

They All Hate Math: Getting Beyond our Stereotypes of Prospective Elementary School
Teachers

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They All Hate Math: Getting Beyond our Stereotypes of Prospective Elementary School Teachers

In their efforts to improve the preparation of prospective elementary school teachers to teach mathematics, mathematics educators have endeavored to understand the beliefs that prospective teachers bring with them to their coursework. Mathematics educators care about beliefs because of their powerful effect on learning (Pajares, 1992). These endeavors to understand beliefs have led to a list of beliefs that are associated with prospective teachers.

Deborah Ball (1991) attributed the following assumptions about mathematics to prospective teachers:

- Doing mathematics means following set procedures step-by-step to arrive at answers.
- Knowing mathematics means knowing “how to do it.”
- Mathematics is largely an arbitrary collection of facts and rules.
- Most mathematical ideas have little or no relationship to real objects and therefore can be represented only symbolically.

For ease of communication, I call these beliefs *traditional*. Many researchers have elaborated on this list. Gellert (2000) wrote, “It is well established that elementary teachers tend to feel uncomfortable with mathematics” (p. 251). Foss and Kleinsasser (1996) noted that prospective teachers believe that “teaching is telling and learning is memorization.” Schuck (1999) suggested that prospective teachers believed that “being good at mathematics was not necessarily helpful for teaching mathematics in primary school” (p. 118). Raymond and Santos (1995) pointed out that this set of beliefs is a natural outgrowth of the prospective teachers’ experiences in mathematics classes that follow the predictable pattern of lecture followed by seat work. I do not believe that Ball’s initial list was intended to apply to all prospective teachers, but in building on this list, others have overgeneralized it and in so doing have developed a stereotype of prospective teachers.

How accurate is this picture of prospective teachers’ beliefs? Do all prospective teachers hold all these beliefs? Asking these questions 13 years after the initial publication

of the National Council of Teachers of Mathematics *Standards* (NCTM, 1989) is particularly appropriate. At least some teachers have changed their practices, and some of our prospective teachers may have been taught by mathematics teachers who implemented at least some of the Standards. If some prospective teachers have had some experiences in problem solving, then their beliefs about mathematics may not be exclusively traditional. It is time for us as mathematics educators to reexamine our assumptions about the beliefs of prospective teachers in light of the changing practices of teachers in the United States.

In this paper I address these questions by examining data from a large-scale study of prospective elementary school teachers at the beginning of their preparation for mathematics teaching. I first outline my interest in these questions and then present data to address the questions. I conclude with observations on revising our expectations about prospective teachers.

I began to wonder about the pervasiveness of prospective teachers' beliefs while I worked with a group of colleagues on the Integrating Mathematics and Pedagogy Project. We were designing a research instrument to assess changes in prospective teachers' beliefs as the result of participating in an experimental course (see Thanheiser, 2001, for a description of the course). In designing our instrument, we developed items to capture the traditional beliefs outlined above. We expected that prospective teachers would want mathematics instruction that focused on memorizing standard algorithms. We expected that they would have little interest in conceptual understanding. In anticipating the prospective teachers' responses to our questions, we expected that the responses would be fairly similar to one another because the research literature depicted prospective teachers as homogeneous in their beliefs and attitudes toward mathematics.

We piloted the instrument with 100 prospective teachers who were at various points in their undergraduate programs. The prospective teachers in the pilot provided more varied responses than we had expected. This finding led us to revise our instrument to be more sensitive to the differences within the group. The data reported in this paper reveal the

diversity in one group of prospective teachers. I fear that we mathematics educators are not preparing our prospective teachers as well as we might, because we are blind to some positive attributes they bring to their teaching-preparation work. In this paper I attempt to dispel the inaccuracies in the stereotype outlined above and to help the mathematics education community to see the diversity that exists within the ranks of prospective teachers.

