Negotiating a Complex Landscape: On Practical Rationality and Dilemmas in Teaching for MKT

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Randy Philipp, Discussant
San Diego State University

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“The take home point will be that the norms of instructional situations are regulations that affect the work of the teacher by prescribing their role vis-a-vis students and content.”

“We can say then that the work of the teacher is shaped by, on the one hand, the obligations that define her position of mathematics teacher and, on the other hand, the demands on her role in each of the situations that she needs to manage.”
The teacher’s actions might have been consequences of dispositions such as...

D1: A proved statement is a true statement.
D2: True statements can be used to justify new statements.

D3: Theorems are socially sanctioned: True statements deemed usable may become theorems.
D4: It is good for children to realize that they could use a proposition they know to be true.

D5: Shorter proofs are mathematically preferable to longer proofs.
D6: Sometimes it is useful that children prefer to do less work than to do more work.

D7: Different students may have thought differently about the same problem.
D8: All students deserve the right to share how they thought about a problem.
D9: If one student publicly noticed something, all students could benefit from that noticing.
How do these dispositions come to be?

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We could think of these dispositions as deriving from higher level principles.

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Disciplinary Principle 1: The logic of mathematical truth should underpin the logic of school mathematical truth.

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Disciplinary Principle 2: The social construction and values of mathematical knowledge should be reproduced in school mathematics.

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Individual Principle 1: Children can figure things out on their own.

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Interpersonal Principle 1: A group can benefit from several individuals’ sharing their work on the same problem.

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### Key Differences (from Dan’s view)

<table>
<thead>
<tr>
<th>Practical Rationality</th>
<th>Dilemmas in Teaching</th>
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<tbody>
<tr>
<td>Focus on the practice—situations and the role of the teacher—not on the individual.</td>
<td>Individuals have commitments that lead them to teach in different ways (TKOT) &amp; have different dilemmas.</td>
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</table>
Remember, we haven’t figured out the individual yet.
What is in the teacher’s head?

Knowledge

Beliefs

Values

Dispositions/Stances

Identity?

But each of these is complex!
Consider One Construct: Knowledge
Another Approach

What is the structure of mathematical knowledge for teaching?

- **MCK** (CCK and SCK)
- **PCK** (KCS)

Subject Matter Knowledge

<table>
<thead>
<tr>
<th>Common Content Knowledge (CCK)</th>
<th>Specialized Content Knowledge (SCK)</th>
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</thead>
<tbody>
<tr>
<td>Knowledge at the mathematical horizon</td>
<td>Knowledge of Content and Students (KCS)</td>
</tr>
<tr>
<td>Knowledge of Content and Teaching (KCT)</td>
<td>Knowledge of curriculum</td>
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</table>
Looking at the mathematics through the lens of children’s mathematical thinking changes what teachers (and PSTs) see.
The Strands of Mathematical Proficiency

The Strands of Mathematical Proficiency

- The tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics.

- The capacity to think logically about the relationships among concepts and situations, including the ability to justify one’s reasoning both formally and informally.

- The ability to formulate mathematical problems, represent them, and solve them.

- Knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.
Example of

*Mathematical Disposition*

70

- 23
Example of *Mathematical Disposition*

\[
\begin{array}{c}
70 \\
-23 \\
\hline
53
\end{array}
\quad \begin{array}{c}
76 \\
-23 \\
\hline
53
\end{array}
\]

“*Yes, math is like that sometimes.*”
The Teacher is *in* Something

In a class…
In a school…
In a state…
In a country…

Thought experiment—What will change in how the teacher thinks/believes/teaches/feels if she were in a different …

What will not change?
We ask teachers to start where the students are.
Where do we start with the teachers?

“We should prepare our students to be agents of change.”

“We should prepare our students not to teach in the schools we have but in the schools we want to have.”
Where do we start with the teachers?

“We want to understand teaching deeply, even the teaching that we don’t like.” -P. H.

“Instead of starting with the math, start with the work of teaching.” -D. B.
Finally, who are we?

What is in our heads?
Where are we standing?
How does one affect the other?
Thank you
Dan, Deborah, and Pat.

Discussion