

A Fifth-Grade Student Reflecting Upon Learning Fractions Without Understanding: A Video Example¹

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Most people, be they teachers, educators, or parents, when asked whether students should learn mathematical concepts or mathematical procedures, will answer that students should learn both. Authors of recent reports concur, calling for students to develop conceptual understanding *and* procedural facility (NCTM, 2000) or procedural fluency (National Research Council, 2001). Whatever the labels, educators seem to agree that students' learning both concepts and procedures is critical. Less clear, however, is whether students should learn concepts before, concurrently with, or after learning procedures. Pesek and Kirshner (2000) addressed the question of whether the order in which concepts and procedures are learned matters. In their study, one group of students was taught a 3-day geometry-concepts unit, and another group of students was taught a 5-day geometry-procedures unit followed by a 3-day unit focused only on geometry concepts. The 3-day conceptual unit, which focused on students' constructing relationships through "concrete materials, questioning, student communication, and problem solving" (p. 529), was identical for both groups. That is, the only difference between the two groups was that one group first received 5 additional days of instruction focused on "memorization and routine application of the formulas" (p. 529). One would expect the students who were exposed to 8 days of instruction to learn more than the students who were exposed to 3 days of instruction. However, results of a 37-question paper-and-pencil assessment together with follow-up interviews indicated just the opposite! Students receiving the procedural instruction prior to the conceptual instruction learned less than students who received only the conceptual instruction. Somehow exposure to routine application of formulas inhibited or interfered with students' subsequent conceptual learning.

The students in Pesek and Kirshner's study were unaccustomed to receiving conceptual instruction on a regular basis. But for students who regularly learn mathematics meaningfully, what effect would a lesson during which a procedure was taught with no focus on understanding (hereafter referred to as "procedure-only") have? Would they recognize the difference? Would they find a way to compensate for the missing conceptual ideas? How would they be affected by a subsequent conceptual lesson? We answer these questions in this video paper, focusing upon one particular student's reactions and reflections.

What We Did

We asked a teacher who normally teaches mathematics with a focus on understanding to teach a lesson on converting between mixed numbers and improper fractions using a procedure-only lesson. To make sense of what happened with the students, we videotaped the lesson, administered a paper-and-pencil assessment directly after the lesson, and videotaped interviews of several students 3 weeks later. Five weeks after the first lesson, the teacher taught the same content again, but in the conceptually oriented way she usually teaches. Again we videotaped the lesson, administered an assessment directly after the lesson, and videotaped interviews of several students 3 weeks later. This study is unusual in that we worked with students who were normally taught conceptually but were taught a procedure-only lesson. We share our findings through video clips of the teacher and of one student and through class data.

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Procedural Lesson

The procedural lesson for converting between improper fractions and mixed numbers was based upon a lesson drawn from a California state-adopted textbook. For example, to convert $2\frac{3}{4}$ to an improper fraction, students were taught to multiply the whole number (2) by the denominator (4), add the numerator (3), and write this total (11) as the numerator of a fraction with 4 as the denominator. To convert from an improper fraction, say $\frac{13}{5}$, to a mixed number, students were taught to divide the numerator by the denominator, with the quotient representing the whole number portion of the answer, the remainder representing the numerator of the fraction part, and the divisor representing the denominator. In the following video clip, the teacher describes her thinking about the procedural-only lesson she taught. She refers to this lesson as *traditional*.

Video Clip 1: 0:52

(The teacher talks about the procedural lesson.)

The teacher was aware that students might learn these procedures by focusing upon individual digits instead of reasoning about fractions as quantities. She realized that instead of thinking of $\frac{13}{5}$ (or $2\frac{3}{5}$) as a single quantity, the students might think of $\frac{13}{5}$ (or $2\frac{3}{5}$) as 2 (or 3) separate numbers. She also noted that many students recognized that this lesson was different from most of their lessons, and they wondered why it was being taught in this way.

After the lesson, the students were assessed with a paper-and-pencil instrument. The items appear in Table 1, which also shows that the class scores fell an average of 21% between the post and delayed procedural assessments. The scores fell only 2% between the post and delayed conceptual assessments.

Table 1. Class Percentages of Correct Assessment Responses Immediately (Post) and 3 Weeks After (Delayed) the Procedural and Conceptual Lessons

Item	Problem	Post procedural (n = 28)	Delayed procedural (n = 28)	Post conceptual (n = 29)	Delayed conceptual (n = 29)
1	$\frac{8}{3} =$	89	79	89	89
2	$\frac{10}{6} =$	85	71	86	89
3	$6\frac{4}{5} =$	71	36	83	70
4	$3\frac{3}{8} =$	85	32	70	62
5	How many eighths are in five wholes?	67	71	89	93
6	Circle the larger number or put = if they are the same. $\frac{5}{3}$ $1\frac{2}{3}$	75	57	79	79

We focus on Rachel, one student who scored well on the paper-and-pencil assessments immediately following both the procedural and the conceptual lessons. Rachel scored 83% on the post-procedural exam, although her delayed-procedural score 3 weeks later was only 16%. Her post-conceptual lesson score was 100%; her score 3 weeks later was 67%. In an interview after the delayed-procedural assessment, we asked Rachel to solve a problem like those she had correctly solved immediately after the lesson. Video Clip #2 shows that she could not remember the procedure she had successfully applied 3 weeks earlier. Rachel is a reflective child who not only noted that this mathematics lesson was different from most mathematics lessons she encountered in her current classroom but also talked about the way this procedure-only lesson impeded her learning.

