Helping Students Develop Confidence to Learn Mathematics

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Your current thinking

• Name something you currently do when teaching that helps your students develop confidence to learn mathematics.

• Is there anything you currently do that might (unintentionally) inhibit students’ developing confidence?
Border Problem

A castle is divided into 36 square sections to form a 6x6 grid (see diagram below). The king and queen tell their children that they should not play in the border regions of the castle because there is danger of attack from hostile neighbors. How many squares are in the forbidden border region?

Can you solve the problem more than one way?
Define Mathematical Proficiency

• Concepts
• Procedures
• Problem Solving
• Reasoning and Justifying
• Positive Outlook
Define Mathematical Proficiency

- Concepts (Conceptual Understanding)
- Procedures (Procedural Fluency)
- Problem Solving (Strategic Competence)
- Reasoning and Justifying (Adaptive Reasoning)
- Positive Outlook (Productive Disposition)

The Strands of Mathematical Proficiency

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Integrated and functional grasp of mathematical ideas

The ability to formulate mathematical problems, represent them, and solve them.

The capacity to think logically about the relationships among concepts and situations, including the ability to justify one’s reasoning both formally and informally.

Knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

The tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics.

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3. The ability to formulate mathematical problems, represent them, and solve them.

4. Knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Integrated and functional grasp of mathematical ideas
Two Teachers’ Experiences Supporting Students’ Productive Dispositions

Teachers (1st and 4th Grades)
Master’s Program Students (SDSU K–8 Mathematics Education Program)
Action Researchers
Our Hopes For Our Students

Mathematics makes sense and is accessible to them.

They see themselves as able to be successful in mathematics.
Our Work

- Initial survey
- Interviews, teacher journaling, and reflections
- Effects on our colleagues
Our Learnings-1

Paying attention to what students say and think helps us by

• enabling us to better know our students;
• tailor how we teach mathematics;
• tailor how we teach *about* mathematics.
Our Learnings-2

Reflections help students be more aware.

Today in math, I …

- learned
- realized
- understood
- discovered
- said
- noticed
- heard
- didn’t understand
- didn’t understand
- thought
- ____________
Our Learnings-3

Effects of Our Dispositions on Our Students

“I learned that sometimes rules can trick you. So be very careful.”

“I learned how to use different strategies to get the same answer.”

What students profess to believe does not always match their behavior.
Our Learnings-4

An Unexpected Personal Connection

“Today in math I learned math can let you down and it can prove you wrong or right and sometimes it’s awesome but other times it frustrates me.”

“Today in math I noticed equal signs are evil.”

“Today in math I noticed cubes are awesome!”

“I hate shapes! They complicate my life.”
Our Learnings-4

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“Today in math I noticed cubes are **awesome**!”

“I **hate** shapes! They complicate my life.”
Relationships Among Traits/Beliefs

- Confidence
- Teacher as holder of knowledge
- Success determined by self
- Math understanding takes time or focus
- Hard work leads to success
Figure 1. Co-occurrence of mistakes as a site for learning with other traits.

Relationships Among Traits/Beliefs

- **teacher as holder of knowledge** (83%)
- **success determined by self** (83%)
- **confidence** (83%)
- **hard work leads to success** (100%)
- **math understanding takes time or focus** (100%)
- Question mark (100%)
Figure 1. Co-occurrence of mistakes as a site for learning with other traits

Relationships Among Traits/Beliefs

- mistakes as a site for learning
- confidence
- teacher as holder of knowledge
- success determined by self
- hard work leads to success
- math understanding takes time or focus

100% 83% 83% 83% 83%
Relationships Among Traits/Beliefs

- **confidence**
  - mistakes as a site for learning
  - teacher as holder of knowledge
  - success determined by self
  - hard work leads to success

- math understanding takes time or focus
Figure 1. Co-occurrence of mistakes as a site for learning with other traits:

- **Hard work leads to success**: 88%
- **Math understanding takes time or focus**: 75%
- **Confidence**
- **Teacher as holder of knowledge**: 75%
- **Success determined by self**: 63%
- **Mistakes as a site for learning**: 75%
Our Questions

Reasons students engage in mathematics (the buy-in)

• Interest (or enjoyment)
• Usefulness (or purpose)
• General attitude toward learning

Are all of these necessary for a child to have productive disposition?

