

Helping Prospective and Practicing Teachers Focus on the Children's Mathematical Thinking in Student-Work Examples¹

Victoria R. Jacobs & Randolph A. Philipp
Center for Research in Mathematics and Science Education
San Diego State University.

Ms. S. has 2,694 bunnies. Ms. C has 186. How many more bunnies does Ms. S. have than Ms. C?

$$\begin{array}{r} 2000 - 0 = 2000 \\ 600 - 100 = 500 \\ 80 - 80 = 0 \\ 14 - 6 = 8 \end{array} \rightarrow 2,508$$

Ms. S. has 4010 bunnies. Ms. C has 999. How many more bunnies does Ms. S. have than Ms. C?

$$\begin{array}{r} 2000 - 0 = 2000 \\ 1800 - 900 = 900 \\ 10 - 90 = 20 \\ 100 - 9 = 91 \end{array} \rightarrow 3,011$$

Figure 1: Misha's (Grade 2) solutions for two versions of a comparison problem.

How did Misha solve these problems, and what mathematical understandings do her strategies reflect? Misha's written work provides a rich context in which prospective and practicing teachers

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can discuss issues of mathematics, teaching, and learning. For example, discussions might include conversations about the similarities and differences between Misha's two strategies, what she must have understood about place value to generate these strategies, and what type of instruction likely preceded and should follow this problem solving effort.

Teacher educators have found the use of student work, like Misha's, to be a useful discussion catalyst and a beneficial component of preservice teacher education and professional development for practicing teachers. Student work is well received by both prospective and practicing teachers because of its authentic link to classroom practice. Research has also supported the use of student work by identifying positive changes in teaching practices and student learning when professional development has included examining student work to explore children's thinking (Little, in press). Helping prospective and practicing teachers learn how to carefully analyze student work has additional benefits that are long-term and generative. Because student work is a daily part of teaching, practicing teachers can use student work as a tool to help them grow continually throughout their careers.

Despite these benefits, little guidance exists for *how* to use student-work examples with prospective and practicing teachers, and in this article, we provide a questioning framework to enhance conversations about student work. In particular, we identify questions to focus attention on children's mathematical thinking and its role in instruction, because research has shown that students benefit when teachers learn to use children's thinking to inform their instructional decision making (Wilson & Berne, 1999). In our framework, we offer three categories of questions that we have found to be valuable discussion catalysts across multiple student-work examples:

- ∑ questions to prepare teachers for understanding the child's thinking,
- ∑ questions to encourage teachers to explore the child's thinking in depth, and
- ∑ questions to help teachers identify instructional "next steps" to extend the child's thinking.

We present this framework to support the facilitation efforts of mathematics educators in a variety of instructional settings with prospective and practicing teachers. Although we acknowledge that these two audiences differ, we have found that both benefit from the use of student work to spark discussions about mathematics, teaching, and learning. Therefore, throughout this paper, when we describe the use of our framework with *teachers*, we envision audiences of both prospective and practicing teachers. We also propose that our framework could be used as a reflection tool by individual practicing teachers who want to investigate the nuances in student work from their classrooms; however, our article focuses on the use of the framework by teacher educators.

We begin by explaining each of our questioning categories in the context of Misha's written work. We then explore the questioning opportunities made possible by other types of student-work examples. Specifically, we apply our framework to a written student-work example that showcases a wrong answer and a video student-work example of a child solving a problem in a one-on-one interview.

Misha's Nontraditional Strategies for Subtraction

As shown in Figure 1, Misha solved each comparison problem by breaking the problem into four partial subtraction problems in which she treated the place-value digits, starting with the thousands, in terms of their values. She began solving the first problem by finding the difference between the quantities indicated by the thousand's place ($2000 - 0$) and the hundred's place ($600 - 100$). The action Misha performed in the ten's place indicated that she had thought ahead and had realized that she would need to use one of the tens when working with the ones. Therefore, she computed $80 - 80$ (instead of $90 - 80$) and used the extra ten with the ones, computing $14 - 6$ (instead of $4 - 6$). To arrive at her final answer, she added the four differences she had computed.

For the second problem, Misha's actions again indicated that she had begun with the thousands, but she thought ahead and realized that she would need to decompose the 4010 so that she could use some of those thousands in subsequent computations. Specifically, she decomposed the 4010 into 2000, 1800, 110, and 100. This distribution allowed her to straightforwardly complete her subtractions within each place value, computing $2000 - 0$, $1800 - 900$, $110 - 90$, and $100 - 9$. As she did when solving the first problem, Misha added the four differences to arrive at her final answer. Misha's nontraditional strategies are interesting not only because they differ from the way most teachers probably solved the problem but also because they reflect strong place-value

understanding. In the following sections, we apply our framework to identify questions to help prospective and practicing teachers focus on Misha's mathematical thinking.