Video Clip 2: 1:02

(Rachel reflects upon why she does not remember the procedure.)

Note that Rachel articulated not only that she was unable to remember the procedure but also that she could draw a causal link back to the way the procedure-only instruction impeded her learning. She said that she “didn’t figure it out for herself.” She went on to explain, “So when I figure that out, it’s easier, and, um, once *I* figure it out, it’s, it *stays* there ‘cause I was the one who brought it there. So, and it is just easier to do when you figure it out yourself, instead of having teachers telling you.”

Conceptual Lesson

Five weeks after the procedural-only lesson, the teacher presented a conceptual lesson on the same topic. To begin this lesson, students used pattern blocks to explore relationships among fractions (see Figure 1). The teacher guided the exploration with questions designed to support the students’ development of their own algorithms for conversion. For example, she asked them to “show five thirds.” The students held up various blocks representations for $\frac{5}{3}$, and during the discussion designed to compare different approaches, students pointed out that if three of the thirds were grouped as one whole, then $\frac{5}{3}$ could also be expressed as $1\frac{2}{3}$ (see Figure 2). The teacher extended the students’ thinking by having them consider fractional amounts, such as fourths and eighths, not easily represented by the pattern blocks. The written assessment also included problems with fractions not easily represented with the pattern blocks.

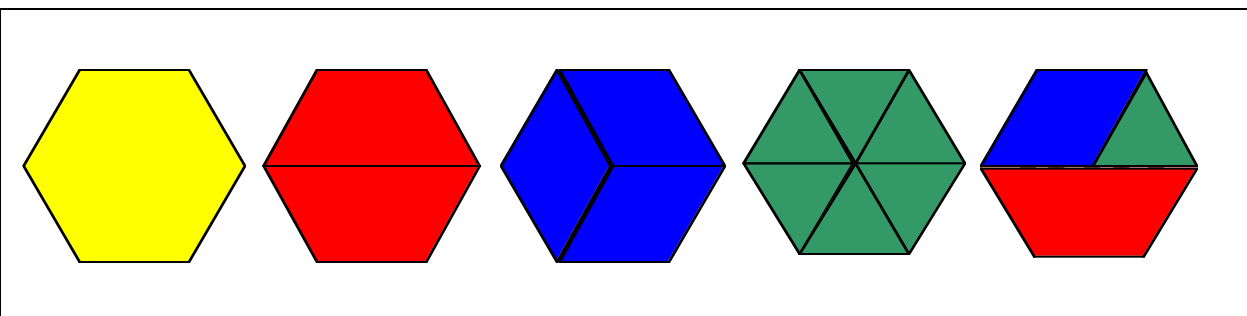


Figure 1. Pattern-blocks representations for one whole.

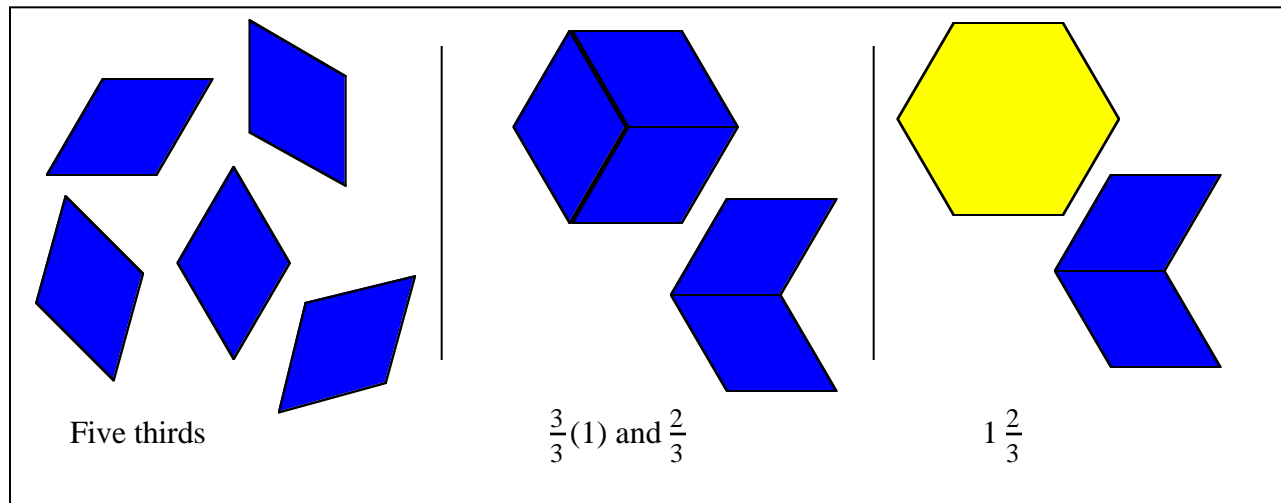


Figure 2. Pattern-blocks representations for $\frac{5}{3}$.

