Productive Disposition: The Missing Component of Mathematical Proficiency

Randy Philipp, John (Zig) Siegfried, Laura Cline, Alison Williams, Vicki Jacobs, & Lisa Lamb

We will use video clips of teachers engaged in small-group discussions to highlight productive disposition, the missing component of mathematical proficiency. Attendees will discuss research issues related to defining and measuring productive disposition. Two elementary school teachers will share what they have learned by studying their students' productive dispositions.
Productive Disposition: The Missing Component of Mathematical Proficiency

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Fay Elementary
1st-Grade Teacher
Plan for the Session

• Solve and Discuss a Task
• Productive Disposition (a strand of Mathematical Proficiency)
• Teachers Studying and Supporting Children’s Productive Disposition
• Discussion
Principal Investigators

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What knowledge do teachers need to teach mathematics?
Common Content Knowledge
the mathematical knowledge teachers are responsible for developing in students

Specialized Content Knowledge
mathematical knowledge that is used in teaching but not directly taught to students

Pedagogical Content Knowledge
“the ways of representing and formulating the subject that make it comprehensible to others” —(Shulman, 1986)
Common Content Knowledge
the mathematical knowledge teachers are responsible for developing in students

Evaluate and understand the meaning of $12 \div 3$.

Specialized Content Knowledge
mathematical knowledge that is used in teaching but not directly taught to students

Write a real-life story problem that could be represented by the expression $12 \div 3$.

Pedagogical Content Knowledge
“the ways of representing and formulating the subject that make it comprehensible to others” —(Shulman, 1986)

How might children think about the problem you wrote?
Directions—After you look over Savannah’s (3rd-grade) work, we would like for you to start your conversation by discussing what you find noteworthy in this work, but feel free to take the conversation in any direction you choose.

38 + 19 = □

38 + 2 → 40
19 + 1 → 20
40 + 20 = 60
60 - 2 = 58
58 - 1 = 57

She explained her work as follows:
“I added 2 to the 38, and I added 1 to the 19. I got 40 + 20, which is 60. Then I subtracted the 2 and the 1 that I had added. The answer is 57.”

38 − 19 = □

38 + 2 → 40
19 + 1 → 20
40 − 20 = 20
20 − 2 = 18
18 − 1 = 17

She explained her work as follows:
“I added 2 to the 38, and I added 1 to the 19. I got 40 − 20, which is 20. Then I subtracted the 2 and the 1 that I had added. The answer is 17.”
Questions We Asked*

1. Pretend that you are Savannah’s teacher. What problem might you pose next to Savannah?
2. Savannah’s approach for addition was correct. Explain why.
3. Savannah’s approach for subtraction was incorrect. Explain why.
4. Do you have any thoughts about why Savannah’s answer for the subtraction problem is off by 2?

*After the group discussed what they found interesting, these questions were posed one at a time. Each subsequent question was posed only when the group requested it.
Define Mathematical Proficiency

- Concepts
- Procedures
- Problem Solving
- Reasoning and Justifying
- Positive Outlook
Define Mathematical Proficiency

• Concepts (Conceptual Understanding)
• Procedures (Procedural Fluency)
• Problem Solving (Strategic Competence)
• Reasoning and Justifying (Adaptive Reasoning)
• Positive Outlook (Productive Disposition)

The Strands of Mathematical Proficiency

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Integrated and functional grasp of mathematical ideas

The ability to formulate mathematical problems, represent them, and solve them.

The capacity to think logically about the relationships among concepts and situations, including the ability to justify one’s reasoning both formally and informally.

Knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

The tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics.

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Andrew Task

In March, Andrew, a second grader, solved $63 - 25 = ?$ as shown below.

Explain why Andrew’s strategy makes mathematical sense.

Please solve $432 - 162 = \square$ by applying Andrew’s reasoning.

\[
\begin{array}{c}
\phantom{0}63 \\
- \phantom{0}25 \\
\hline
\phantom{0}38 \\
\end{array}
\]

\[
\begin{array}{c}
\phantom{0}432 \\
- \phantom{0}162 \\
\hline
\phantom{0}270 \\
\end{array}
\]
Andrew Task

In March, Andrew, a second grader, solved $63 - 25 = ?$ as shown below.