Preparing to understand the child's thinking

Before sharing a student-work example, we encourage teachers to think about the mathematical problem in the example from their own perspectives. This sequence gives teachers an opportunity to first engage with the mathematics and, perhaps, to extend their own understanding of the mathematics, before trying to interpret a child's strategy that may reflect a different approach to solving the problem.

How could you solve this problem using two different strategies? By asking teachers to first solve the child's problem themselves, we encourage them to make sense of the mathematics and to begin with strategies that are most comfortable for them. For example, with Misha's work, many teachers may initially apply the traditional subtraction algorithm to subtract 186 from 2694 or 999 from 4010. By requesting that they use a second strategy, we encourage teachers to *think again*. This think-again idea highlights an implicit but important value we hold: The purpose for solving mathematical problems is not only to find correct answers but also to develop powerful ways of reasoning that might be applied to other similar, or even very different, problems.

Once teachers have generated multiple strategies, we have found their sharing of these strategies to be beneficial. Sharing provides teachers with opportunities to appreciate how other teachers solve problems in ways different from their own. Engaging with multiple adult strategies provides a starting point for understanding children's strategies because children so often think about problems in ways that are different from those adults would expect, an idea exemplified nicely by Misha's second solution. After exploring multiple strategies, teachers are better poised to think about children's unusual but creative thinking.

How might a child solve this problem? We again highlight the importance of considering strategies different from one's own by asking teachers to predict strategies that a child might use. Sometimes we ask for a variety of children's strategies whereas other times we ask teachers to identify strategies that children with a particular understanding (or lack of understanding) might use. Although we have found that practicing teachers are generally better able than prospective teachers to respond to these questions, we are convinced that prospective teachers also benefit from considering how a child might solve the problem. We do not expect teachers to predict exactly the upcoming strategy in a student-work example, but having them do so is not our goal. Instead, we want teachers to grapple with the mathematics and to consider how children might make sense of the mathematical situation.

Children and adults often view mathematical situations differently, as with this problem: "Juan has 18 stickers. How many more stickers does he need to buy to have 34 stickers?" Whereas most adults would approach this problem by subtracting, most young children would explicitly follow the (buying more stickers) action, thereby adding on from the smaller number to find the answer (Carpenter, Fennema, Franke, Levi, & Empson, 1999). By asking teachers to consider how a child may solve the problem, we encourage them to step outside of their own shoes and into a child's shoes to consider the mathematics embedded in the problem. After reviewing their own understanding of the mathematics and children's hypothetical strategies, teachers are better poised to engage with a specific child's thinking.

Exploring the child's thinking

By giving teachers opportunities to explore the details of a child's strategy, we underscore the importance of looking beyond whether the child answered the problem correctly or used a strategy adults might have used. By understanding how the child thought about the problem, teachers can focus on what the child *knows* rather than on what he or she *does not yet know*. This difference is subtle yet critical for teachers using children's thinking as a starting point from which they build their instruction. We have found that an effective approach is first to focus discussions exclusively on understanding the child's strategy and then to ask teachers to step back and think about the

relationship between the child's thinking and the underlying mathematics. Below, we provide three specific questions that have facilitated these goals for us.

How did the child solve this problem? To help teachers make sense of a child's strategy that may be different from their own, we begin discussions with a general question such as "How did the child solve this problem?" Teachers may need time to examine and discuss strategies such as Misha's, not only because the strategies are nontraditional but also because the child's way of recording strategies may be novel. In addition, even when written work is explicit, teachers may initially be unfamiliar with articulating a child's strategy in detail. They may focus on the operation ("she added") or the tool ("he solved the problem with blocks") instead of describing, step-by-step, how the child solved the problem. Follow-up questions to encourage description of the entire strategy can be helpful. For example, we might ask, "What did the child do first?" (and then next and so on) or "Can you describe each of the steps the child went through to solve this problem?"