Data Source

The belief-measurement instrument was administered on the computer and contained nine free-response items that were scored using 17 rubrics. Respondents were asked to react to various situations involving children's learning elementary school arithmetic. Respondents were sometimes asked to interpret a situation, for example, by evaluating which of two subtraction procedures reflects greater mathematical understanding. In other items they were asked to make instructional decisions, for example, to choose which of several approaches they would have students in their classroom share. We chose a free-response format to measure the strength of the prospective teachers' beliefs because we believe that constructing an answer is a better belief indicator than agreeing or disagreeing with a statement, as one does in response to Likert -scale prompts.

The rubrics used to score responses are associated with particular beliefs about mathematics and about mathematics teaching and learning. Different aspects of items are scored with different rubrics. Graduate students served as readers to score the responses. To ensure that they were not biased in their scoring, students we hired had no other relationship to our project. Two readers used the same rubric to score responses; 20% of the responses were scored by both readers to obtain a reliability measure. The teams maintained at least 75% reliability for all the rubrics.

While piloting the instrument, to validate that responses on the survey were consistent with oral answers to the same question, we interviewed respondents who had completed the instrument. In the interviews, respondents were pressed to clarify their responses and to elaborate on them if they could. Students' comments during these

interviews provided additional evidence of the diversity of beliefs held by prospective teachers.

The prospective teachers completed our instrument at the beginning and end of their first mathematics-for-prospective-teachers course. To show the mathematics beliefs of prospective teachers before their beliefs have been affected by their course work, I report here on the data from the beginning of the semester when 208 students—58 freshmen, 47 sophomores, 56 juniors, 11 seniors, 10 post BA students, and 26 students who did not report their class standing—completed the computer-based survey. All were students at a regional university and were intending to become teachers. The students completed the survey before classes started or during the first week of classes and were paid for their participation. A copy of the entire survey is available at <http://66.111.76.36/imap/>.

Survey Results

This report is focused on three rubrics for items designed to measure the beliefs of prospective teachers and on the degree to which they align with the traditional view outlined above. I analyze the findings from two of the rubrics to show that prospective teachers' beliefs are more diverse than earlier studies indicated and that many hold some nontraditional views about mathematics and mathematics learning. I analyze results from a third rubric to show that most preservice teachers hold some traditional views as well. I begin by examining responses we found surprising in their diversity.

Interpretations of Alternative Approaches

We developed several items to assess the prospective teachers' beliefs about concepts and procedures. For two sets of items the prospective teachers compared various approaches for solving multidigit problems: one addition and one subtraction. I focus on the item for which respondents evaluated two approaches to multidigit subtraction. This item was included because it relates to the view that “Doing mathematics means following set procedures step-by-step to arrive at answers,” identified by Ball (1991).

Place Figure 1 about here

All of a respondent's answers to the questions in this item were considered in assigning scores. Responses were scored according to the degree to which the respondent was willing to consider approaches other than the standard algorithm for subtraction. We assumed that prospective teachers often underestimate the difficulty that children have in learning and understanding the subtraction algorithm typically taught in U.S. schools. We thought of this item as a strategy-appreciation item. Prospective teachers who believed that the focus in learning mathematics is the memorization of standard algorithms would not consider Ariana's approach worthy of attention. Prospective teachers who valued some aspect of Ariana's approach showed that they believed that mathematics is more than memorizing standard procedures.

We were surprised that the majority of the prospective teachers showed some appreciation for Ariana's alternative procedure. The prospective teachers' responses were more varied than we had expected. Several prospective teachers expressed interest in having children use strategies that made sense to them instead of exclusively using the standard algorithm. A closer examination of the distribution of scores on the item reveals the diversity of thinking among the prospective teachers in our sample (see Table 1).

Place Table 1 about here

The 36% of the prospective teachers who scored 0 expressed the traditional views that we were expecting. They thought that children would be more successful when they used the standard algorithm and wanted children to use this approach. We expected that most prospective teachers would respond much as this student responded:

Lexi [has the better understanding]. She put it in the right format and subtracted the right way, borrowing from the right numbers; 10 [out of 10 children would be successful using the standard algorithm] because it is easier to see and understand

as long as they understand the process of borrowing and subtracting single digits from right to left; 0 [out of 10 would be successful using Ariana's approach] because it is too complicated with too many different numbers. I'd want students in my class to use Lexi's; it's easier to understand and see with two numbers instead of the 8 Ariana used.

