# The Role of a Children's Mathematical Thinking Experience in the Preparation of Prospective Elementary School Teachers

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Abstract

Historically, content preparation and pedagogical preparation of teachers in California have been separated. Recently, in integrating these areas, many mathematics methodology instructors have incorporated children's thinking into their courses, which are generally offered late in students' undergraduate studies. We have implemented and studied a model for integrating mathematical content and children's mathematical thinking earlier, so that prospective elementary school teachers (PSTs) engage with children's mathematical thinking while enrolled in their first mathematics course. PSTs' work with children in the Children's Mathematical Thinking Experience (CMTE) may enhance their mathematical learning. Preliminary study results indicate that the sophistication of CMTE students' beliefs about mathematics, teaching, and learning increased more than the sophistication of beliefs held by students enrolled in a reform-oriented early field experience and that experiences considering children's mathematical thinking provided PSTs with increased motivation for learning mathematics.

# The Role of a Children's Mathematical Thinking Experience in the Preparation of Prospective Elementary School Teachers

In the United States, education in general and teacher education in particular are state responsibilities. Historically, university students planning to become teachers first study mathematics content, and only later do they study teaching. We agree with the rationale for this sequence: One cannot teach what one does not know, and so a prospective teacher must understand mathematics to be in a position to teach it. However, we find problematic the idea's translation into practice, so that prospective elementary school teachers first learn mathematics content and only later consider issues of teaching and learning. First, we know that developing deep understanding of the mathematics of elementary school is far more difficult than was once thought (Ball, 1990; Ma, 1999; Sowder, Philipp, Armstrong, & Schappelle, 1998). Second, our experience has been that even when PSTs attend a thoughtfully planned course designed to engage them in rich mathematical thinking, too many of them go through the course in a perfunctory manner. Many PSTs' expectations about what mathematics is—a fixed set of rules and procedures—along with their perceptions of how children and adults learn mathematics—by being shown how to solve problems in a prescribed step-by-step fashion—clash with the more conceptual, meaning-making goals of mathematics courses designed for PSTs. We thus believe that there is a better place to begin working with prospective elementary school teachers.

At one public, comprehensive, urban university in the state of California, we have implemented and studied a new model for integrating mathematical content and children's mathematical thinking: Prospective elementary school teachers (PSTs), while enrolled in their first mathematics course for elementary school teachers, engage with children's mathematical thinking years before they begin student teaching. In the first section of this manuscript, we present a framework guiding our approach to the preparation of PSTs in general and their mathematical preparation in particular. The second section shows how, in what we refer to as a Children's Mathematical Thinking Experience—Live (CMTE-L[2]), we integrate content and children's mathematical thinking for PSTs. This work is part of a large-scale quantitative study of PSTs' changing beliefs and content knowledge, and although the presentation of our extensive quantitative data is beyond the scope of this paper, in the third part of the paper we provide one quantitative example of the effects of the CMTE-L on the PSTs' beliefs about children learning mathematics. We also present PSTs' comments on their experiences in the CMTE-L.

## Why Integrate Mathematics and Children's Mathematical Thinking?

The widely accepted notion that children learn by building upon their existing knowledge might be taken as an axiom of learning. David Ausubel stated, "If I had to reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows" (quoted in Hiebert & Carpenter, 1992, p. 80). We adapt Ausubel's comment to include not only what the learner knows but also about what the learner cares. One hundred years ago John Dewey (1990) wrote that every subject has two aspects, "one for the scientist as a scientist; the other for the teacher as a teacher" (p. 351). He wrote, "[The teacher] is concerned, not with the subject-matter as such, but with the subject-matter as a related factor in a total and growing experience [of the child]. Thus to see it is to psychologize it" (p. 352). In our approach to working with PSTs, we assume that they can come to care (Noddings, 1984) about mathematics, not as scientists, but as teachers.

