Engaging Prospective Elementary School Teachers in Mathematics Through Children’s Mathematical Thinking*

Joint MAA/AMS Meeting

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San Diego State University

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Start With an Example

What do teachers need to understand about rational number division to help develop their students’ mathematical proficiency?
What is *mathematical proficiency*?

- Concepts
- Procedures
- Problem Solving
- Reasoning and Justifying
- Positive Outlook
Define Mathematical Proficiency

- Concepts (Conceptual Understanding)
- Procedures (Procedural Fluency)
- Problem Solving (Strategic Competence)
- Reasoning and Justifying (Adaptive Reasoning)
- Positive Outlook (Productive Disposition)

The Strands of Mathematical Proficiency

The Strands of Mathematical Proficiency

Integrated and functional grasp of mathematical ideas
The Strands of Mathematical Proficiency

Knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.
The Strands of Mathematical Proficiency

The ability to formulate mathematical problems, represent them, and solve them.
The Strands of Mathematical Proficiency

The capacity to think logically about the relationships among concepts and situations, including the ability to justify one’s reasoning both formally and informally.
The Strands of Mathematical Proficiency

The tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics.
Example of Mathematical Disposition

\[
\begin{array}{c}
70 \\
\underline{- 23} \\
53
\end{array}
\quad \begin{array}{c}
76 \\
\underline{- 23} \\
53
\end{array}

“Yes, math is like that sometimes.”
Which of these strands do your students develop?
What Mathematics Do PSTs Need to Learn?

Mathematical Knowledge for Teaching

(Ball, Hill, & Bass, 2005; Hill, Sleep, Lewis, & Ball, 2007)

*Common content knowledge*—the mathematical knowledge teachers are responsible for developing in students

*Specialized content knowledge*—mathematical knowledge that is used in teaching but not directly taught to students.
Common Content Knowledge

Divide $1\frac{1}{2} \div \frac{1}{3}$
Common Content Knowledge

Divide $\frac{1}{2} \div \frac{1}{3}$

Specialized Content Knowledge

How might students make sense of these tasks?

$1 \div \frac{1}{3}$  
$1\frac{1}{2} \div \frac{1}{3}$
Content Knowledge of Interest to Us

• Place Value
• Meanings of Operations
Andrew Task

In March, Andrew, a second grader, solved $63 - 25 = ?$ as shown below.

Explain why Andrew’s strategy makes mathematical sense.

Please solve $432 - 162 = \square$ by applying Andrew’s reasoning.

\[
\begin{array}{c}
432 \\
- 162 \\
\hline
270
\end{array}
\]
STEP Participant Groups

Practicing Teachers
(average of 15–16 years of teaching experience per group)

• Emerging Teacher Leaders (ETLs): At least 4 years of sustained professional development
• Advancing Participants (APs): 2 years of sustained professional development
• Initial Participants (IPs): 0 years of sustained professional development

College Students
• Prospective Teachers (PSTs): Undergraduates enrolled in a first mathematics-for-teachers content course
• Mathematically Strong Students (MSSs): Students enrolled in advanced college mathematics courses
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<thead>
<tr>
<th>Type</th>
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Andrew Task  
(Alternative Subtraction Algorithm)

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\text{PSTs} & \quad n = 36 \\
\text{IPs} & \quad n = 32 \\
\text{APs} & \quad n = 31 \\
\text{MSSs} & \quad n = 33
\end{align*}$
### Andrew Task

(Alternative Subtraction Algorithm)

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Division Task

The teacher needs to put 15 flowers in vases. Each vase can hold 5 flowers. How many vases does she need?
## Division Task

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<td>APs</td>
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- PSTs: $n = 36$
- IPs: $n = 32$
- APs: $n = 31$
- ETLs: $n = 33$
- MSSs: $n = 33$
## Division Task

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<td>MSSs</td>
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<td>2.48</td>
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</table>
One’s Task

Below is the work of Terry, a second grader, who solved this addition problem and this subtraction problem in May.

Problem A

\[
\begin{align*}
    &\phantom{+}259 \\
+ &\phantom{+}38 \\
\hline
    &\phantom{+}297
\end{align*}
\]

Problem B

\[
\begin{align*}
    &\phantom{-}429 \\
- &\phantom{-}34 \\
\hline
    &\phantom{-}395
\end{align*}
\]
1) Does the 1 in each of these problems represent the same amount? Please explain your answer.

2) Explain why in addition (as in Problem A) the 1 is added to the 5, but in subtraction (as in Problem B) 10 is added to the 2.
# One’s Task

(Place Value in Context of Algorithms)

<table>
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## One’s Task

*(Place Value in Context of Algorithms)*

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- **IPs**: $n = 32$
- **APs**: $n = 31$
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- **MSSs**: $n = 33$
# One’s Task

(Place Value in Context of Algorithms)

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<tr>
<td>MSSs</td>
<td>$n = 33$</td>
<td>0.94</td>
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</tbody>
</table>
1) Does the 1 in each of these problems represent the same amount? Please explain your answer.