Responses that received this score reflected the view that standard algorithms are at the heart of mathematics and should be the focus of instruction. People receiving this score saw no value in Ariana's approach and wanted to avoid it.

Many responses in this category indicated a superficial examination of Ariana's strategy. One student in the pilot test stated, "Ariana has I don't think any clue what she's doing.... I think she made it a lot more difficult than it needed to be." We expected that most of the prospective teachers would be similar to this group in viewing an unfamiliar strategy as unworthy of their attention. Our expectation was based on the literature that indicated that prospective teachers are not interested in engaging in critical examination of mathematics.

We were surprised that 64.6% of the prospective teachers saw potential in Ariana's approach. This group of students did not reject it as being both too difficult and unworthy of attention in the classroom; 37.5% of the prospective teachers received a 1 according to our rubric. Their responses included *one* positive impression of Ariana's approach. Some indicated that children would be as successful using Ariana's approach as using the standard algorithm. The responses of the others in this group indicated that using Ariana's approach was appropriate for children who preferred it. The following is an example of a response scored 1:

Ariana [has the better understanding]. Ariana breaks the problem down into four problems, making it easier for her to subtract. This estimation of the problem and combination of the numbers means she knows how to break stuff down. Nine [out of 10 would be successful using Lexi's approach]. I think this is the easiest way to solve a problem for a child and that 9 out of 10 will get it right. One [out of 10 would be successful using Ariana's approach]. I think too many students would be

confused on how to break down the number into hundreds, tens and ones, and many would lose their attention or train of thought. I think both of these approaches are useful to students. Lexi's approach is simple and straightforward. I also think Ariana's approach to using estimation is helpful to students and will be especially helpful in their later math classes.

This response shows some appreciation for the thinking involved in Ariana's approach. Although this student views Ariana's approach as more difficult and confusing, she believes that children using this approach have better mathematical understanding, and she thinks that for her students to learn this way of solving problems would be valuable. This kind of response illustrated the shortcomings in our initial assumptions about the ways that prospective teacher's view alternative approaches. Even before the students enrolled in their mathematics class, some were willing to consider Ariana's alternative approach and saw merit in it.

On the presurvey 27% of the prospective teachers had *two* positive impressions of Ariana's approach. They thought that children using it would be as successful as, if not more successful than, if they had used the standard algorithm. They also thought that having children using both approaches in the classroom was appropriate. Some of these students went so far as to indicate that they felt that Ariana had greater mathematical understanding. The following is an example of a response that received a score of 2.

Lexi has the better understanding. She answered the problem more straightforward. In doing the problem without breaking it down, she has shown a greater comprehension of the material. Five [out of 10 would be successful using Lexi's]. I think that because her approach is the more difficult one. I think that it would be a difficult concept to borrow twice. I anticipate half of the students to get the problem correct. Eight [out of 10 would be successful using Ariana's approach]. Her way seems to be a more simplistic way of looking at subtraction. I think for the beginner, this would be an easier concept to grasp. I think for the beginning students that are new to subtraction, Ariana's way is a bit easier to understand. Her logic is clear and she breaks it down. Lexi's approach is more advanced. It assumes that the student is comfortable and competent in strategies (i.e., borrowing twice).

This response showed an appreciation for Ariana's approach as easier to learn and understand. In discussing the difficulty of borrowing across zeroes, the respondent showed that she recognized that the standard algorithm is difficult to understand. Prospective teachers receiving this score demonstrated a willingness to closely examine Ariana's alternative strategy, suggesting their interest in mathematics and their willingness to examine unfamiliar approaches. We were impressed that more than 25% of the prospective teachers had the mathematical disposition to contemplate the benefits of Ariana's approach.