Many PSTs report having had bad experiences learning mathematics (Ball, 1990). They do, however, care about children (Darling-Hammond & Sclan, 1996). We believe that we can facilitate the process by which PSTs come to learn mathematics by beginning with what PSTs care most about—children (see Figure 1). We place children (rather than

[2] The Children's Mathematical Thinking Experience—Live (CMTE-L) stands in contrast to a separate treatment, the Children's Mathematical Thinking Experience—Vicarious (CMTE-V). In the CMTE-V, students watch video clips but do not actually work with children.

children's thinking, for example) at the center of caring because we believe that for most PSTs, the initial caring is a phenomenological act of concern for the whole child versus for a particular characteristic of the child. For example, following is a PST's written response regarding what she found most valuable in a course in which she learned about children's mathematical thinking while working with children:

> I think it was just amazing. I had never ever taken a class like this one, so I am very surprised and happy that I took it. The most valuable (I think) is the contact with the kids. (Marisol, 12/5/01)

Tapping into PSTs' caring about children is the first step, but then we hypothesize that when PSTs engage children in mathematical problem solving, the PSTs' circles of caring expand to include children's mathematical thinking. For example, the PST who responded below indicated that she not only found her work with children valuable but also appreciated the chance to understand their mathematical thinking:

> This experience with working with the children, itself, is what I think makes this class valuable. Also, I think that analyzing the way children solve problems is valuable information for us to reference to when we, finally, become teachers, especially to become more effective in teaching mathematics. (Isabel, 12/4/01)

PSTs begin to see how children think about mathematics and come to recognize that children solve problems in varied and sometimes mathematically powerful ways. It is then, we predict, that many PSTs' circles of caring will extend to mathematics, because they realize that to be prepared to understand the depth and variety in children's mathematical thinking, they must themselves grapple with the mathematics. Figure 1 shows a model that captures our view of the growth of PSTs' caring.

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Insert Figure 1 About Here

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## How Do We Integrate Mathematics and Children's Mathematical Thinking?

Our approach to integrating mathematics content and children's mathematical thinking is through the CMTE-L we created to provide PSTs opportunities to interview and tutor children in mathematics and to reflect upon the process. The CMTE-L is a course held weekly for 2 1/4 hours in a classroom on-site at a local elementary school. The course addresses children's mathematical thinking and mathematics, and although it is neither a mathematics course nor a mathematics methodology course, it combines aspects of both. The CMTE-L is different from a mathematics course because the mathematics studied is not an end in itself but instead arises from the PSTs' work with children. Furthermore, the CMTE-L is different from a mathematics methodology course because we do not attempt to help students learn to teach a group of students. For example, we do not discuss lesson or unit planning, textbooks, testing, or classroom management. We do not consider how children might interact and learn from one another, because a PST works with only one child at a time, a model selected for two reasons. First, by working with only one child, the PST can focus solely on the child's mathematical thinking, because discipline issues seldom arise. Second, when experiencing difficulties understanding the child's thinking or considering ways to support the child, the PST must grapple with the child's reasoning and cannot turn to other children for help (cf. Thompson & Thompson, 1994). In the CMTE-L, PSTs work directly with children in about half of the 13 sessions; other times they analyze previous sessions with children, plan subsequent sessions, or consider more general issues related to children's thinking or mathematics. The PSTs in the CMTE-L are concurrently enrolled in the first of four mathematics content courses required of PSTs. Simply stated, the CMTE-L is a course on children's mathematical thinking and the related mathematics. We describe a CMTE-L we recently taught and studied.

In the first part of the course, the PSTs examined the mathematical thinking exhibited in young children's solution strategies for various types of mathematics problems (Carpenter, Fennema, Franke, Levi, & Empson, 1999). The PSTs learned to conduct an assessment interview by examining and reflecting upon video clips of interviews, by observing a role-play of an interview, and by watching the instructor interview a primary-grade child. Each pair of PSTs then conducted an interview with a primary-grade student. One PST took the interviewing lead while the other PST took detailed notes; they alternated roles from interview to interview. One interview was videotaped. After each interview, the instructor led a class discussion, focusing upon what the PSTs had learned, had struggled with, or had found surprising during their interviews. This discussion each day was supported by clips the instructor chose, because of their potential to raise issues the instructor deemed important, from the videotaped interview.

In the interviews during the second part of the course, the PSTs investigated third-grade students' place-value understanding; the instructors then helped the PSTs connect their interview experiences with mathematical ideas of place value before the PSTs provided the same primary-grade students opportunities to think more deeply about place value concepts. (These three sessions are described in detail in the next section.) During the remainder of the course, the PSTs interviewed and tutored fifth-grade students several times to delve into the students' thinking about rational numbers. The interview sessions were interwoven with rational-number discussions set in the context of facilitating a child's rational-number understanding.