"In Problem A you are adding one. In problem B you are adding 10."

2) Explain why in addition (as in Problem A) the 1 is added to the 5, but in subtraction (as in Problem B) 10 is added to the 2.

"Because in A you need to carry the 1 in 17 over. But in B you are taking the 1 from the 4."
1) Does the 1 in each of these problems represent the same amount? Please explain your answer.

“No, in the addition problem it represents a ten because $9 + 8 = 17$, but in the subtr. problem it represents 100 or 10 tens.”

2) Explain why in addition (as in Problem A) the 1 is added to the 5, but in subtraction (as in Problem B) 10 is added to the 2.

“In the addition problem it is a ten from the 17 so we add one more ten to the 50 + 30. In the subtraction problem it’s not 1 more ten it’s 10 tens so we are taking 3 tens away from 12 tens.”
The Problem

Even when students attend a thoughtfully planned mathematics course for prospective elementary school teachers (PSTs), too many go through the course in a perfunctory manner.

(Ball, 1990; Ma, 1999; Sowder, Philipp, Armstrong, & Schappelle, 1998)

Many PSTs do not seem to value the experience.
Why?

Many PSTs hold a complex system of beliefs that supports their not valuing the learning of more mathematics. For example, consider the effect of believing the following:

“If I, a college student, do not know something, then children would not be expected to learn it. And if I do know something, then I certainly don’t need to learn it again.”
Should we try to get PSTs to care more about mathematics for mathematics sake?

This is one starting point.

We took a different starting point, building on the work of Nel Noddings (1984).
Why not start with what PSTs care about?

Children!
-Darling-Hammond & Sclan, 1996
Circles of Caring

Start with that about which prospective teachers care.
When PSTs engage children in mathematical problem solving, their circles of caring expand to include children’s mathematical thinking.
And when PSTs engage with children’s mathematical thinking, they realize that, to be prepared to support children, they must themselves grapple with the mathematics, and their circles of caring extend.
This idea is at least 100 years old.

John Dewey, 1902

Every subject has two aspects, “one for the scientist as a scientist; the other for the teacher as a teacher” (p. 351).

“The teacher] is concerned, not with the subject-matter as such, but with the subject-matter as a related factor in a total and growing experience [of the child]” (p. 352).

Looking at mathematics through the lens of children’s mathematical thinking helps PSTs come to care about mathematics, not as mathematicians, but as teachers.
We tested this theory.

Major Research Question—Does learning about children’s mathematical thinking early in their undergraduate experience affect PSTs’ beliefs and mathematical content knowledge? If so, how?
The Role of Beliefs

A Guiding Question—
Imagine that you are teaching a mathematics course for prospective elementary school teachers. If you could identify one belief that, if held by your students, might increase the likelihood that they would be poised to benefit from what you had to offer, what belief would you change?
Design of Study

- We conducted a large-scale experimental study \((N = 159)\) of PSTs enrolled in the first mathematics courses for Liberal Studies majors. The PSTs were randomly assigned to one of four treatments or to a control.
- In two treatments, the students observed classrooms (“convenient” or “select”) and wrote reflections.
- In two treatments, the students learned about children’s mathematical thinking (by only viewing video or by a combination of viewing video and working with children).
- PSTs in a control group only took the mathematics course.
- All students completed pretests and posttests using a specially developed web-based beliefs survey with open-ended responses and a paper-and-pencil content instrument.
- All data coding was done by trained outsiders; data were blinded as to pre/post and treatment.
Major Result

Prospective elementary school teachers who studied children’s mathematical thinking while learning mathematics improved their mathematical content knowledge and developed more sophisticated beliefs about mathematics, mathematical understanding, and learning than prospective elementary school teachers who did not learn about children’s mathematical thinking.
Average Percentages of Students in Each Beliefs Change-Score Category, by Treatment

![Bar chart showing the average percentages of students in each beliefs change-score category, by treatment. The categories are: Large Increase, Small Increase, No Increase. The treatments are: CMTE-L, CMTE-V, MORE-R, MORE-T, Control.](chart.png)
<table>
<thead>
<tr>
<th></th>
<th>CMTE-L</th>
<th>CMTE-V</th>
<th>MORE-S</th>
<th>MORE-C</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Increase</td>
<td>38.6%</td>
<td>36.1%</td>
<td>18.6%</td>
<td>6.9%</td>
<td>13.3%</td>
</tr>
<tr>
<td>Small Increase</td>
<td>30.0%</td>
<td>32.7%</td>
<td>27.9%</td>
<td>26.9%</td>
<td>30.3%</td>
</tr>
<tr>
<td>No Increase</td>
<td>31.4%</td>
<td>31.1%</td>
<td>53.6%</td>
<td>66.3%</td>
<td>56.4%</td>
</tr>
<tr>
<td>Ratio Large/No</td>
<td>1.23</td>
<td>1.16</td>
<td>0.35</td>
<td>0.10</td>
<td>0.24</td>
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*Note.* Because of rounding, some treatment totals do not sum to 100%.
## Content Data