Five percent of the prospective teachers thought that Ariana's approach was superior in *all* ways. They thought that she had greater mathematical understanding than Lexi, that children using her approach would be more successful than those using Lexi's, and that children should have opportunities to use Ariana's approach to solve subtraction problems. The following is one such prospective teacher's response, scored 3:

Ariana has the better understanding. Even though Lexi used the way most teachers would prefer. Ariana understood that $30 + 50 = 80$, and she was able to divide the problem up in a way that was easier for her. So I think that shows a good understanding of how numbers can be made simpler. Three [out of 10 would be successful using Lexi's approach] because they may not understand that you cannot go directly to the seven and borrow from it, you have to go through the other zero and then borrow from that. Six [out of 10 would be successful using Ariana's approach] because in this method of solving you don't have to borrow and you break each number down into smaller ones which makes it much easier to understand and solve. I would want the kids to first try using Lexi's approach and then go back and check their answers using Ariana's method.

We were impressed by this response. It reflected a close examination of Ariana's strategy and a recognition of the decomposition of number apparent in Ariana's strategy. The respondent not only recognized the difficulty of borrowing across zeroes but also saw that Ariana's flexible manipulation of the numbers revealed a greater understanding than the

mechanical approach used by Lexi.¹ Four of the 10 students responding in this way were freshmen who, before taking any mathematics courses at the university, recognized the limitations of standard procedures.

In summary, when comparing approaches for subtraction problems, the prospective teachers responded in a variety of ways. Many students said that they would be willing to have children use Ariana's alternative approach in their future classrooms. Many thought that children would be successful when using this approach. Many students said that Ariana had a greater understanding than the child using the standard algorithm. Slightly more than one third of the students responded in a traditional manner. The majority of the sample responded in other than a traditional manner.

Autonomous Thinking

Gellert (2000) suggested that prospective teachers want to create a safe environment in mathematics for their future students so that the students will become neither anxious nor discouraged. Along with this wish goes a desire to provide direct instruction to students, "teaching as telling," as some would say. According to Lappan and Even (1989), prospective teachers tend to believe that their role is to transmit knowledge to students rather than to provide students with opportunities to think for themselves.

One item in our instrument is related to these traditional beliefs. The prospective teachers were asked whether they would provide their future students with opportunities for autonomous thinking. They were asked to respond to the following question: "When you are a teacher, will you ever ask your students to solve a new kind of problem without first showing them how to solve it? Please elaborate on why or why not. How often will you ask your students to do this?"

¹ We recognize that the standard algorithm can be used with understanding. Many children do not, however, understand the principles underlying the procedure; their lack of understanding contributes to the many errors they make when using it. We asked the prospective teachers to choose the child with the better understanding to see whether they would acknowledge that the standard algorithm is often used by children who do not understand it.

Prospective teachers responded to this item in a variety of ways. The majority of responses reflected a willingness to provide children with opportunities to think for themselves. The distribution of responses is shown in Table 2.

Place Table 2 about here

Scored according to our rubric, about one third (32%) of the responses received a score of 0 on this item; prospective teachers giving these responses would never ask children to solve problems without first showing them how to solve them. Some likened this approach to throwing a child into a pool without first teaching him how to swim. Some suggested that this approach would lead to children's hating mathematics. Some indicated that a teacher who did not show the children how to solve problems was failing to do her job. Responses of this group reflected the traditional view that direct instruction is the best way to teach mathematics. The following is a response scored 0:

I think that being a teacher is showing and teaching the students how to do things. I think that it is a lot harder to learn without direction and the students all need to be directed in a way for each to understand the problems. How are students supposed to learn without direction and showing how things are done?

Clearly, this prospective teacher does not intend to provide her future students autonomy in their problem solving. We expected responses like this, and we expected even more of them than we found. The stereotype of prospective teachers is that they believe mathematics involves “lecture followed by seat work ... regurgitation of what [one's] teachers have spelled out in class” (Raymond & Santos, 1995, p. 58).