We consider our PSTs' circles of caring when we initiate them into the interview process. Most have yet to consider how a child's point of view toward mathematics differs from their own; they have thought little about the mathematics under consideration; and they certainly have not considered the intersection of the two areas. Most enter the course believing that mathematics teaching consists of showing and explaining procedures and that children must be shown how to solve mathematics problems in a prescribed step-by-step fashion. To address these beliefs, we initially constrain the role of the PSTs so that they assess children's understanding of concepts by using carefully selected tasks. In the process, the PSTs see that many primary-grades children understand mathematics they *have not* been taught and that many intermediate-grades children do not understand mathematics they *have* been taught. The PSTs recognize that children, when given opportunities, do solve problems in a variety of appropriate, creative, and mathematically powerful ways. Many PSTs further recognize that if they want to understand children's mathematical thinking, they themselves must understand the mathematics in powerful ways. By the time the PSTs tutor fifth-grade children during the last half of the CMTE-L, most have started to think differently about the mathematics they are teaching, and they have begun to understand how children make sense of mathematics. These beliefs in turn influence the mathematics the PSTs believe they must know in order to teach the children about whom they care deeply.

To provide the reader with a sense for the particular tasks and situations we ask our PSTs to explore, we describe a detailed 3-session sequence of the CMTE-L to illustrate how these experiences are integrated to help the PSTs develop deeper understanding of mathematics. The instructors focus parts of the course explicitly on mathematical topics, for example, place value; they help the PSTs come to view place value as much more than knowing the value of a place in a numeral or being able to regroup numbers. One major place-value idea on which we focus is that a number can be reconstituted in many ways; 27 can be thought of in the standard way as 2 tens and 7 ones or, for example, as 1 ten and 17 ones. This kind of flexibility of grouping and regrouping a number is an important prerequisite for understanding the standard procedure for subtraction (Ross, 1989). This grouping ability develops slowly in children, and in the CMTE-L we present place-value understandings in three levels, discussed below. Another focus of the place value discussion is that the position of a digit in a number determines its value. For example, the digit *3* represents 3 ones in the number 463 but the same digit represent 3 hundreds in the number 376. Furthermore, the value of any place in a number increases multiplicatively by a factor of 10 as we move to the left in a number, so that the 6 (or the 4 or the 8) on the left in the number 664,488 represents 10 times as much as the 6 (or 4 or 8, respectively) on the right (Kamii, 1986; Ma, 1999). Because of this relationship, we can regroup in our standard algorithms. Using a multiple-representations model, the instructors relate many aspects of place value. As Wearne and Hiebert (1994) pointed out,

> Understanding place value involves building connections between key ideas of place value—such as quantifying sets of objects by grouping by ten and treating the groups as units—and using the structure of the written notation to capture this information about grouping. (p. 273)

## Overview of the Three Sessions

This three-session sequence took place during the 5th, 6th, and 7th weeks of the class. During the first and last sessions, PSTs worked with the same third-grade student, assessing the child's place-value understanding in the first and tutoring to enhance the third grader's understanding of place value in the third. During the second session, the PSTs were presented with the multiple-representations model of place value and with a description of place-value understandings at three loosely defined levels. Also, several video clips of children working on place-value tasks were shown and discussed.

The sessions were sequenced to help the PSTs become invested in grappling with issues of place value. By first assessing the place-value understanding of a third grader, the PSTs, during the second session, could draw from an experience with a child, and they valued coming to understand the complexities of the domain of place value because that understanding was situated in the context of understanding the thinking of a child about whom they cared.

## Three Consecutive CMTE-L Sessions

### Session 1

In their first session with a third grader, PSTs assessed the child's place-value knowledge. The PSTs worked in pairs, each pair interviewing one child. After this interview, many PSTs thought that the children had exhibited good place-value understanding if they were able to regroup in using standard algorithms. The following excerpts are representative of the PSTs' brief interview reflections, written immediately after their interviews.

- "He knew place value and knew why you carry and borrow.”
- ”I was really excited to see that the child knew place value. When he solved 70 minus 23, he wrote it vertically, and instead of writing a one on top where he borrowed, he wrote a ten.

