Making the Most of Story Problems

Victoria R. Jacobs
San Diego State University

Rebecca C. Ambrose
University of California – Davis

This manuscript will appear as an article in Teaching Children Mathematics.
Story problems are an important component of the mathematics curriculum, yet many adults shudder when remembering their experiences with them, often imagining elusive train problems in algebra. In contrast, research has shown that story problems can be powerful tools for engaging young children in mathematics, and many children enjoy making sense of these situations (National Council of Teachers of Mathematics, 2000; National Research Council [NRC], 2001). Of critical importance is honoring children's approaches to story problems so that they are constructing strategies that make sense to them rather than parroting strategies they do not understand.

To explore how teachers can capitalize on the power of story problems, we chose to study teacher-student conversations in problem-solving interviews in which a K–3 teacher worked one-on-one with a child. The skills needed for productive interviewing are the same as those needed in the classroom: Teachers must observe, listen, question, design follow-up tasks, and so on. We focused our investigation on interviews because interviews isolate these important teacher-student conversations from other aspects of classroom life.

After analyzing videotaped problem-solving interviews conducted by 65 teachers interviewing 231 children solving 1018 story problems, we identified eight categories of teacher moves (i.e., intentional actions) that, when timed properly, were productive in advancing mathematical conversations. We separately considered (a) the supporting moves that a teacher used before a child arrived at a correct answer and (b) the extending moves that a teacher used after the correct answer was given. We want to be clear that the eight categories of teacher moves we present are not intended to be a checklist that a teacher executes on every problem. Instead, we consider these moves to be a toolbox from which a teacher can draw, after consideration of the specific situation and instructional goal(s). The most effective moves arise, in the midst of instruction, in response to what a child says or does, and therefore these moves
cannot be preplanned. Because strategically responding to a child's mathematical thinking is challenging, we identified our eight categories of teacher moves in an effort to assist teachers in this decision making.

**Supporting a Child's Mathematical Thinking Before a Correct Answer Is Given**

When a child struggles or has the wrong answer, a teacher must determine how and when to intervene to facilitate the child's moving forward without taking over the child's thinking. Supporting a child's mathematical thinking requires the teacher to “enter the child's mind” (Ginsburg, 1997) as much as possible to determine what the source of difficulty might be, and then the teacher’s hypotheses about a child’s thinking should drive the choices made. Because “entering the child’s mind” can be quite difficult, a teacher needs to be flexible and prepared to explore various approaches for supporting. In our analysis, we identified four categories of moves that teachers regularly used to support a child's thinking before arrival at a correct answer (see Table 1).

**Make Sure That the Child Understands the Problem**

A teacher can provide support by helping a child develop an understanding of the problem to be solved. Typical teacher moves include rereading a problem multiple times and asking a child about specific quantities in a problem (e.g., "How many puppies are in the park?"). A twist on this repetition is to ask a child to explain a problem in his or her own words. In listening to a child describe a story problem in its entirety, a teacher can pinpoint what a child does and does not understand.

Rephrasing or elaborating a story can also help to engage a child. Often, this elaboration involves use of a more familiar context or personalization so that the child (or the child's friends) are characters in the story. For example, a kindergartner was asked to solve this problem: The teacher has 12 pencils and three baskets. If she wants to put the same number of pencils in each
basket, how many pencils should she put in each basket? The child made a pile of 15 cubes and kept rearranging them. In response, the teacher (Mr. Reynolds) decided to elaborate and personalize the problem by involving their classroom and making himself the teacher in the story.

Let me change it a little bit. Let’s try this. Mr. Reynolds has three baskets. I have three baskets, and I have 12 pencils in my hand, and I say, “I’ve got to do something with these pencils. I can’t walk around with them all day! What am I going to do with these pencils? Oh, here’s what I’ll do. I’ll put some in each basket so the kids can come get them.” But then I think, “I’d better put the same number in each basket. Because if I put like 2 in one basket and 10 in one basket, that’s not fair. So I have to put the same number of pencils in each basket.” How many pencils would I put in each one of those baskets so that all the baskets would have the same number of pencils inside?