Why might the child have done ... (insert some specific aspect of the child's strategy)? Once teachers have generally described the child's strategy, asking specific questions about that strategy can be useful to make the details of the mathematics and the child's thinking explicit. For example, we might ask, "Why might Misha have started with the thousands instead of the ones?" or "Why might Misha have written $80 - 80$ in her first strategy?" or "Why might Misha have used 110 in her second strategy?" Because teaching involves continuous observation and decision making, teachers need the ability to notice and interpret important details in classroom interactions, in particular, in children's strategies (van Es & Sherin, 2002). Engaging teachers with student work and asking questions about the details of children's thinking can help teachers grow in this expertise.

What is the mathematics embedded in this strategy? After identifying a child's strategy, teachers can reexamine the strategy to further explore the underlying mathematics. We want teachers to consider not only *how* a strategy works but also *why* it works and to what sorts of problems it would generalize. For example, we might ask teachers to explain why Misha's strategies are appropriate for these two problems and, in particular, how the numbers in the problem might have affected her thinking. To help teachers explore whether Misha's strategies are appropriate for solving other problems, we might specifically ask them to try to write a problem that could *not* be solved with Misha's strategies. The goal of these types of questions would be to generate a discussion about the generality of Misha's form of reasoning and how it could be used to solve any subtraction problem.

Exploring the mathematics of a strategy can also include identifying the mathematical understandings a child needs to use a particular strategy. For example, we might ask, "What place-value understanding did Misha need to meaningfully use her strategies?" This question might invite teachers to more broadly discuss the definition of place-value understanding and how we can know when children have that understanding. In Misha's case, place-value understanding was reflected in her ability to decompose numbers, manipulate them, and then recombine the results. Thus, student-work examples can serve as entry points that motivate teachers to engage in discussions about broader mathematical issues. After teachers have spent time exploring the child's strategy and the mathematics underlying it, we ask teachers to think about how they can use this understanding to plan subsequent instruction.

Extending the child's thinking

Our last category of questions focuses on helping teachers use what they know about the child's mathematical thinking and the related mathematics to consider how they might help the child learn. We have found that three questions, in particular, have been useful for engaging teachers in discussions about how to extend a child's thinking.

What questions could you ask to help the child reflect on the strategy?

Children need opportunities to articulate and reflect on their strategies as a way to deepen their knowledge and clarify their thinking for teachers. Many of the questions that we asked teachers to help them clarify their understanding of the strategy would be effective questions for teachers to consider asking children as well. For example, a teacher could ask Misha why she computed $80 - 80$ in her first strategy or why she used 110 in her second strategy. We also encourage teachers to probe the reasoning behind the child's strategy. For example, they might ask whether Misha could identify the 186 or 2694 in her first strategy, or why she decided to decompose 4010 into 2000, 1800, 110, and 100 (instead of, for example, 3000, 900, 100, and 10) in her second strategy?

What questions might encourage the child to consider a more efficient strategy? Children need opportunities to improve their efficiency when problem solving, and teacher questioning can help. For example, teachers might consider encouraging a child to simplify elaborate drawings or use groups of tens (instead of ones) to build or count quantities. Increased efficiency can take many forms because teachers' thoughts about efficiency will depend upon the context, including the child, the problem, and the embedded mathematics. Given that Misha's solutions were both correct and displayed strong place-value understanding, teachers might consider encouraging Misha to solve the second problem without paper and pencil by taking advantage of number relationships, in this case, the idea that 999 is only 1 away from 1000. After teachers have identified several questions that could help the child reflect on an existing strategy or generate a more efficient strategy, we then broaden the discussion to consider different but related problems.

On the basis of the child's existing understandings, what problem might you pose next and how might the child solve it? After exploring the child's work, we would encourage teachers to consider logical next steps. We might ask them to identify specific follow-up problems, to generate hypotheses about how the child might solve those problems, and to consider what they and the child might learn from the experience. Teachers may consider using the structure of the original problem and varying the numbers, or they may consider writing problems about different but related mathematical concepts.

Generating related problems is often difficult for teachers, especially prospective teachers, but can be facilitated by the use of student work like Misha's. Because we have Misha's work for more than one problem, we can ask teachers to compare and contrast what they learned from analyzing her work on these two problems. From Misha's first solution, we learned how she thought about regrouping when it was needed only once. From her second solution, we learned how she could flexibly regroup to accommodate multiple regroupings. Examining a series of related problems solved by the same child can help teachers better understand how to design follow-up problems that provide additional information about a child's thinking. In Misha's case, teachers might consider designing a problem like $1000 - 998$ to ascertain whether she could use her number sense in other ways. They might wonder whether Misha could immediately recognize the answer of 2, or whether she would decompose the subtraction problem into subproblems as she did in solving the comparison problems. We have found that having teachers identify a *specific* next problem they might pose is critical because it gives teachers opportunities to translate their understandings into instructional tasks. Discussing the general reasoning by which they generated this problem is also beneficial.