Lisa and Jeannette were videotaped conducting their interview of Estella; after the PSTs had written interview reflections, the instructor chose excerpts from the videotaped interview to support discussion of aspects of place value, asking the PSTs to compare their child's thinking with Estella's. The goals were (a) to show that children who are able to regroup may neither understand the associated concepts nor be able to complete other place-value tasks and (b) to set the stage for the next session's focus on place value. For example, in one clip, Estella, when asked to subtract 8 from 17 (written vertically), regrouped by crossing out the 1 and writing 1 to again make 17; she then subtracted the 8 and wrote *9*.

$$\begin{array}{r} {}^{0}\not{1}\,{}^{1}7 \\ \underline{-\quad 8} \\ 9 \end{array}$$

Lisa pointed out to Estella that she had not simplified the problem because “it was already 17.” Estella did not immediately understand that she had replaced 17 with 17. After viewing this video clip, other PSTs reported that several other children had done just what Estella did. This experience helped the PSTs recognize that many children, when approaching the subtraction algorithm, treat 17 as two numbers (1 and 7) instead of as a single quantity. This excerpt also illustrates that children often apply a procedure before they think about how to solve a problem.

Because the PSTs fundamentally cared about the children they interviewed, they become more vested in the mathematics they wanted the children to understand. This first interview and the discussion of a “live” child set the stage for the second session.

Session 2

The second session was designed for students to recognize the complexities inherent in understanding place value. Although the PSTs did not interview children during this session, the discussion of the mathematics was situated in terms of understanding both the children they had interviewed and the children they watched in the video clips. After discussing how children come to learn about numbers, the instructor talked about three levels of place-value understanding: *pre place-value understanding, beginning place-value understanding,* and *intermediate place-value understanding* (these ideas are drawn from others, particularly Bowers, personal communication, Feb. 2001; Fuson et al., 1997; and Kamii, 1986).

- A student at *a pre place-value-understanding* level is able to form groups only with 1 as a unit. For example, whereas a child at this level may be able to count to 10 and even group 10 ones, this child is not yet able to conceive of that quantity as one group of 10.
- A student at the *beginning place-value-understanding* level can conceptualize 10 ones as 1 ten. This student can then treat the 10 as though it is a single composite unit. For example, a child at this level can conceive of 40 + 70 as comprised of 11 tens, 110.
- A student at the *intermediate place-value-understanding* level is able to reconceptualize the unit and see a number in multiple ways. A student at this level understands the multiplicative relationship among the places, so that, for example, the hundreds place has 10 times the value of the tens place. A student at this level should be able to simultaneously conceive of 320 as 3 hundreds and 2 tens or as 32 tens.

After this discussion of place value, video clips of elementary school children and classes of children working on place-value tasks were shown to illustrate the levels of place-value understanding. In one clip, children in a Grade 3–4 combination classroom discussed how to rename the number 192,000. Some students suggested "192 *thousands*," at which point Daniel spoke up and said, "It is one thousand nine hundred and twenty *hundreds*." At the request of the teacher, Daniel attempted to justify his reasoning, but he struggled to articulate his thinking. Bert jumped in and suggested that the number is "nineteen thousand two hundred *tens*," justifying this result by pointing out that 19,200 multiplied by 10 is 192,000. After Bert explained his thinking, Daniel grappled with the idea of multiplying the number of *hundreds (*1920) by 10 to get the number of *tens* (19200). During a class discussion, one PST (Tamara) responded to this video clip:

> I just feel like that each kid has a different way of seeing things and conceptualizing math. And, like, I have learned it, basically, a few certain ways. And I feel like they would surpass my knowledge and be a lot smarter than me. And [they] would sit there and say these things, explain the way they know it, and I would say, "Ahm, I think that's right." You know. I just feel like they would—kids are learning nowadays like how to do math their own way, that different ways are right, not just their own basic way—which is good. I just feel like that I have learned it in such a set way that it is, that it is hard to open my mind to, like, know all the different ways that you could do it in order to be able to teach them if it was right or wrong. [Excerpted from class video 10/23/01]

At the end of this session many PSTs realized that place-value understanding is more complex than they had previously thought and that children do think about problems in many ways. They also realized that following a child's explanation can be difficult. For example, Alexandra said, "I saw how difficult it can be to understand where a child is going with math concepts."

At the end of class the PSTs wrote about what they had learned or thought about during this lesson and the lesson highlights. The following are some of their thoughts:

*S1:* I didn't realize that there are stages to place value. It is obviously a very deep and involved concept, more than just 10s, 100s, 1000s place. Different methods of grouping numbers are very interesting to me.

*S3:* I was able to open my mind to think about looking at numbers more than one way. …

*S4:* I learned that kids think about problems differently. Therefore, when teaching a child, you should know your basics, and you as the teacher should be able to understand where the child is coming from.