This elaborated story did not change the mathematical structure of the problem but did make the problem more real for the child, and in this case, she solved the problem correctly by using trial and error to create three piles of four cubes each. Elaborating a story may seem counterintuitive because it goes against the traditional approach of helping a child identify keywords or irrelevant information in story problems. However, when elaboration is designed to make a problem more meaningful, a child is more likely to avoid mechanical problem-solving approaches and instead work to make sense of the problem situation.

**Change the Mathematics in the Problem to Match the Child's Level of Understanding**

When a child does not understand a problem, even after attempts to rephrase or elaborate it, changing the problem itself can be productive. One type of change is to use easier numbers. Specifically, using smaller or friendlier numbers (e.g., decade numbers) can help a child gain access to the mathematics underlying a problem. After making sense of an easier problem, a child generally gains confidence and, in many cases, can then make sense of the original problem.
Similarly, because research has shown that children have more difficulty with problems having some structures than others, another type of change is to use an easier mathematical structure (Carpenter, Fennema, Franke, Levi, and Empson, 1999). For example, a first grader was asked to solve this problem: *Twelve mice live in a house. Nine live upstairs. How many live downstairs?* Because part-whole problems (like this mice problem) do not have an explicit joining or separating action, children often do not know how the quantities relate. This child made a set of 9 cubes, made a set of 12 cubes, and joined them to get 21. After several unsuccessful attempts to help the child understand the problem, the teacher chose to change the problem to include an explicit separating action. Specifically, the teacher explained, “Nine of those mice are going to go upstairs and watch TV.” In response, the child separated 9 mice from her set of 12, leaving a group of 3. This change in mathematical structure did more than allow the child to solve a problem correctly. By providing the child access to an easier but related problem, the teacher created opportunities for discussing the quantities and relationships in both problems. Thus with further skilled questioning, the teacher could use the child's understanding of the second problem to help the child understand the original problem and, more generally, problems with a part-whole structure.

**Explore What the Child Has Already Done**

When struggling with a problem, a child can sometimes determine what went wrong if he or she is encouraged to articulate partial or incorrect strategies. General questions such as "Can you tell me how you solved it" or "What did you do first?" can be helpful for starting conversations, but follow-up questions require a teacher to ask about the details of a child's strategy and thus cannot be preplanned. For example, a first grader was asked to solve this problem: *One cat has four legs. How many legs do seven cats have?* The child (C) put out seven teddy-bear counters. He saw teddy-bear counters as having two legs and two arms, and therefore counted only two
legs on each teddy bear, answering 14. The teacher (T) recognized that the child’s confusion was linked to the counters he had chosen, and she posed questions to clarify how his work related to the problem context.

T: How many legs on a bear?
C: Two.
T: How many legs on a cat?
C: Four.
T: How many did you count? How many legs each did you count?
C: Two.
T: Is that how many legs cats have?
C: No, cats have 4 and bears have 2.
T: Ok, could you do that again for me?
C: First I get one cat [puts out one teddy bear counter], and then I get a bear [puts out another teddy bear counter], and this cat has 4 legs and the bear has 2 legs.
T: Are there bears in the story?
C: No, there’s cats.

This dialogue continued for some time before the child solved the problem correctly by counting four legs on each bear and then again by using a different tool. The support the teacher provided began with what the child had already done, and through specific questioning, she helped him make sense of how his initial strategy was related (and not related) to the problem. Note that she could not have preplanned this conversation, because it grew out of her careful observation of the child's way of using the teddy-bear counters.

**Remind the Child That Other Strategies Can Be Used**

Sometimes a child gets lost in a particular strategy, and instead of abandoning that strategy for a more effective one, the child persists in using it in unproductive ways. A teacher can help by nudging a child to think more flexibly and to try alternative approaches. A simple suggestion to try a different tool or a different strategy can sometimes give a child permission to move on and self-redirect. At times, a teacher may also find suggesting a particular tool or reminding a child of strategies used in the past to be beneficial. For example, a first-grade child was asked to solve this problem: *Let’s pretend we’re out at the snack tables, and 4 seagulls came to the snack*
tables. And then 7 more seagulls came to the snack tables. How many seagulls are at the snack tables? The child first counted to 4, raising one finger with each count. She then put those 4 fingers down. Next, she counted to 7, raising one finger with each count. At this point, the child was baffled, staring at her fingers. The teacher suggested, “Want to try it with cubes?” The child immediately made a stack of 4 unifix cubes and a stack of 7 unifix cubes, and then counted them altogether to get an answer of 11. She was confident and efficient once she started using the unifix cubes. The teacher did not tell the child how to solve the problem but did encourage her to consider using a tool that was more conducive to representing both sets; the child did not have enough fingers to show 7 and 4 at the same time! This support reflected the teacher’s understanding of children’s direct-modeling strategies in which they represent both sets before combining them.