Summary of Questioning Framework

Table 1 summarizes the questioning framework we applied to Misha's student-work example. This framework is designed to identify questions that can serve as starting points for discussions among prospective or practicing teachers. We are not suggesting that all three categories of questions need to be used with every student-work example. Instead, we regularly select and modify questions on the basis of the goals we hold for a particular instructional session, our knowledge about the audience, and the mathematics embedded in the specific example of student work under consideration. In the following sections, we provide two additional examples of how our framework can be applied to student work. We selected different types of student

work, including one example with an incorrect solution and one captured on video, to highlight different questioning opportunities.

Table 1. Discussion Questions for Student-Work Examples

<p>1) Questions to prepare teachers for understanding the child's thinking How could you solve this problem using two different strategies? How might a child solve this problem?</p> <p>2) Questions to encourage teachers to explore the child's thinking in depth How did the child solve this problem? Why might the child have done ... (insert some specific aspect of the child's strategy)? What is the mathematics embedded in this strategy?</p> <p>3) Questions to help teachers identify instructional "next steps" to extend the child's thinking What questions could you ask to help the child reflect on the strategy? What questions might encourage the child to consider a more efficient strategy? On the basis of the child's existing understandings, what problem might you pose next and how might the child solve it?</p>

Heidi's Incorrect Solution

The image shows a handwritten student solution for the problem $4002 - 199 + 199 = \underline{\hspace{2cm}}$. The student has written the following work:

$$\begin{array}{r} 399 \\ \cancel{4002} \\ - \quad 199 \\ \hline 3803 \\ + \quad 199 \\ \hline 4092 \end{array}$$

The student's work shows a subtraction of 199 from 4002, resulting in 3803, followed by an addition of 199 to 3803, resulting in 4092. The final answer is 4092.

Figure 2: Heidi's (Grade 5) solution to an algebraic problem.

Heidi correctly used the standard subtraction algorithm to subtract 199 from 4002; however, she made a computational error when using the standard addition algorithm to add 199 to 3803. We conjecture that she added $9 + 1$ (in the ten's column), realized that the answer was 10, carried 1 to the hundred's column, but wrote 9 instead of 0 in the ten's column of her answer. Heidi's incorrect answer masked the fact that the correct answer is the same as the initial number. Her strategy is interesting, therefore, not only because of her computational error but also because of her failure to recognize that subtracting then adding 199 from a number leaves that original number unchanged. Attending to these types of number relationships is foundational for algebraic understanding.

In the following sections, we apply our questioning framework to Heidi's work. Each student-work example provides different questioning opportunities, and Heidi's work is different from Misha's in part because her strategy resulted in an incorrect answer. When faced with a child's error, a teacher must decide what type of support is warranted and whether or how to address that error. When considering next steps with Heidi, teachers could choose to focus on correcting her error, encouraging the algebraic thinking that the problem was designed to promote, or both. To make this decision, teachers would need to consider whether Heidi simply made an isolated, computational error or whether she was more generally confused about whole number addition.

Preparing to understand the child's thinking

Common approaches to this problem include computing from left to right, as Heidi did, or recognizing that computation is unnecessary because $-199 + 199 = 0$. Teachers need opportunities to consider the strategies they would use to solve this problem and the strategies children are likely to use. For example, teachers might wonder whether children would more easily see the relationship between adding and subtracting the same number if the positive number were presented first ($+199 - 199$ instead of $-199 + 199$, as in Heidi's problem). After investigating the mathematics and children's hypothetical strategies, teachers should be better poised to consider Heidi's error in the context of this problem and the type of algebraic thinking it was designed to promote.

Exploring the child's thinking

Heidi used standard algorithms for both addition and subtraction, and her approach is valid. However, from her written work, we cannot determine whether she understands why those algorithms work and whether her error is careless or conceptual. Written student work is often an incomplete reflection of a child's understanding, underscoring the need for teacher questioning to elicit children's thinking. Teachers can benefit from discussions in which they first consider the limitations of what they can learn from children's written work and then brainstorm possible questions to gather further information.

When considering the mathematics embedded in this problem, we recognize that Heidi missed an opportunity to look at the number relationships before beginning her computations. We might ask teachers to explore this issue by asking, "What understanding would Heidi have needed to recognize so that she could solve this problem without performing any addition or subtraction?" or "Why do these numbers encourage algebraic thinking?"