Significant Differences Between CMTEs and Non-CMTEs  
(Effect Size = 0.26)

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Pretest average</th>
<th>Posttest average</th>
<th>Average change score (Standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMTEs ((n = 77))</td>
<td>37.6</td>
<td>51.9</td>
<td>14.3 ((7.87))</td>
</tr>
<tr>
<td>Control/MOREs ((n = 82))</td>
<td>36.3</td>
<td>48.5</td>
<td>12.2 ((8.21))</td>
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</table>
Implications

The notion that we first teach prospective elementary school teachers mathematics content and later address issues of teaching and learning was turned on its head in this study.

Integrating PSTs’ learning of content with working with children supports the development of their beliefs and knowledge.

When thinking about connecting PSTs and children (EFEs), it is not whether we put them together, it is how.

There is no holy grail in teacher education. Some PSTs will learn in almost any environment, and some PSTs will not learn in almost any environment.
Where Might We Infuse Children’s Mathematical Thinking?

• Education Courses
• Mathematics Courses
• Hybrid Courses
How did the focus on children’s thinking help the PSTs?
Effects of CMTE-L on Learning in Math 210 and in Subsequent Courses

Phil: (strong mathematics student) One thing I got out of 296 [CMTE-L]—if I hadn’t taken 296, I probably would have gone through 211, 312, 313 [the subsequent mathematics courses] focusing on the thing that I already knew, the algorithm that I already knew, and thinking, “All right, that’s the best.” But now I realize that I have to take it all in, everything that the class is teaching, not just what I think is the most important. Because all of this is important. I probably wouldn’t have realized that if I hadn’t taken 296.

RP: Why is it important?

Phil: Because people think in different ways, and not everyone thinks like me.

RP: And you don’t think you would have gotten that from [Math 210] alone?

Phil: No. No way.
An Example Task

NAEP ITEM: 13-year olds

Estimate the answer to $\frac{12}{13} + \frac{7}{8}$. You will not have time to solve the problem using paper and pencil.

a) 1  b) 2  c) 19  d) 21  e) I don’t know

b) 24%

e) 14%

a) 7%
PSTs Speak for Themselves

Directly after working with students

T: How did you feel during the interview today?
S: Stupid.
T: What was that?
S: No, I felt---I---well, on the estimation problem, my kid---or my kid. Michael figured out the problem before I did. Like Kimmie asked him---she's all, "Estimate the sum of nine tenths plus six sevenths." And she said, "Shut your eyes." And he had his eyes shut. And at the same time, I was trying to estimate it, but I was thinking like, "Common denominator." I was like, "Oh," thinking. And then he was like, "Two." And I was like, "Two?"

S: I was like, "Why is it 2?" And I had him explain it to me. And he explained it to me. And I was just like, "Oh, yeah. Thanks."
S: I felt stupid. I was like, "Oh, my gosh." But yeah.
T: But you were willing to listen to your child's thinking. That's great.

Author’s note. Because the video is not available online, the transcript is provided.

PSTs Speak for Themselves

Reflecting upon the class

CMTE Clip, 0:14–0:56

For people who are going to take [the first mathematics class]—just because *a lot* of the times in class . . . people get *so* mad and *so* frustrated as to why they are learning what they are learning. And then you come [to the CMTE-L], and you see a kid do exactly what you are learning in [the mathematics] class. And it just makes sense, and it eliminates that whole frustration of feeling like “Why am I learning this? Where am I going to ever use this?” So by taking this class, you see how ... the children actually apply what you are learning, the different styles or the different methods for solving problems.

Author’s note. Because the video is not available online, the transcript is provided.
Some Examples of Children’s Mathematical Thinking

Each example was selected to raise particular issues with which we wanted the PSTs to grapple.
Javier, Grade 5

At the time of this interview, Javier had been in the United States about one year, and he did not speak English before coming to this country.

See Javier video available as Clip 1 at http://www.sci.sdsu.edu/CRMSE/IMAP/video.html

One Representation of Javier’s Thinking

\[ 6 \times 12 = (5 \times 12) + (1 \times 12) \quad \text{(Distributive prop. of x over +)} \]
\[ = \left(\frac{1}{2} \times 10\right) \times 12 + 12 \quad \text{(Substitution property)} \]
\[ = \left[\frac{1}{2} \times (10 \times 12)\right] + 12 \quad \text{(Associative property of x)} \]
\[ = \left[\frac{1}{2} \times (120)\right] + 12 \]
\[ = 60 + 12 \]
\[ = 72 \]

Question for PSTs
What would a teacher need to know to understand Javier’s thinking?
Ally, End of Grade 5
An average 5th-grade student at a high-performing local school

Questions for PSTs
How did Ally come to hold these conceptions?
What do teachers need to know to reduce the occurrence of these conceptions in their students?