**Extending a Child's Mathematical Thinking After a Correct Answer Is Given**

Solving a story problem correctly using a valid strategy is an important mathematical endeavor. However, we view problem solving as a context for having mathematical conversations, and this conversation need not end when the correct answer is reached. Instead, a teacher can pose additional questions to help a child deepen his or her understanding of work completed and connect it to other mathematical ideas. We have identified four categories of moves that teachers regularly used to extend a child's thinking after arrival at a correct answer (see Table 2).

**Promote Reflection on the Strategy the Child Just Completed**

Once a child has correctly solved a problem, a teacher can ask for a strategy explanation or for clarification about how a particular strategy used makes sense with the quantities and mathematical relationships expressed in the problem. Articulation of these ideas can reinforce a child's understandings and give a teacher a window into those understandings. Again, attention
to details matters. Similar to the supporting questions intended to explore a child's partial or incorrect strategies, teachers' extending questions were most productive when they were specific and in response to the details of what a child had already said or done.

For example, a second grader was asked to solve this problem: *This morning I had some candy. Then I gave you 5 pieces of candy. Now I have 6 pieces of candy left. How many pieces of candy did I have this morning before I gave some to you?* The child quickly solved this problem mentally and explained, “Five plus 5, if you took 1 away, is 10 and then 1 more is 11, so you had 11.” Children often provide correct answers to problems with this structure, in which the initial quantity is unknown, without really understanding what they are finding. In this case, the teacher probed the child's thinking in relation to this issue.

T: So how did you know to add them together?
C: I don’t know. I just added them I guess.
T: Well, think about it. Why does that make sense for the problem?

The child thought about this question for some time, and eventually used unifix cubes to act out the story and convince himself (and the teacher) that 11 was the correct answer and made sense with the story. By asking the child to reflect further on his strategy, the teacher ensured that he was making sense of the mathematics.

**Encourage the Child to Explore Multiple Strategies and Their Connections**

A child needs opportunities not only to solve problems but also to explore the mathematical connections among multiple strategies for the same problem. One approach is to ask a child to generate a second strategy—any strategy—to a problem that he or she has already solved. Another approach is to ask for a second strategy that is connected to a child's initial strategy in deliberate ways. For example, a third grader using base-ten blocks to represent 12 pages of 10 spelling words per page put out 12 ten rods but counted all 120 blocks by ones! The teacher
built on this initial strategy by asking her to count the blocks another way. The child responded by counting by tens and even shared that this second strategy was easier.

Another way a teacher can deliberately build on a child's initial strategy is to ask for a mental strategy that is an abstraction of work with manipulatives. For example, a third grader was asked to solve this problem: *There are 247 girls on the playground and there are 138 boys on the playground. How many kids are on the playground?* The child initially represented both quantities with base-ten flats, rods, and single cubes. Next he combined the hundred flats (3), combined the ten rods (7), combined some of the single cubes to make 10, traded the 10 single cubes for 1 ten rod making a total of 8 ten rods, and finally counted the remaining single cubes (5) to answer 385. The teacher then asked, “Doing just what you did with the materials, could you solve that problem in your head?” The child looked at the numbers and abstracted what he had just done with the cubes. Specifically, he explained that he could add 100 to 200 to get 300 and then 30 to 40 to get 70. Next he put 2 from the 7 with the 8 to get another 10, which made 80, and had 5 ones left, so the answer was 385. When executing this mental strategy, the child articulated the underlying mathematical idea of both strategies: combine like units and, when necessary, regroup (i.e., decompose the 7 into 5 and 2 so that 2 can be combined with the 8 to make a new 10).