Extending the child's thinking

When determining how to extend a child's thinking on the basis of an incorrect strategy, teachers must consider whether they will focus on correcting the error or helping the child more generally reason about the problem. With Heidi's work, we might help teachers consider the possibility of ignoring, at least temporarily, the computational error to instead promote algebraic thinking. Specifically, we might ask teachers, "What problems could orient Heidi toward looking at the whole problem before immediately computing from left to right?" The goal would be to have teachers generate other number sentences that could entice Heidi (and other children) to simplify calculations by taking advantage of the relationships between numbers (e.g., $75 + 99 + 25 = \underline{\quad}$; $6000 + 105 - 105 = \underline{\quad}$; $247 + 326 = 246 + \underline{\quad}$).

We might also help teachers consider how to support Heidi in correcting her error by asking teachers, “How could you help Heidi recognize her error without directly pointing out her mistake?” Our goal in posing this question would be to generate suggestions such as asking Heidi to compare her work with that of another student or to solve the problem using a different strategy so that she could compare her own answers. Teachers might also consider the benefits of having Heidi read the original number sentence aloud with her (incorrect) answer. Because reading the entire problem can help children recognize number relationships that they otherwise might miss, Heidi might see that $-199 + 199 = 0$, realize that the answer should have been 4002, and, therefore look for errors in her original computation.

Summary of Questioning With Children’s Written-Work Examples

We have discussed two written student-work examples to illustrate the generalizability of our questioning framework to a variety of written work. However, we also recognize that the *specific* questions that can be asked are always influenced by the particular features of the student work under consideration. Key features include the problem, the strategy, and whether the strategy is presented alone or with other related work from the same child or different children. Because each student-work example involves different mathematics, different reflections of children’s thinking, and different instructional opportunities, teacher educators must consider these differences when selecting which examples to use. Student-work examples can be borrowed from many sources. For example, we have found figures illustrating children’s strategies in most issues of *Teaching Children Mathematics*, especially in the “Problem Solvers” section. When working with practicing teachers, we also encourage teachers to bring student work samples from their classrooms. We have found that almost all student work provides something of interest, and using participants’ student work can increase the authenticity of discussions.

Our questioning framework can also be a useful tool for structuring discussions of video examples of student work, and we address these video-based discussions next. For video resources, we have found several useful commercially available materials, some of which are published by the National Council of Teachers of Mathematics (for example, see their new *ON-Math* online journal).

Javier’s Solution in a Video Example

Javier is a fifth-grade Spanish-speaking student who had been in the United States only one year at the time of this one-on-one interview. He mentally solved 6×12 by thinking of the problem as $5 \times 12 + 12$ and thus added $60 + 12$ to find the answer of 72. He also explained that 5×12 was 60 because he knew that it was half of 10×12 , or 120. Javier’s videoclip is interesting not only because it demonstrates how a child with well-developed number sense can approach problems flexibly but also because it highlights the importance of teachers’ questioning in understanding how children solve problems. This questioning is particularly important when children use mental strategies because, for example, we would not know how Javier solved 5×12 if the interviewer had not thought to pose that question.

In the following sections, we apply our questioning framework to Javier’s example. In comparison to the two written-work examples, the use of video provides additional questioning opportunities. With video, we can investigate a greater variety of children’s strategies. Because we can learn about children’s thinking from the verbalization of their strategies, as well as from any written work, we gain access to non-written strategies including mental or manipulative-based strategies that children do not represent on paper. Similarly, children’s explanations give us further information that can clarify written strategies that are incomplete or unclear.

Video provides additional information by capturing the conversation between a teacher and child, thereby making their explanations and questions available for investigation. Specifically, teachers may explore whether a child clearly described his or her strategy and how they could support the child in improving that articulation expertise. Teachers may also work to improve their own questioning by analyzing the follow-up questions posed by the teacher in the video.

Interviewer: You know how eggs come in a carton?
 Javier: Yeah.
 Interviewer: And there are usually—
 Javier: Twelve.
 Interviewer: —a dozen. There are 12, in there?
 Javier: Yeah.
 Interviewer: How many eggs would you have if you had six dozen—so if you had six of those cartons?
 Javier: Okay. Let me see. Six times 12. That would be ... 72.
 Interviewer: And how did you figure that out?
 Javier: Because when I say, "Five times 12," that equals 60. If I add 12 more, that will be 72 [writing 72].