Explain why Andrew’s strategy makes mathematical sense.

Please solve $432 - 162 = □$ by applying Andrew’s reasoning.

\[
\begin{align*}
432 \\
-162 \\
\hline
260 \\
\hline
270
\end{align*}
\]
Andrew Task

In March, Andrew, a second grader, solved $63 - 25 = ?$ as shown below.

Conceptual Understanding
Integrated and functional grasp of mathematical ideas

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Please solve $432 - 162 = \square$ by applying Andrew’s reasoning.

\[
\begin{array}{c}
432 \\
-162 \\
\hline
380 \\
\end{array}
\]
Andrew Task

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Integrated and functional grasp of mathematical ideas

Procedural Fluency
Knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Adaptive Reasoning
The capacity to think logically about the relationships among concepts and situations, including the ability to justify one’s reasoning both formally and informally.

Strategic Competence
The ability to formulate mathematical problems, represent them, & solve them.
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Introduction

Teachers (1st and 4th grades)
Master’s Program Students (SDSU K–8 Mathematics Education Program)
Action Researchers
Our Hopes for Our Students

Mathematics makes sense and is accessible to them.

They see themselves as able to be successful in mathematics.
Our Work

• Initial survey
• Interviews, teacher journaling, and reflections
• Effects on our colleagues
Our Learnings — 1

Paying attention to what students say and think helps us by enabling us to

• better know our students,

• tailor how we teach mathematics, and

• tailor how we teach about mathematics.
Our Learnings—2

Reflections help students be more aware.

Reflect starters include:

Today in math, I…

learned  noticed
realized  heard
understood  didn’t understand
discovered  thought
said  
____________
Our Learnings—3

Our dispositions affect our students.

Student comments

“I learned that sometimes rules can trick you. So be very careful.”

“I learned how to use different strategies to get the same answer.”

What students profess to believe does not always match their behaviors.
Our Learnings-4

We found unexpected personal connections between students and mathematics:

“Today in math I learned math can let you down and it can prove you wrong or right, and sometimes it’s awesome but other times it frustrates me.”

“Today in math I noticed equal signs are evil.”

“Today in math I noticed cubes are awesome!”

“I hate shapes! They complicate my life.”
Our Learnings—4

Intensity of Personal Connections to Mathematics

“Today in math I learned math can let you down and it can prove you wrong or right, and sometimes it’s awesome but other times it frustrates me.”

“Today in math I noticed equal signs are evil.”

“Today in math I noticed cubes are awesome!”

“I hate shapes! They complicate my life.”
Relationship Among Traits/Beliefs

- confidence
- teacher as holder of knowledge
- success determined by self
- hard work leads to success
- math understanding takes time or focus
Relationship Among Traits/Beliefs

- math understanding takes time or focus
- hard work leads to success
- teacher as holder of knowledge
- success determined by self
- confidence

Figure 1: Co-occurrence of traits or beliefs with the trait.
Relationship Among Traits/Beliefs

- mistakes as a site for learning
- teacher as holder of knowledge
- success determined by self
- hard work leads to success
- math understanding takes time or focus

83% confidence
100% mistakes as a site for learning
83% math understanding takes time or focus
83% teacher as holder of knowledge
83% success determined by self
100% hard work leads to success
Relationship Among Traits/Beliefs

- mistakes as a site for learning
  - math understanding takes time or focus
    - hard work leads to success
- confidence
- teacher as holder of knowledge
- success determined by self

- 100%
- 83%
Hard work leads to success

Math understanding takes time or focus

Confidence

Teacher as holder of knowledge

Success determined by self

Mistakes as a site for learning

Relationship Among Traits/Beliefs
Our Questions

What are the reasons students engage in mathematics (the buy-in)?

• Interest (or enjoyment)
• Usefulness (or purpose)
• General attitude toward learning

Are all of these necessary for a child to have a productive disposition?

Are productive dispositions stable?
Discussion