Through experiences with multiple strategies, a child can gain flexibility and ability to change strategies when use of one strategy is unsuccessful. A teacher can also use multiple strategies to highlight underlying mathematical ideas by asking a child to explicitly compare and contrast strategies. At times, a teacher may even ask a child to compare a successful strategy to a previous unsuccessful attempt, because, in many cases, the child will discover the reason it failed.
Connect the Child's Thinking to Symbolic Notation

When solving a story problem by drawing, using manipulatives, or computing mentally, a child may not use any symbolic notation. A teacher can encourage a child to connect his or her work with mathematical symbols by asking the child to either generate a number sentence that "goes with" the problem or record the strategy used to solve the problem. Although requesting a number sentence that "goes with" the problem is perhaps the more typical request, asking for a strategy representation can be powerful. Young children often begin recording their strategies in unconventional ways that include a mix of symbols and drawings. For example, they might draw pictures of the manipulatives they used and then add number labels to parts of those pictures. Over time, children's recordings become progressively more abstract until they are completely symbolic.

Generating a symbolic strategy representation can help a child develop meaning for, and facility with, mathematical symbols because the representation is linked with his or her interpretation of the problem. For example, a second-grade child solved the problem about the number of legs on seven cats by first putting out seven tiles (cats). Next he moved two tiles to the side and said, "Four plus 4 equals 8." He then moved another tile to the side and said, "Eight plus 4 equals 12." He continued moving one tile at a time until he had used them all, each time adding 4 more to his running total. When asked to write a number sentence to show what he had done, he wrote the following: $4 + 4 \rightarrow 8 + 4 \rightarrow 12 + 4 \rightarrow 16 + 4 \rightarrow 20 + 4 \rightarrow 24 + 4 \rightarrow 28$. Unlike the number sentence that "goes with" the problem ($7 \times 4 = 28$), this symbolic representation reflects how this child thought about and solved it. Note that the child's use of arrows instead of the equal sign avoids the incorrect use of the equal sign between expressions.
Requesting links between a child's strategies and symbolic notation is important so that the child sees the mathematics done on paper as connected to the solving of story problems. In addition, once a child becomes facile with symbolic notation, the notation itself can become a tool for problem solving and reflection. We offer a final note of caution. A teacher may initially need to support a child in recording each step of a strategy so that parts are not omitted. However, a teacher needs to be vigilant in providing support to record the child's—not the teacher's—ways of thinking about a problem.

Generate Follow-Up Problems Linked to the Problem the Child Just Completed

By carefully sequencing problems, a teacher can create unique opportunities for mathematical discussions. Although we recognize the importance of practice, we are suggesting something beyond simply giving a child additional problems to solve. We advocate that, in the midst of instruction, a teacher can consider a child's existing understandings and then modify the initial problem or create a new problem to add challenge or to encourage use of more sophisticated strategies. For example, a first grader was asked to solve this problem: *The Kumyee woman was collecting acorns. She had nine baskets, and she put 10 acorns in each basket. So how many acorns did she have altogether?* The child quickly responded, "Ninety," and explained that he had counted, "Ten, 1," putting up 10 fingers and then 1 finger. He continued, "Twenty, 2," again putting up 10 fingers and then this time, 2 fingers. He continued with this pattern of counting and finger use: "Thirty, 3; 40, 4; 50, 5; 60, 6; 70, 7; 80, 8; 90, 9." The teacher then decided to extend the child’s use of 10 by posing a related problem and asking the child to consider the connections.

T: So that’s how you got 90. What if she had nine baskets, but she put 11 in each basket instead of 10? [Child thinks for a while.] Could you use some of the work that you’ve already done—that we did during the afternoon—or would you have to start all over again? She still has nine baskets, and there’s still 10 acorns in each basket, and then she puts 1 more so that each basket has 11.
C: Ohhhhh! I get it. Well, there’s already 10 in each basket so that’s 90. So I count up 9, one more 9. I mean 9 ones. I’m going to add 9 ones. So there’s already 90 so 91, 92, 93, 94, 95, 96, 97, 98, 99.

By strategically selecting numbers and by drawing attention to the link between problems, the teacher was able to further this child's base-ten understanding by helping him recognize and use the 10 in the number 11.

Summary

Our project builds on the work on teacher questioning (see, for example, Mewborn and Huberty, 1999; Stenmark, 1991), in which lists of potential questions are provided. These lists can be important starting points for eliciting a child's thinking, but we hope that our eight categories of teacher moves (four designed to support a child's thinking and four to extend it) can help a teacher further customize questioning to make the most of story problems. These moves do not always lead to correct answers, and we reiterate that not all eight are intended to be used in every situation. However, together, they form a toolbox from which a teacher can select means to help a child solve problems and explore connections among mathematical ideas. Engaging in mathematical discussions about story problems is challenging, and we offer three final thoughts to guide participation.