Session 3

In the third session students worked with the same child they had assessed in Session 1, but this time they also tutored to further the child's understanding. The PSTs were given a carefully sequenced set of place-value tasks to use with the children. In one task, the third grader was introduced to a new algorithm for addition, an expanded algorithm we called *Julio's method* (see Figure 2). The third grader was then asked to use Julio's method to solve other addition problems. Again one tutoring session (Alexandra tutoring Felisha) was videotaped, and excerpts of the video were shown and discussed in class after the PSTs had written reflections on their individual interviews. When asked to add 73 + 46 using Julio's method, Felisha knew that she needed to add 70 and 40; she counted out 7 ten-rods and 4 ten-rods then put them together and counted the 11 ten-rods: "Ten, 20, 30, 40, 50, 60, 70, 80, 90, 100, 101[*sic*]." She wrote 101 for the first step in her use of Julio's method. She completed the algorithm as shown in Figure 2. She read her answer (110) as "eleven hundred."

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Insert Figure 2 About Here

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In the CMTE-L discussion of Felisha's interview, the instructor noted how the symbols, manipulatives, and words were disconnected for Felisha. Several students responded to this assertion. Beatrice stated, "[Watching and discussing the tape] was valuable because it showed me how all the different parts of thinking about place value connect. It also showed me how the child can be missing one or two of those connections." Lucy commented, "It is good to see the missing connections between place value concepts." Others agreed, and one PST stated, "I was able to see that she had *some* place value knowledge, but it is very fragile."

Summary

Initially many PSTs thought that to understand place value one need know only the value of a place and the processes for regrouping. In their first interview they encountered students who demonstrated that level of knowledge but did not understand the procedure they had used or the value of the regrouped 1. After working with children and watching and discussing excerpts of another interview, the PSTs were prepared to delve into the complexities of place value in the second session. They were motivated to learn about and discuss place value in preparation for tutoring a child with whom they had developed a caring (albeit a brief) relationship. The PSTs could ground the discussion about place value in their reflections on the child they had interviewed and consider how the discussion would affect their upcoming work with the child.

## Conclusions

The combination of experiences in the CMTE-L helped the PSTs see the complexities of place value.[3] We hypothesize that no single CMTE-L experience would have the same effect as the collection of experiences incorporated into the CMTE-L. The reflection piece is particularly important, and we have found that using videotape of one of the PST's interviews prompts deeper and more specific discussion than occurs without such video. Focused viewing and discussion of the just-completed videos support PSTs in attending to the specifics we believe are important.

The individual interviews and tutoring sessions motivate the PSTs and help them to see that mathematics is a complex subject to teach and to learn. When they see, firsthand, a child struggle to make sense of some of the ideas and then see the children on the videos struggle with the same ideas, they begin to recognize the difficulty of the topic and realize that it merits their attention. Without both sources, the PSTs could dismiss their own experiences as anomalous. We hope that this compelling evidence shapes their beliefs and ultimately motivates them to attend to the mathematical content. On the basis of the PSTs'

[3] We should acknowledge that this sequence provided only a beginning point for our PSTs to grapple with the complexities of place value; we believe that, like children, the PSTs will come to understand these complexities if given a variety of opportunities to learn the mathematics over an extended period of time.

written reflections and comments, we concluded that after this interview, most PSTs thought that knowing place value required more than knowing the value of a place and being able to regroup in using standard algorithms.

## Methodology

### Research Design

The data described in this section are a subset of data collected as part of a large-scale study involving five treatments and more than 160 PSTs. The description of *all* treatments and data collected is beyond the scope of this paper. Because of the project timeline and scope of this paper, the analysis provided here is preliminary, but it does include quantitative and qualitative data.

### Data Sources

To investigate the CMTE-L's effects on our students' beliefs about mathematics and children's thinking, we analyzed data from two sources. The first source was a belief survey designed to assess seven beliefs held by PSTs about mathematics and children's mathematical thinking. The data described herein include data on one belief for change from presurvey to postsurvey. In addition, we collected end-of-course written-survey data from 47 PSTs enrolled in our two CMTE-Ls, and we followed up with interviews of 10 of these students. The survey included five open-ended questions about the CMTE-L and the mathematics content course in which the students were concurrently enrolled.