We were surprised that 68% of the prospective teachers expressed a willingness, for various reasons, to provide children with problems without first showing them a way to solve the problems. Scores were assigned on the basis of both the prevalence of autonomous problem solving in the prospective teacher's statements about future teaching and the rationale for the practice. Higher scores were given to responses that indicated that children's solution strategies would be an integral part of mathematics instruction.

On the item concerning willingness to let children work problems before the teacher showed them how to solve the problems, 52% of the prospective teachers scored *1*. Many of them included comments about assessment. Some suggested that the teacher could find out whether any students already had an approach to this kind of problem and not waste time teaching something the children already knew. Some suggested that the teacher might find that children had interesting or surprising ways to solve the problem. Some indicated that children would develop self-esteem by having opportunities to solve problems on their own. These respondents suggested neither that students figuring out their own approaches would understand the concepts better nor that children's solution strategies would be central to instruction.

The following is an example of a response scored *1*:

Just to see what they come up with, so they can think about it first. As a teacher I would like my students to try and solve things without knowing how at first. This way, when they are confronted with something new, they will try doing it instead of giving up simply because they don't know how to do it.

This prospective teacher expressed interest in having her students develop an adventurous disposition. Although whether she will use her students' strategies as an integral part of instruction is unclear, she saw value in providing her students opportunities for problem solving.

On this item, 15.5% of the prospective teachers received the highest score for expressing a willingness to ask children to solve problems without first showing them how to solve them. These respondents would use the children's approaches as an integral part of instruction. Many of their responses included a strong rationale for basing instruction on problem solving. In some cases they indicated that the children's approaches would be as legitimate as the teacher's and would be useful to other children.

The following is an example of a response scored *2*:

I think it is important to let students look at things in the way that makes the most sense to them first. If a child is simply shown how to do a problem without being

allowed to think about it themselves, they may memorize, but not understand. We are/will be teachers, not flash cards. I will do this every time a new concept is introduced

This prospective teacher equated understanding with having the students make sense for themselves. She pointed out that this approach was superior to students' mimicking an approach provided by the teacher. We were impressed that she had developed this view before beginning her teacher-preparation classes.

The majority of the prospective teachers expressed at least a willingness to provide children with problem-solving opportunities. Many students expressed appreciation for multiple solution strategies. Some noted the importance of assessment and finding out how children think. Some saw this approach as an opportunity for children to develop confidence in their mathematical abilities and become self-reliant. This collection of responses indicates that the majority of the prospective teachers hold nontraditional views about the role of problem solving in mathematics teaching.

The distribution of scores based on this rubric has implications for instructors of prospective teachers. The instructor can expect that about one third of the class will balk at a problem-solving focus and expect that the instructor should provide explanations. About half the class will see value in providing students with problem-solving opportunities but might also expect lectures to go with the problem-solving opportunities. The other sixth of the students, who value problem solving and see merit in focusing instruction on it, can be a resource to the instructor who engages students in problem solving. These students can articulate the power of this kind of activity and perhaps persuade some of their peers that it is worthwhile.

Multiple Representations

Prospective teachers are interested in hands-on activities, but they are uncertain how to use manipulatives to help children create meaning (Foss & Kleinsasser, 1996).

This is one of the few aspects of the stereotype of prospective teachers' beliefs that applied to most prospective teachers in the study.

Prospective teachers' understanding of the importance of manipulative models in promoting understanding was assessed using a video-based item. The students watched video of a child being shown the invert-and-multiply algorithm for division of fractions. The explanation was entirely procedural; the teacher explained the steps for dividing fractions. The child then practiced a problem on her own. The prospective teachers were asked whether the child would be able to solve a similar problem in 3 days. After answering some additional questions, the prospective teachers watched a second clip, filmed 3 days after the first, that showed that the child was unable to solve a division-of-fractions problem given then. The prospective teachers were then asked to "Provide suggestions about what the teacher might do so that more children would be able to solve a similar problem in the future." The prospective teachers' responses tended to be alike. The distribution of responses is shown in Table 3.