Javier has written

$$\begin{array}{r}
 12 \\
 \times 5 \\
 \hline
 60 \\
 + 12 \\
 \hline
 72
 \end{array}$$

Interviewer: Wow! And how did you know that 5 times 12 is 60?
 Javier: Because 12 times 5 equals 60. And if I take the half of 120, that would be 60.
 Interviewer: Wow! Great explanation.

Figure 3: Problem-solving interview with Javier (Grade 5). This video clip is available online at http://www.sci.sdsu.edu/CRMSE/IMAP/vid_mult.html.

Video also makes possible the sharing of partial interactions because video can be stopped at select points to allow teachers to share reactions, make predictions, or discuss what they might do next before seeing what actually transpired. For example, we might stop the video after Javier explained his strategy of $5 \times 12 + 12$ but before the interviewer asked how he knew $5 \times 12 = 60$. Holding a discussion with teachers about Javier's thinking before (and then after) they watch how Javier explains how he solved 5×12 would underscore the necessity for questioning children about the details of their strategies.

Preparing to understand the child's thinking

Many teachers will either know 6×12 as a memorized fact or compute it quickly by applying the standard multiplication algorithm. Considering the underlying mathematical concepts embedded in alternative strategies for multiplication could help prepare teachers for Javier's thinking. For example, when asked to think again to generate additional strategies, some teachers might use repeated addition for 6×12 , whereas others may take advantage of the implicit use of the distributive property in the standard multiplication algorithm thereby computing $6 \times 2 + 6 \times 10$.

We have found that Javier's solution is even more interesting and thought provoking to teachers after they have first considered various strategies on their own.

Exploring the child's thinking

Because Javier's work is on video, we might ask teachers to describe not only his strategy but also the clarity of his explanation and the role of the teacher in helping us understand his thinking. We might also showcase the mathematical properties embedded in Javier's strategy. One symbolic representation of Javier's thinking is presented below:

$$\begin{aligned} & 6 \cdot 12 \\ &= (5 \cdot 12) + (1 \cdot 12) \text{ (Distributive property of multiplication over addition)} \\ &= \left[\frac{1}{2} \cdot (10 \cdot 12)\right] + 12 \\ &= \left[\frac{1}{2} \cdot (10 \cdot 12)\right] + 12 \text{ (Associative property of multiplication)} \\ &= \left[\frac{1}{2} \cdot (120)\right] + 12 \\ &= 60 + 12 \\ &= 72 \end{aligned}$$

We are not suggesting that Javier would have represented his strategy as we have done. However, by representing it in this formal manner, we provide another tool for investigating the mathematics reflected in Javier's reasoning. We have found that by starting with children's thinking, like Javier's, teachers are often more motivated to discuss mathematical properties than when we begin the discussion with a formal presentation of the mathematics.

Extending the child's thinking

When helping teachers identify instructional next steps for Javier, we would encourage teachers to consider questions designed to have Javier reflect on his strategy and its generalizability. For example, in the video, Javier shared that he knew that 5×12 was the same as $1/2$ of 10×12 . Teachers might wonder whether Javier understood that any number multiplied by 5 could be calculated quickly by calculating half of the product of 10 and that number. We might ask them to design a specific problem that would assess whether Javier could apply his strategy with different numbers. Teachers might also wonder whether Javier's use of these number relationships was flexible enough to help him solve such related division problems as "You have 120 cookies, and 5 cookies fit into each bag. How many bags do you need?"

Final Comments

We have found discussions around student work to be particularly productive for helping teachers understand both how children think about mathematics and how teachers can use children's thinking to help them decide what question or problem to pose next. Student-work examples ground the conversations in the specific and important details of children's thinking, and well-selected discussion questions can help teachers attend to these details while also developing more general understandings about mathematics, teaching, and learning.

We also recognize that any time we look at only one slice of a child's thinking, we lose the context needed for a deep understanding of that child. For example, when we look at a child's written work, we may not be able to determine in what order he or she wrote what appears, nor may we know whether what is written exactly reflects what the child thought when solving the problem. These concerns are mitigated by using video examples of student work, yet additional questions remain. Even with video we cannot know contextual information about the classroom or the child that could be pertinent to the child's mathematical understanding. Although recognizing these limitations is important, they do not detract from the value of using student work with prospective and practicing teachers because the primary goal is not to understand the particular child but is

instead to provide a common context around which teachers may engage in rich mathematical and pedagogical discussions. Our questioning framework is designed to support these rich discussions, helping prospective and practicing teachers grow in their knowledge, beliefs, and instructional practice to the ultimate benefit of students in their classrooms.

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