### Participants and Treatments

All PSTs who participated in the treatments were concurrently enrolled in the first of four mathematics content courses designed for PSTs. They were paid to participate in the project and were randomly assigned to one of five treatments. We have described the CMTE-L, and we compare results from PSTs in the CMTE-L ($n$ = 50) to results from PSTs in a second treatment, an early field experience referred to as the Mathematical

Observation and Reflection Experience in a Reform Classroom (MORE-R[4], $n = 22$). Participants in the MORE-R observed mathematics lessons of teachers known to engage in reform teaching practices. We compared the data from the CMTE-L group and the MORE-R group for two reasons: (a) The MORE-R provided PSTs access to children, teachers, and classrooms, and so one might expect that this experience would positively shape the PSTs' beliefs about children's mathematical thinking, and (b) the teachers selected for the MORE-R were known to engage in mathematics-reform teaching practices, and so one might expect that when PSTs observed these teachers teaching mathematics in a conceptually oriented way, their beliefs about mathematics would change as well.

## Data Analysis

The belief survey data were blinded and coded by trained coders external to the project. The interrater reliability for the data presented here was, on average, 78%. The authors jointly determined codes for the end-of-course-survey data, triple-coded a subset of responses to check for interrater reliability, then single coded the remaining data.

## Results of Quantitative Data

Our belief instrument surveys seven beliefs about mathematics, mathematical thinking, teaching, and learning; across all seven beliefs measured, more CMTE-L PSTs than MORE-R PSTs increased their belief scores: More than two thirds of the CMTE-L PSTs' belief scores but fewer than one half of the MORE-R PSTs' belief scores increased. Furthermore, the CMTE-L PSTs' increases were greater than those of the MORE-R PSTs. Each of the seven beliefs was assessed with multiple items, and we share preliminary results of one item selected from Belief 2: One's knowledge of how to apply mathematical procedures does not necessarily go together with one's understanding of the underlying mathematical concepts. We selected Item 3 from Belief 2 because it addresses the content area of place value, which we discussed in describing the CMTE-L, and because the results

[4] The Mathematical Observation and Reflection Experience in a Reform Classroom (MORE-R) stands in contrast to a separate treatment, the Mathematical Observation and Reflection Experience in a Traditional Classroom (MORE-T).

are representative of the larger belief-data set. In this item, three children's strategies for adding 149 + 286 were presented (see Figure 3.) Carlos solves the problem by applying the traditional multidigit addition algorithm used in the United States. Elliot applies an alternative algorithm; he adds the hundreds, then the tens, and, last, the ones. Because Elliot's algorithm is not taught in the United States, one might infer that Elliot's ability to apply this algorithm reflects base-ten understanding. Sarah uses a strategy that illustrates her number sense. Survey respondents are asked whether Carlos can be expected to make sense of and explain Elliot's strategy or Sarah's strategy. Written responses to this item indicating no evidence, weak evidence, some evidence, or strong evidence of Belief 2 were assigned scores of 0, 1, 2, or 3, respectively. For example, rubric score 0 was assigned to responses indicating that Carlos could explain and use the alternate strategies or that Carlos could not explain the alternate strategies because the strategies were too confusing. Rubric score 3 was assigned to those whose responses indicated that Carlos may not understand underlying concepts and that Elliot's and Sarah's explanations were well developed.

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Insert Figure 3 About Here

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Fifty CMTE-L students and 22 MORE-R students completed both the presurvey and the postsurvey. Table 1 shows the percentages of students, by treatment, who increased 1, 2, or 3 levels and the total percentages who increased. Although their presurvey scores were nearly identical, 56% of the CMTE-L PSTs' belief scores increased at least 1 point, compared with 31.8% of the MORE-R PSTs' scores. The percentage of CMTE-L students whose scores increased 2 points (22%) was more than 2 1/3 times the corresponding percentage of MORE-R students (9.1%), and the ratio was even greater for those whose scores increased 3 points (20% for CMTE-L vs. 4.5% for MORE-R).

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Insert Table 1 About Here

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These results illustrate that more PSTs who learned about children's mathematical thinking while taking the mathematics course than the PSTs who visited reform-oriented teachers while taking the same mathematics course increased their scores on the belief survey. Furthermore, the increases were greater for the CMTE-L students than for the MORE-R students. Evidently, studying children's mathematical thinking is more effective for PSTs' belief development than visiting even exemplary classrooms.

## Results of Qualitative Data

The CMTE-L had two components: (a) work with children to investigate their mathematical thinking and reflect upon the experience and (b) explicit discussions about mathematics grounded in the context of children's mathematical thinking. The quantitative data provide evidence that the sophistication of beliefs held by CMTE-L students increased more than the sophistication of beliefs held by the MORE-R students, but these data do not indicate why. We turn to the qualitative data for some possible explanations.