Place Table 3 about here

On the basis of our rubric, a large majority of the students (87%) received a 0 on responses for this item because they suggested only that the teacher provide the child with more practice or additional explanation of the procedure, or both. These students did not mention that the teacher might include representations in addition to the symbols to help the child successfully solve such problems. The following are three students' responses to this item:

Continue to review it a couple of minutes each day so it sticks in their head.

Teacher may have the student practice more or maybe make a rhyme to help the child learn the concept.

She needed to explain what the steps were doing and why to change the sign, and flip the fraction.

Although some students mentioned the importance of explaining why the procedure works, none in this group (a) mentioned the value of seeing, for example, $1/2 \div 1/3$ as "How many $1/3$ s are in $1/2$?" (b) suggested that a manipulative or drawing might help the child understand the concepts underlying the procedure, or (c) noted that a real-world context might help the child. The prospective teachers' responses reflected the traditional view that learning mathematics involves memorizing procedures. We wondered whether this item elicited this belief because the prospective teachers themselves did not understand this procedure. We suspected that responses to this item would change after the students had completed their mathematics course and developed some conceptual understanding of this procedure.

Some students (13%) received *1* or *2* on this item and indicated that the teacher should go beyond the symbols to help the child be successful with similar problems. The following is an example of response scored *1*:

Using pictures and figures so that you can actually show visually what happens when you divide fractions. That way they can picture things in their head, and they would have a better understanding of the concept as a whole, not just numbers and symbols on a paper.

This response reveals an interest in using multiple representations to develop understanding. This prospective teacher recognized the limitations of a focus on symbols.

The scores on this item reflected the traditional view, identified by Ball (1991), of mathematics learning as "knowing how to do it." The students, for the most part, failed to bring up multiple representations as a vehicle for teaching. This result is in keeping with others' findings that although prospective teachers like to have children use manipulatives, they are unsure how to use them to promote conceptual understanding (Foss & Kleinsasser,

1996). At least a few students generated the idea that models and contexts would help children.

Overall Responses

The belief survey included 17 rubrics with scales ranging from 0–3 to 0–5. Responses that reflected traditional views were scored 0. We categorized the overall responses of a student as *traditional* if the respondent received nine or more 0 scores and no more than one score of 2. In other words, students we considered traditional received the lowest possible score on more than half the rubrics and only one high score, a fairly liberal definition of a *traditional student*. Prospective teachers who gave several nontraditional responses were included in this *traditional* group.

Using this liberal definition of *traditional student*, we found that 40% of the students were traditional. Considering these data another way, 60% of the students were nontraditional, responding to some survey items in ways that reflected more reform-minded views of mathematics and mathematics teaching and learning. This is not to say that 60% of the students had all the beliefs that we are hoping our students will develop. It is fair to say that aspects of many of their answers showed hints of the reform-minded orientation we hope to come to see in students.

The diversity in the data is also noteworthy. On the basis of 6 of the 17 rubrics, more than half the students received scores greater than 0. For items scored according to 10 of the 17 rubrics, at least 40% of the students received scores greater than 0. In general, many students scored greater than 0 on many items. This finding challenges the notion that prospective teachers hold similar views about mathematics teaching and learning and that these views tend to be traditional.

Conclusions

The data presented here indicate that prospective teachers are not all alike. Some hold traditional views, but many do not. Many prospective teachers hold some traditional

views and at the same time have some reform leanings. These data show that many prospective teachers are open to multiple solution strategies and will at least allow, if not encourage, children to use alternatives to standard algorithms. Many see value in offering children problem-solving opportunities, and a few recognize that problem solving can be an important element in the construction of mathematical understanding. Most of these prospective teachers have not yet learned that visual representations and real-world contexts help children to develop conceptual understanding. This image of the group is different from mathematicians' and mathematics educators' images, the genesis of which I explore and discuss in relation to gender.

In the late 1980s, mathematics educators began to pay attention to the beliefs of preservice teachers. Thompson (1984) initiated this interest with her work on teachers' beliefs. Ball's dissertation and her articles based on her dissertation data (see, for example, Ball, 1990) laid the groundwork for discussion of the problematic nature of prospective teachers' subject-matter knowledge and beliefs. Ball inferred the prospective teachers' beliefs from their responses to interview questions about mathematical topics. She helped others in the field to recognize the importance of attending to beliefs and knowledge and described some beliefs of some prospective teachers.