Are productive dispositions stable?
Typical Classroom Discourse

IRE

- Initiate—Teacher poses a question
- Respond—Student provides an answer
- Evaluate—Teacher provides evaluation, then moves to the next problem or question
Discourse That Supports Development of Productive Dispositions
Wait Time

• **Wait-Time 1 (WT1)**—a pause after a teacher utterance

• **Wait-Time 2 (WT2)**—a pause after a student utterance
Wait-Time Benefits

- Providing students with time to process a question or response
- Allowing students to formulate a question about the mathematical topic at hand
- Encouraging broader participation
- Holding students accountable for thinking and doing mathematics
- Affording the teacher an opportunity to consider what teacher move to make next
REVOICING and RESTATING

Partial Revoicing involves two parts:
1) A student contributes an idea.
2) The teacher repeats or restates some or all of what the student said.
REVOICING and RESTATING

**Full Revoicing** involves four parts:

1) A student contributes an idea.

2) The teacher repeats or restates some or all of what the student said.

3) The teacher then checks back with the student to see whether they want to amend the teacher’s contribution.

4) The student confirms or amends the idea, on the basis of whether she or he agrees or disagrees with the teacher’s restatement.
In **restating**, a teacher asks students to put into their own words what another student said.

**EXAMPLES OF RESTATING**

“Can anyone put Marie’s explanation into his or her own words?”

“Carlos, would you please share what you heard Rachel say?”
EXTENDING (IDEAS OR DISCUSSIONS)

The teacher initiates a question, statement, or invitation to provide additional information with the intention of increasing the quantity and quality of student participation in the discussion.
EXTENDING (IDEAS OR DISCUSSIONS)

“What do other people think?”

“Does anyone have a question about Olivia’s method?”

“What do you think of his idea?”

“Do people agree or disagree with that explanation, and why?”

“Recently I saw a student solve the problem the following way. What do you think of this?”
Dweck

Fixed Mind Set Versus Growth Mind Set

“You are smart.”

Praising students’ innate abilities leaves children vulnerable to failure, fearful of challenges, and unmotivated to learn.
Dweck

Fixed Mind Set Versus Growth Mind Set

Praising students’ effort, strategies, focus, and persistence encourages a growth mind set.

“You did a good job drawing. I like the detail you added to the faces.”

“I like the way you tried different strategies.”

“Oh, sorry. That was too easy. Let’s do something more challenging that you can learn from.”
A Problem

Nineteen children are taking a minibus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has seven seats. How many children will have to sit 3 to a seat, and how many can sit 2 to a seat?
A Secondary-School Algebra Solution

Nineteen children are taking a minibus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has seven seats. How many children will have to sit 3 to a seat, and how many can sit 2 to a seat?

\[ x = \# \text{ of seats with 2 children}; \quad y = \# \text{ of seats with 3 children} \]

\[ x + y = 7 \quad \text{and} \quad 2x + 3y = 19 \]

\[
\begin{align*}
2x + 3y &= 19 \quad \text{and} \quad y = 7 - x \\
2x + 3(7 - x) &= 19 \\
2x + 21 - 3x &= 19 \\
-x &= -2 \\
x &= 2 \quad \text{and} \quad y = 5
\end{align*}
\]
A Problem

Nineteen children are taking a minibus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has seven seats. How many children will have to sit 3 to a seat, and how many can sit 2 to a seat?

In six classes, 51% of kindergarten students (36 of 70), correctly solved this problem in May.

How do you think they did that?

“Throughout the year children solved a variety of different problems. The teachers generally presented the problems and provided the children with counters, . . . but the teachers typically did not show the children how to solve a particular problem. Children regularly shared their strategies. —Carpenter et al., 1993, p. 433. (JRME, Vol. 24)
What will you take back to your classroom? What can you try?
Discussion

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Handouts

• Title Page
• Summary of Ideas (including strands)
• Journal Topics
• Bus Problem
• Dweck Paper
Mistakes

A) Are bad. They are to be avoided. Students will learn wrong, and it will take longer to fix.

B) Are unavoidable, darn-it. But bad.

C) Are natural, and we learn from them.

D) We encourage thinking, even if it may lead to mistakes!