An analysis of students' responses to the question "Have your experiences in Math 296 (the CMTE-L) affected your experiences in or thinking about Math 210 (the mathematics content course)?" indicated that 94% percent of the PSTs in our survey responded affirmatively. Of these students, about half thought that their experiences in the CMTE-L affected their thinking in the mathematics content course because of their experiences with children. Many of the students thought of the CMTE-L as a kind of laboratory course designed to support their learning in the mathematics content course. Pat wrote

> Taking the [CMTE-L] has enabled me to make a practical application to what I learned inside my [mathematics course], and use it and see what function it serves in the elementary classroom. (Pat, 12/4/01)

As a lab course, the CMTE-L course seemed to support the mathematical learning of students by providing a reason for the PSTs to learn the content.

In responding to the question "In what ways have your experiences in the CMTE-L affected your experiences in or thinking about Math 210 [the mathematics content course]," about half attributed the effects to discussions explicitly centered on mathematics content. For some students, discussions of mathematics grounded in children's thinking positively affected their experience in Math 210, whereas for other students, the work with children played the critical role.  For example, following is a student's response to the survey question, "When you think about your experience in Math 210, do you think it was different from the experiences of others who were not in the CMTE-L this semester? If so, how was your experience different?":

> I do think my experience in Math 210 was different. I think I am able to see how the children actually deal with the math we are learning. What I mean is I am able to utilize what we are using in Math 210 in Math 296 (CMTE-L). Instead of wondering how it will help in teaching, I am able to see it (Julie, 12/5/01)

## Conclusion

We have provided a framework for one way to address problems resulting from separating the study of mathematics content from the study of children's mathematical thinking for undergraduate PSTs. We integrate mathematics content and children's mathematical thinking earlier for PSTs by building upon the interests that drew PSTs to teaching: interest in children. Our initial data are promising. These deliberately chosen and highly structured experiences in the CMTE-L support PSTs' engagement in and motivation to learn mathematics to prepare them to support their future students' mathematical thinking. These experiences resulted in PSTs' beliefs about mathematics and mathematics learning and teaching developing more than the beliefs of PSTs who made visits to classrooms of reform-oriented teachers. Qualitative data indicated that these experiences provided motivation for many of the PSTs to attend differently to the mathematics they were learning in their content courses.

We end with one implication for our work. We recognize the difficulty of adding a CMTE course to an already large list of undergraduate requirements for PSTs. However, we have evidence that mathematics instructors are willing to infuse a focus on children's mathematical thinking into their existing courses. We have created a large collection of video clips of children's mathematical thinking, and we have shared some clips with college mathematics instructors. The following comments from two professors who infused video clips into their college mathematics courses for PSTs support our finding that the study of children's mathematical thinking can motivate PSTs in their mathematics learning:

> I have used the tape to show my prospective elementary teachers the kind of creative and "different" thinking students use to reason and make calculations. The video clips became motivational clips and saved me having to make the argument for PUFM (Profound Understanding of Fundamental Mathematics, Ma, 1999) (George Poole, personal communication, November 12, 2001).

> I used the video you provided last summer with my content course—the first 6 or 7 parts. It had a wonderful effect. One student just remained sitting after class when others were leaving. She finally said, "This is my last math class [we require two]. I don't think I know enough math to teach these students we saw!" (Mary Ann Lee, personal communication, March 12, 2002).

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## Biographical Sketches

Randolph A. Philipp is professor of mathematics education in the School of Teacher Education at San Diego State University and associate director of CRMSE, the Center for Research in Mathematics and Science Education. He has written many research articles and chapters, coauthored a research monograph, and has also written for practitioners. He teaches elementary and secondary mathematics methods courses and graduate courses in mathematics education and supervises mathematics education doctoral students. He is currently directing the Integrating Mathematics and Pedagogy Project (IMAP) designed to investigate the effects of integrating mathematics content and pedagogy for preservice elementary school teachers earlier in their education programs.

Eva Thanheiser is a doctoral student in the mathematics and science education joint doctoral program between San Diego State University and the University of California San Diego. Her research interests are in prospective elementary school teachers' understanding of multidigit numbers. She serves as a researcher on the Integrating Mathematics and Pedagogy Project (IMAP), and she has helped develop and has taught the children's mathematical thinking experience course.