Researchers in subsequent studies of prospective teachers have examined beliefs in various ways. Many depend upon qualitative data to discuss prospective teachers as a group. These studies have confirmed Ball's findings on prospective teachers' beliefs. Foss and Kleinsasser (1996), writing about 22 prospective teachers in their mathematics methods course, said, "A sketch of what the entire group thought about mathematics will give an idea as to their perceptions and beliefs about mathematical content knowledge" (p. 434). These authors presented a single image of the beliefs of prospective teachers to make the point that prospective teachers want to make learning mathematics fun for their students and are unsure how to engage the children in substantive mathematics.

Using data from 50 students, Schuck (1999) treated all the students as if they were similar, presenting a single image of the prospective teacher rather than looking for differences within the group. To show that prospective teachers want to make mathematics learning fun and that they think knowledge of mathematics is a disadvantage to teachers in the primary school, she used quotes from individual students as if they were representative of the whole group. I have no doubt that many students in her sample held these views. I also believe that if Schuck had looked for differences within the group, she would have found that at least some students held different views.

Taking a similar approach, Gellert (2000) used data from 42 students to make a case that prospective teachers are so concerned with making children feel safe that they want to “shelter them from mathematical shock” (p. 266). He, like Schuck, treated the students as if they were all similar and presented a single image. He pointed up no differences within his group and portrayed all prospective teachers as if they lacked subject-matter knowledge and wanted to avoid “explorations and adventures into mathematics.” Again I have no doubt that many students in the sample held these views. I suspect, however, that at least some of the students could be characterized differently. Gellert’s portrayal is particularly troubling, because I think that he denigrated the prospective teachers’ desires to make mathematics learning enjoyable. He related this to the prospective teachers’ caring and mothering orientations and suggested that these orientations would interfere with the prospective teachers’ abilities to teach mathematics. I was troubled that Gellert characterized these laudable, typically feminine traits as problematic.

As the above three studies illustrate, mathematics educators, in their research, have tended to use the whole group of prospective teachers as their unit of analysis in an effort to find ways to improve their instruction. They use qualitative data from a few individuals to make their case about the whole group. The resulting portrayal of prospective teachers may not be accurate for the whole group. The survey results I report here indicate that the portrayal is limited.

I recognize that characterizing the group as a whole can serve a purpose, but I believe that as far as prospective teachers are concerned, this purpose has been served. We mathematics educators know the tendencies of the group. We know less about within-group differences. We know less about the prospective teachers who enter their teacher-preparation programs feeling comfortable with mathematics and eager to engage in problem solving with students. We know less about the students who are open to multiple solution strategies.

This analysis leads to the relationship between the stereotype of prospective teachers and gender. Researchers trying to understand mathematics and gender have conducted studies over more than 25 years and have shown mean differences between males and females. We know that females' attitudes toward mathematics tend to be different from males', that females tend to be less confident than males in their abilities to do mathematics, and that they tend to perform less well than males on some spatial tasks and on some tasks that involve higher order thinking. Unfortunately these studies may have contributed to a stereotype of females as less mathematically capable and less mathematically oriented than males. Stereotypes lead people to assume that all members of group are alike and will behave similarly. Because the group of prospective teachers is predominantly female, these stereotypes about women and mathematics are applied to the whole group of prospective teachers. The stereotype is maintained because it fits with existing ideas about women and mathematics and holds for some prospective teachers. When one looks for evidence to support the stereotype, one can find it. If the group were mixed in gender, I wonder whether it would be treated as if it were homogeneous.