The teacher encapsulates her goal for the lesson when she speaks about the importance of students' creating their own algorithms.

Video Clip 3: 0:27

(The teacher describes the purpose of the second lesson.)

Immediately after the conceptual lesson and 3 weeks later, the students completed the paper-and-pencil assessment. After the delayed-conceptual assessment, we again interviewed several students. Rachel had forgotten the correct procedure, and she incorrectly converted $3\frac{3}{8}$ to $\frac{17}{8}$. However, after applying the incorrect procedure, Rachel chose to solve the problem a second way, arriving at a different answer.

Video Clip 4: 1:52

(Rachel incorrectly applies the procedure for converting $3\frac{3}{8}$ to an improper fraction, then immediately notes that she could also have solved the problem by drawing a picture, and then becomes confused when her answers from the two solutions differ.)

The interviewer recognizes Rachel's confusion and asks her how many eighths are in each whole. Rachel works through the problem and arrives at the correct answer. Her incorrect procedural answer is written above the correct conceptual answer.

Video Clip 5: 0:48

(Rachel follows her understanding and finds the correct answer.)

Notice that when the interviewer asked Rachel which answer she thought was correct, Rachel chose the answer she arrived at by drawing a diagram. In the next clip, the interviewer investigates whether Rachel can use her understanding of the concepts to arrive at a correct procedure for converting mixed numbers into improper fractions.

Video Clip 6: 0:45

(Rachel uses her understanding to correct her incorrect procedure.)

When asked whether she could try again to make sense of a procedural approach, Rachel was able to figure out a procedure and to explain why it worked. We have worked with many children from different schools, and we think that Rachel's willingness to try to make sense of the procedure is a testament to the sense-making approach she takes with mathematics. In the final clip, the interviewer asks Rachel to reflect upon the reason she applied the procedural approach even though she did not seem to remember it. Rachel's response shows a great deal of insight about her learning process.

Video Clip 7a: 0:14

(Rachel reflects upon the reason she applied the incorrect procedure before resorting to her underlying conceptual understanding.)

Video Clip 7b: 0:19

(Rachel explains the order in which she would prefer to learn concepts and procedures.)

Rachel says that the reason she applied an incorrect procedure instead of first relying upon her conceptual understanding was that she learned the procedure first. She also stated that she would prefer instruction that focuses on helping her understand the concepts before instruction that shows her a particular procedure.

Summary

Rachel is a sixth-grade student in a class taught by a teacher committed to helping her students understand underlying mathematical concepts. When the teacher deviated from her normal instruction and taught a procedure-only lesson, Rachel could not, after a few weeks, remember the procedure. The subsequent conceptual lesson in which students developed their own connections between mixed numbers and improper fractions was sufficient to enable Rachel to make sense of these problems for herself a few weeks after the lesson. She did so, however, only after first applying an incorrect procedure, an approach she attributed to having been taught the procedure first.

Rachel may seem atypical because of how articulate and reflective she is, but we believe that she is typical in terms of how she makes sense of mathematics. Clearly Rachel is speaking only for herself, but if the manner in which she learns is at all similar to the manner in which other children learn, then the instructional implications for determining the optimal sequence in which we teach procedures and concepts are important.

During interviews with other students we noted that many of the students were not able to reproduce the algorithm correctly 3 weeks after the procedural lesson. The one interviewed student who could was unable to explain how or why the procedure worked. Only after the conceptual lesson could she identify the roles of the numbers in the fractions and explain the reason for multiplying and adding to convert a mixed number into an improper fraction.

Conclusion

How common is Rachel, and how common are the issues raised in this video paper? We think that they are common, and we think that a child like Rachel, who is able to articulate her reasoning, may be in the position to speak for many other children who experience the same mathematical difficulties.

If we had not had the opportunity to sit and talk to Rachel, we would not have known how she was thinking about learning the concepts and procedures. Although teachers may be unable to interview all their students, we suggest that the experience of digging deeply into the thinking of even a few students is powerful. Although not every child will be as reflective about his or her thinking as Rachel, the experience of better understanding any child's mathematical thinking has instructional implications that go far beyond helping the individual child.

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