Lisa Clement is an assistant professor of mathematics education in the School of Teacher Education at San Diego State University and a member of CRMSE, the Center for Research in Mathematics and Science Education. Her research interests are prospective elementary school teachers' beliefs and knowledge for teaching mathematics, and she has published for researchers and practitioners. She teaches elementary and secondary mathematics methods courses and graduate courses in mathematics education and serves on mathematics education doctoral committees. Lisa serves as a researcher on the Integrating Mathematics and Pedagogy Project (IMAP), and she has helped develop and has taught the children's mathematical thinking experience course.

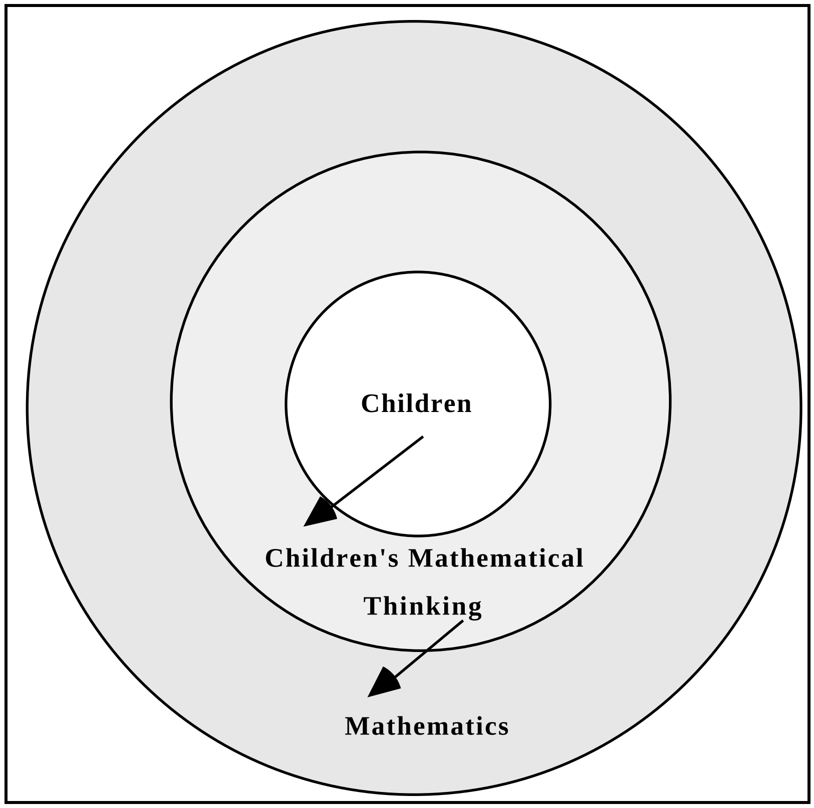


*Figure 1.* A model of growth, by way of children's mathematical thinking, from PSTs' caring about children to caring about mathematics.

| Traditional Algorithm | Expanded Algorithm (Julio's Method) | Felisha's Version (Julio's Method) |
|---|---|---|
| 1 | | |
| 57 | 57 | 73 |
| +34 | +34 | + 46 |
| 91 | 80 | 101 |
| | +11 | 9 |
| | 91 | 110 |

*Figure 2.* Traditional and expanded addition algorithms.

Teachers often ask children to share their strategies for solving problems with the class. Read the following student answers for the question 149 + 286 and respond to Questions 1 and 2 below.

**Carlos's Way**

```
 1 1
 149
+286
----
 435
```

**Elliot's Way**

```
 149
+286
----
 300
 120
  15
----
 435
```

**Sarah's Way**

Sarah says, "149 is only 1 away from 150, so 150 and 200 is 350, and 80 more is 430, and 6 more is 436. Then I have to subtract the 1, so it is 435."

**Question 1** Do you think that Carlos could make sense of and explain Sarah's strategy? Why or why not?

**Question 2** Do you think that Carlos could make sense of and explain Elliot's strategy? Why or why not?

*Figure 3.* Belief 2, Segment 3 task and questions.

Table 1
*Percentage of Students in Each Treatment Whose Scores on Belief 2, Segment 3 Increased 1, 2, or 3 Levels From Presurvey to Postsurvey.*

| | 1-level increase | 2-level increased | 3-level increase | Increased ≥ 1 level |
|---|---|---|---|---|
| CMTE-L (*n* = 50) | 14% | 22% | 20.0% | 56.0% |
| MORE-R (*n* = 22) | 18.2% | 9.1% | 4.5% | 31.8% |