Perhaps the stereotypes persist also because we mathematics educators have been largely unsuccessful in reaching this population of students. Many of the innovations we have tried have been unsuccessful. Instead of blaming this lack of success on our innovations, we have blamed it on the durable beliefs of the students. I suggest that we need to stop generalizing about this group of students and to begin to look beyond our

stereotypes. We need to begin to expect to see differences in the group, whether we are teaching them or studying them. The group of prospective teachers is heterogeneous, despite the fact that most students in the group are female. If we begin to recognize the positive beliefs that some of the students have, we might be able to capitalize on this resource in our teacher preparation work.

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Figure Caption

Figure 1. Subtraction item from belief survey.

Subtraction Item	Ariana
Lexi	Ariana
$\begin{array}{r} 635 \\ - 482 \\ \hline 153 \end{array}$	$\begin{array}{r} 635 - 432 \\ 635 - 400 = 235 \\ 235 - 30 = 205 \\ 205 - 50 = 155 \\ 155 - 2 = 153 \end{array}$
<p>Lexi says, "First I subtracted 2 from 5 and got 3. Then I couldn't subtract 8 from 3, so I borrowed. I crossed out the 6, wrote a 5, then put a 1 next to the 3. Now it's 13 minus 8 is 5. And then 5 minus 4 is 1, so my answer is 153."</p>	<p>Ariana says, "First I subtracted 400 and got 235. Then I subtracted 30 and got 205, and I subtracted 50 more and got 155. I needed to subtract 2 more and ended up with 153."</p>
<p>Which child (Lexi or Ariana) shows the greater mathematical understanding? Why?</p>	
<p>If 10 students used Lexi's approach, how many do you think would be successful at solving the problem $700 - 573$? Why do you think that?</p>	
<p>If 10 students used Ariana's approach, how many do you think would be successful at solving the problem $700 - 573$? Why do you think that?</p>	
<p>If you were the teacher, which approach would you prefer that your students use? Please explain your choice.</p>	

Table 1
Frequencies of Responses by Students Enrolled in the First Mathematics Course for Prospective Elementary School Teachers (N = 208) to Subtraction Item

Frequency	Score	Responses	
36%	0	Respondents receiving this score <ul style="list-style-type: none"> indicated a preference for having children use the standard algorithm and predicted that children would have an easy time using the standard algorithm and would be less successful using Ariana's approach. 	
37%	1	Two types of responses received this score:	
		Respondents <ul style="list-style-type: none"> indicated a preference for having children use standard algorithm and predicted that children would be as successful using Ariana's approach as using the standard algorithm. 	Respondents <ul style="list-style-type: none"> would allow children to use either approach to solve subtraction problems and predicted that children would have an easy time using the standard algorithm and would be less successful using Ariana's approach.
22%	2	Two types of responses received this score:	
		Respondents <ul style="list-style-type: none"> would allow children to use either approach to solve subtraction problems and predicted that children would be as successful using Ariana's approach as using the standard algorithm. 	Respondents <ul style="list-style-type: none"> thought that Lexi had the greater understanding, would allow children to use either approach to solve subtraction problems, and predicted that children using Ariana's approach would be more successful than children using the standard algorithm.
5%	3	Respondents receiving this score <ul style="list-style-type: none"> thought that Ariana had the greater mathematical understanding, would allow children to use either approach to solve subtraction problems, and predicted that children using Ariana's approach would be more successful than children using the standard algorithm. 	

Table 2
Frequencies of Scores for the Problem-Solving Item

Frequency	Score	Type of response
32%	0	Never provide children with problems to solve before showing them how to solve them.
52%	1	Provide students with a little bit of instruction first and then let them think. Let students think on their own then I'll tell them "my way." [May include a good reason for having children solve problems on own] Let students think on their own so teacher can assess understanding. Let children think on their own to develop their self-esteem.
15.5%	2	Let students think on their own. [Provides a rationale for importance of having children figure things out for themselves]

Table 3
Frequencies of Responses to Division-of-Fractions Item

Frequency	Score	Type of response
87%	0	Practice more [with no mention of teacher intervention of any kind]. Practice more of the same kind with some support from the teacher. The teacher should explain why the procedure works along with more practice.
13%	1–2	The teacher should provide a context or visual aids in conjunction with, or instead of, the algorithm.