- **Elicit and respond to a child's ideas.** The most effective teacher moves cannot be preplanned. Instead, they must occur in response to a child's specific actions or ideas. Thus, expertise is less tied to planning before a child arrives and more related to seeding conversations, finding the mathematics in a child's comments and actions, and making in-the-moment decisions about how to support and extend a child's thinking.

- **Attend to details in a child's strategy and talk.** Research on children's developmental trajectories has shown that subtle differences in children's strategies and talk can reflect important distinctions in their mathematical understandings (NRC, 2001). A teacher can
customize instruction on the basis of these distinctions, and by attending to details of a child's explanations and comments, a teacher also communicates respect for a child's ideas.

- **Do not always end a conversation after a correct answer is given.** Important learning can occur after a correct answer is given when a child is asked to articulate, reflect on, and build on initial strategies.

---

This research was supported in part by a grant from the National Science Foundation (ESI0455785). The views expressed are those of the authors and do not necessarily reflect the views of the National Science Foundation.
References


<table>
<thead>
<tr>
<th>Category</th>
<th>Sample teacher moves</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Make sure that the child understands the problem.</strong></td>
<td>• Ask the child to explain what he or she knows about the problem.</td>
</tr>
<tr>
<td></td>
<td>• Rephrase or elaborate the problem.</td>
</tr>
<tr>
<td></td>
<td>• Use a more familiar or personalized context in the problem.</td>
</tr>
<tr>
<td><strong>Change the mathematics in the problem to match the child's level of understanding.</strong></td>
<td>• Change the problem to use easier numbers.</td>
</tr>
<tr>
<td></td>
<td>• Change the problem to use an easier mathematical structure.</td>
</tr>
<tr>
<td><strong>Explore what the child has already done.</strong></td>
<td>• Ask the child to explain a partial or incorrect strategy.</td>
</tr>
<tr>
<td></td>
<td>• Ask specific questions to explore how what the child has already done relates to the quantities and relationships in the problem.</td>
</tr>
<tr>
<td><strong>Remind the child that other strategies can be used.</strong></td>
<td>• Ask the child to consider using a different tool.</td>
</tr>
<tr>
<td></td>
<td>• Ask the child to consider using a different strategy.</td>
</tr>
<tr>
<td></td>
<td>• Remind the child of relevant strategies he or she has used before.</td>
</tr>
</tbody>
</table>
Table 2

*Teacher Moves to Extend a Child's Thinking After a Correct Answer Is Given*

<table>
<thead>
<tr>
<th>Category</th>
<th>Sample teacher moves</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Promote reflection on the strategy the child just completed.</strong></td>
<td>• Ask the child to explain his or her strategy.</td>
</tr>
<tr>
<td></td>
<td>• Ask specific questions to clarify how the details of the child's strategy are connected to the quantities and mathematical relationships in the problem.</td>
</tr>
<tr>
<td><strong>Encourage the child to explore multiple strategies and their connections.</strong></td>
<td>• Ask the child to try any second strategy.</td>
</tr>
<tr>
<td></td>
<td>• Ask the child to try a second strategy connected to his or her initial strategy in deliberate ways (e.g., more efficient counting or abstraction of work with manipulatives).</td>
</tr>
<tr>
<td></td>
<td>• Ask the child to compare and contrast strategies.</td>
</tr>
<tr>
<td><strong>Connect the child's thinking to symbolic notation.</strong></td>
<td>• Ask the child to write a number sentence that &quot;goes with&quot; the problem.</td>
</tr>
<tr>
<td></td>
<td>• Ask the child to record his or her strategy.</td>
</tr>
<tr>
<td><strong>Generate follow-up problems linked to the problem the child just completed.</strong></td>
<td>• Ask the child to solve the same or a similar problem with numbers that are more challenging.</td>
</tr>
<tr>
<td></td>
<td>• Ask the child to solve the same or a similar problem with numbers that are strategically selected to promote more sophisticated strategies.</td>
</tr>
</tbody>
</table>