) Correct explanation of Andrew on top but without attention to place value and correct application on bottom; or (b) correct explanation of Andrew on top and correct application on the bottom but something mathematically incorrect</td>
</tr>
<tr>
<td>3</td>
<td>Score of 4 but either (a) inconsistently talks about the negative 2 vs. subtracting 2, or (b) general sense that the respondent understands the algorithm with attention to place value but either something missing or unclear explanation</td>
</tr>
<tr>
<td>4</td>
<td>Clear explanation of Andrew’s approach on top, correct application on the bottom, no reference to place value as place holders, and consistently approaching the 2 as subtracting 2, adding negative 2, or explicitly making connections between these two ways of thinking</td>
</tr>
</tbody>
</table>

Figure 4. Response characteristics for Andrew’s task.

Results

Terry’s task was designed to probe the participants’ understandings about the standard addition and subtraction algorithms. The goal was not to see whether the participants knew how to use the standard algorithms (the assumption was made that they did) but rather to see what their understandings were about the number issues underlying the algorithm (Thanheiser, 2007). In particular, the purpose was to determine whether the participants saw the differences in the regrouped 1s, as related to issues of place value. In Problem A, the 1 represents 10 ones that have been regrouped into 1 group of ten. Thus, the 1 group of ten is added to the 5 groups of ten and the 3 groups of ten to total 9 groups of ten, represented by 9 in the sum of 297. In Problem B, the 1 represents 1 group of a hundred that was regrouped into 10 groups of ten. Thus, the 2, symbolizing 2 groups of ten (in 429), is added to 10 instead of to 1. In other words, 4 hundreds and 2 tens is the same amount as 3 hundreds and 12 tens. The 3 groups of ten (in 34) are subtracted from the 12 groups of ten giving 9 groups of ten. The 9 in the resulting difference (395) represents these tens.

From the graph shown in Figure 5, note that the majority of preservice teachers (PSTs) scored 1, showing that although the PSTs knew that the 1s were different, they were unable to explain the difference, apart from perhaps talking about the nature of the addition and subtraction algorithms in broad terms. Unlike the PSTs, the initial participants (IPs) had no predominant peak. A higher percentage of IPs than PSTs, however, said that the 1s were the same, perhaps because the PSTs and practicing teachers think about the algorithms differently. A clearer trend can be seen for the advancing participants (APs), only four of whom scored 2 or lower. The vast majority scored 4 or 5, indicating that professional development does affect teachers’ understandings.

Last, we were interested in the results for the teacher leaders (TLs); the majority scored 3, 4, or 5, indicating that professional development has a positive effect on the teachers’ understandings of place value. Although the fact that a small set of TLs scored 0 may seem to contradict the
previous conclusion, we offer an alternative hypothesis: On the one hand, when TLs believe that they have knowledge about a mathematical task, they do, and, on the other hand, if they do not think that they have this knowledge, they are willing to admit it by answering, “I don’t know,” a response scored 0.

![Figure 5. Score distribution for Terry’s task.](image1)

![Figure 6. Score distribution for Andrew’s task.](image2)

The results for Andrew’s task (see Figure 6) are evenly distributed across the five scores, illustrating that the scoring was fair and unbiased. Andrew’s alternative algorithm, for the most part, caused the PSTs confusion, as shown by the peak at score 0. We believe that PSTs have procedural fluency with the traditional method but that these low-scoring PSTs lacked the flexibility to adapt their traditional methods and justifications for the traditional method to Andrew’s strategy. However, we explain the surprising peak at score 4 for the PSTs in that from recently taking high school or college level math courses they might be more familiar than APs and TLs with the concept of negative numbers. IPs scored 0, 1, or 2. Moreover, of 9 IPs, 3 scored 0, 3 received scored 1, and 3 scored 2—a very even distribution. Most APs received a score of 1, 2, or 3. The results for the TLs are in complete contrast with the results for the PSTs. The results indicate that most TLs were able to make sense of and successfully reason about Andrew’s strategy, illustrating adaptive reasoning, conceptual understanding, and procedural fluency. Overall, the years of professional development and the higher scores for this task seem to be correlated.

We noted several trends in comparing the data from the two tasks. On both tasks, the PSTs and IPs scores were relatively low, whereas the APs and TLs scores were relatively high. Also, the TLs scored higher than the APs, on average. The tasks were relatively similar in difficulty for the four groups with a similar distribution of scores.

**Conclusion**

The results of this study indicate that sustained professional development does affect teachers’ analyses of and perspectives on student work. Exemplifying the need for specific content knowledge for mathematics, Ball et al. (2005) stated, “Every day in mathematics classrooms across the country, students get answers mystifyingly wrong, obtain right answers using unconventional approaches, and ask questions … teachers are in the unique position of having to professionally scrutinize, interpret, correct, and extend this knowledge” (p. 17). In this study, we examined how teachers interpret and respond to a mathematically correct answer.
obtained by using an unconventional algorithm as evidenced by Andrew’s task. Furthermore, in Terry’s task, we examined how teachers interpret the place-value concepts underlying the procedures used in standard algorithms. We investigated whether sustained professional development affects teachers’ responses, and, on the basis of the results from this study, we suggest that professional development can improve teachers’ understanding of students’ standard and alternative algorithms.

For both tasks, the results are evenly distributed among all the scores, illustrating that the tasks and the scoring were fair and unbiased. The extra years of professional development seem to allow the participants to explicitly and consistently discuss Andrew’s strategy by making connections and using the correct mathematical language, which are characteristics of flexible and complete conceptual understanding. Similarly, for Terry’s task the extra years of professional development cultivated a better understanding of place-value issues, which otherwise may have been lost in the use of the standard algorithms. The results of this study indicate that sustained professional development positively affects teachers’ analysis of and perspectives on student work. With recent national concern for improved mathematics education, this study, in a small way, illustrates how professional development can improve and contribute to student achievement.

References