Author’s note. Because the video is not available online, the transcript is provided. See transcript of this video on the next three slides.

I: So let's start with these questions. What I'd like you to do, Ally, is look at these two numbers and circle whichever one you think is bigger, or put a---write an equal sign, if you think they're the same.

I: Okay?

C: Okay?

I: Okay?

C: Should I just circle the denominator, or the---?

I: You can circle the whole number. That's fine.

C: Okay.

I: Okay. Why don't---let's go through each pair. And you can tell me how you were thinking about them, so I know.

C: Okay.

I: So what were you thinking here?

C: Well, I kind of messed up. I was thinking one third, more likely that's bigger. Because if you changed the digit down more 3, and then if it was 1 one, then it would have to be ... it would equal to 1. So ... and 1 is a whole number. So it's bigger. And on this one---

I: Wait. So---so which one's bigger here? \[
\frac{1}{6} \quad \frac{1}{3}
\]

C: Umm, one third.

I: Oh, you think one third is bigger?

C: Yes.
• I: Okay. How about here? $\left[ \frac{4}{3} \right]$
• C: One.
• I: And why is that?
• C: I thought 1 was bigger, because it's a whole number.
• I: Aah.
• C: And it's just one group of just a one number.
• I: Okay. And how about here? $\left[ \frac{3}{6} \quad \frac{1}{2} \right]$

• C: Umm, I chose one half. Because if you changed the denominator to 1, just one digit lower, then it would equal to 1. And 1 is a whole number, so ....
• I: Okay. And how about here? $\left[ \frac{1}{7} \quad \frac{2}{7} \right]$

• C: I chose one seventh, because ... I wasn't quite sure about this one. So I chose one seventh, because I thought it was just the smallest number. And usually, you go down to the smallest number to get to the biggest number.
• I: With fractions?
• C: Yes.
Transcript of Ally Video
(Part 3 of 3)

- I: And how about here? $\left[ \frac{3}{10}, \frac{1}{2} \right]

- C: I chose one half, because it's like, umm ... one half in this picture, it's like ... I could just change the bottom number one more digit, then---and it would be 1.

- I: I see. How about if I gave you this, then? $\left[ \frac{1}{2}, \frac{4}{6} \right]

- I: Which would you say is bigger there?
- C: I'd still say one half.
- I: Okay. Do you want to circle that, so we'll remember that?
- I: And then this one, you wanted to change, right?
- C: Yes.
- I: Okay. So why don't I circle this? And that way, we'll remember.
- C: Okay.
- I: Okay?
Felisha, End of Grade 2

Felisha learned fraction concepts using equal-sharing tasks in a small group over 14 sessions (7 days). She had not been taught any procedures for operating on fractions.

See Felisha video available as Clip 2 at http://www.sci.sdsu.edu/CRMSE/IMAP/video.html

**Question for PSTs**

What do teachers need to know to help students make sense of fractions as Felisha has done?

George Poole, personal communication, November 12, 2001

I have used the tape to show my prospective elementary teachers the kind of creative and "different" thinking students use to reason and make calculations. The video clips became motivational clips and saved me having to make the argument for PUFM (Profound Understanding of Fundamental Mathematics, Ma, 1999).
Rachel, Grade 5

Rachel’s teacher valued sense-making. We asked this teacher to teach students to convert between mixed numbers and improper fractions using a lesson based upon a state-adopted CA textbook. After the lesson, we interviewed Rachel.

•I: And this time, I am going to ask you to change this $3 \frac{3}{8}$ into an improper fraction.
•C: Okay.
•C: We did this before. But I don't exactly remember it as well, because I didn't figure it out for myself. So ....
•I: Tell me about that. What do you mean, you didn't figure it out for yourself?
•C: Well, when she like tells us the answer to something, then I try and find out how she got it. And so when I figure that out, it's easier. And, umm ... and once I figure it out, it's ... it stays there, because I was the one who brought it there. So ... and it's just easier to do, when you figure it out yourself---
•I: Is that---
•C: ---instead of having teachers telling you.
•I: Is that how she usually does it, or ... ?
•C: Yeah. Yeah. And then this little time, it was different. And it was harder.

Question for PSTs
What expectations about mathematics learning do you think Rachel holds? Compare her expectations to those you hold.
Thank you.

Questions?
Comments?
“Those who can, do. Those who understand, teach.”

On the office door of Alba Gonzalez Thompson, 1946 – 1996

(originally from Shulman, 1986).